JMAP's IMPROVED MINI GUIDE TO THE INTEGRATED ALGEBRA REGENTS EXAMINATION

Weighted Distributions by Topic of New York Integrated Algebra Regents Exam Questions from Fall 2007 to June 2014

MINI GUIDE

Dear Sir

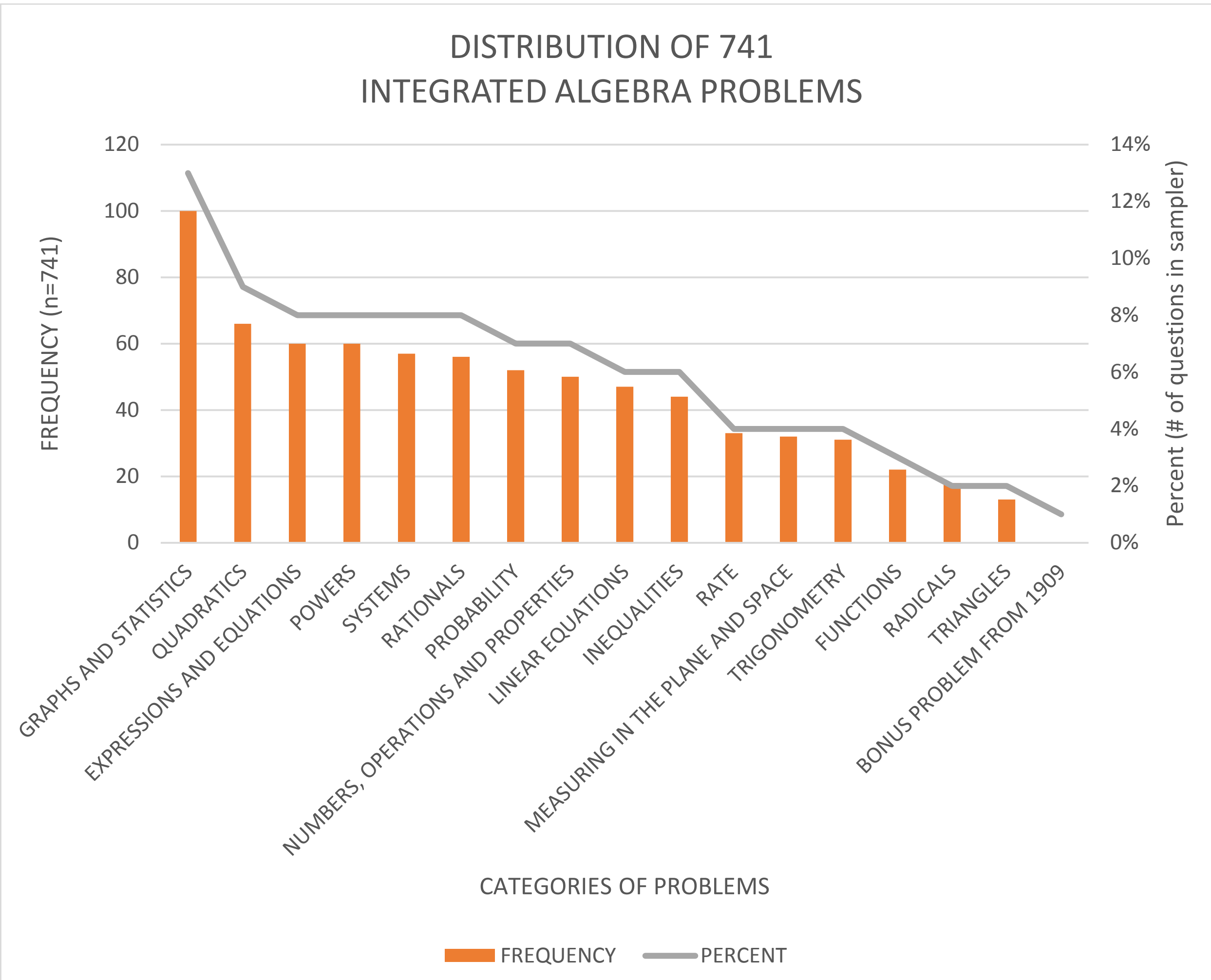
I have to acknolege the reciept of your favor of May 14. in which you mention that you have finished the 6. first books of Euclid, plane trigonometry, surveying & algebra and ask whether I think a further pursuit of that branch of science would be useful to you. there are some propositions in the latter books of Euclid, & some of Archimedes, which are useful, & I have no doubt you have been made acquainted with them. trigonometry, so far as this, is most valuable to every man, there is scarcely a day in which he will not resort to it for some of the purposes of common life. the science of calculation also is indispensible as far as the extraction of the square & cube roots; Algebra as far as the quadratic equation & the use of logarithms are often of value in ordinary cases: but all beyond these is but a luxury; a delicious luxury indeed; but not to be indulged in by one who is to have a profession to follow for his subsistence. in this light I view the conic sections, curves of the higher orders, perhaps even spherical trigonometry, Algebraical operations beyond the 2d dimension, and fluxions.

Letter from Thomas Jefferson to William G. Munford, Monticello, June 18, 1799

TABLE OF CONTENTS

PREFACE

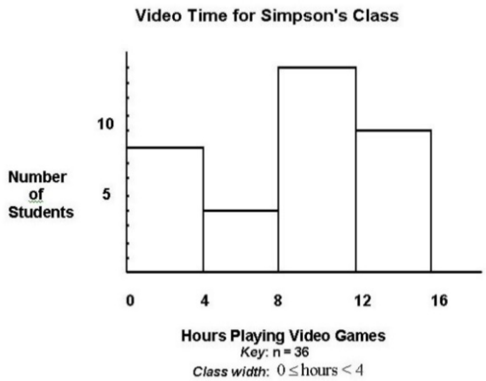
This mini guide to the Integrated Algebra Regents examination is based on analysis of the 741 Integrated Algebra problems that appeared in the first 19 Integrated Algebra Regents examinations. This analysis revealed the following distribution of problem topics.



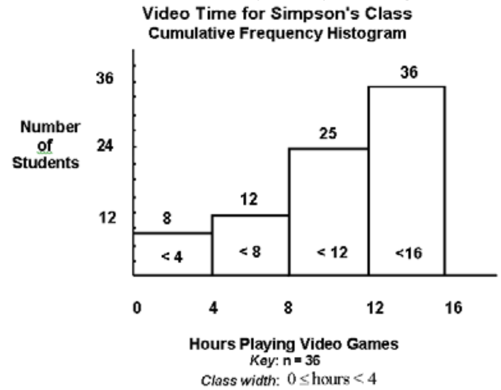
On the following pages, you will find 100 carefully selected samples of questions covering different topics. The number of questions on each topic correspond to the percent of questions on the first nineteen examinations. For example, approximately 13% of the 741 questions on the first 19 Integrated Algebra examinations related to graphs and statistics, so there are 13 carefully selected problems illustrating the ways that graphs and statistics problems are assessed on the examination.

Graphs and Statistics – 13% of the questions

Frequency Histogram



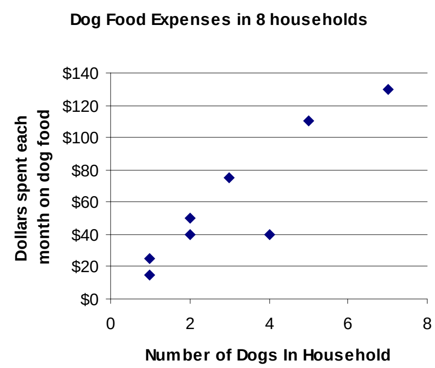
Cumulative Frequency Histogram



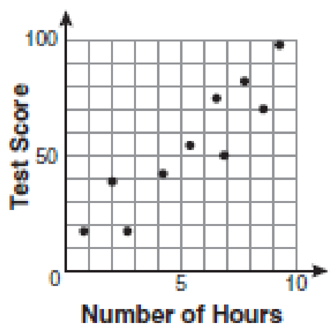
Table

Household	Number of dogs	Dollars spent each month on dog food
Perez	3	75
Jones	2	40
Balcovich	5	110
Parson	1	15
Montego	1	25
Schwartz	7	130
Barton	2	50
Walker	4	40

Scatter Plot

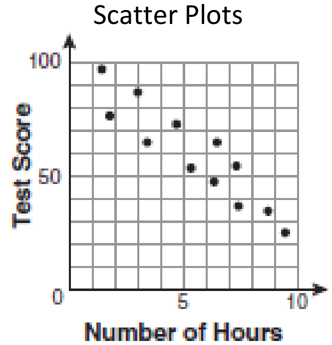


Scatter Plots



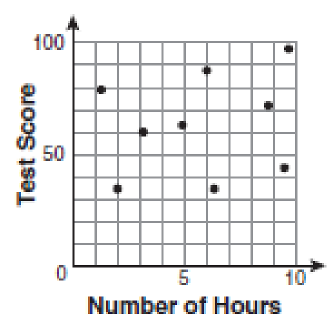
Positive Correlation

In general, both sets of data increase together.



Negative Correlation

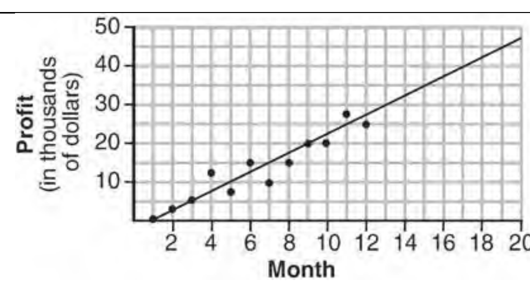
In general, one set of data decreases as the other set increases.



No Correlation

There is no general trend.

Line of Best Fit



Error

Relative Error = $\left|\frac{\text{measured-actual}}{\text{actual}}\right|$ Percent Error = $\left|\frac{\text{measured-actual}}{\text{actual}}\right|(100)\%$

Analysis of Data

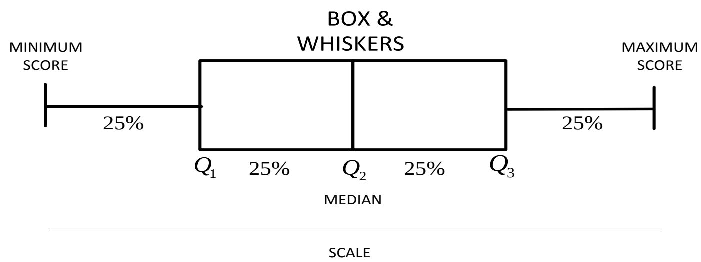
Quantitative Data: Involves numerical data.
Qualitative Data: Involves non-numerical data such as color, type of animal, etc.
Univariate Data is a set of data involving **one** variable.
Bivariate Data is a set of data involving **two** variables
A **biased sample** is a sample has some property that influences the sample.

Examples:

- If you want to know what the general population thinks about soccer, you would not go to a soccer game and take a poll. The soccer fans are biased in favor of soccer.
- A poll to determine whether a stop sign is needed at a school crossing might be biased if the sample polled consisted only of parents who dropped their children off at school.

Correlation: Event A is related to, but does not necessarily cause event B.
Causation: Event A causes event B.

Box and Whisker Plots



The five vertical lines on a box and whisker plot show:

1. minimum (lowest score),
2. lower quartile (Q_1),
3. median (Q_2),
4. upper quartile (Q_3),
5. maximum (highest score).

Measures of Central Tendency

How do I find **quartiles**? (A quartile divides a data set into four equal parts.)

1. Find the median of the entire set. This divides the set into two equal parts.
2. Find the medians of each half of the set. This divides the set into four equal parts.

How do I find the **mean**?	How do I find the **median**?	How do I find the **mode**?
1. Find Sum of Items in Data Set 2. Count Items in Data Set 3. Divide Sum by Count Formula $\overline{X}=\frac{x_1+x_2+x_3+\ldots+x_n}{n}$	1. Put the data set in ascending order. 2. Count the items in the data set and determine if the count is even or odd. a. If odd, median is middle number. b. If even, median is halfway between the middle two.	• Put the data set in ascending order. • Find the number that occurs most frequently. *Note: Data sets can have no mode, one mode, or many modes.*

ANSWERS - See page 75

GRAPHS AND STATISTICS

13% - GRAPHS AND STATISTICS

1 Which phrase best describes the relationship between the number of miles driven and the amount of gasoline used?
1) causal, but not correlated
2) correlated, but not causal
3) both correlated and causal
4) neither correlated nor causal

2 The table below shows a cumulative frequency distribution of runners' ages.

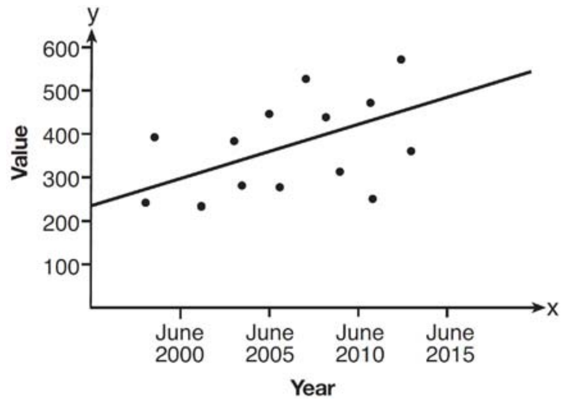
Cumulative Frequency Distribution
of Runners' Ages

Age Group	Total
20–29	8
20–39	18
20–49	25
20–59	31
20–69	35

According to the table, how many runners are in their forties?
1) 25
2) 10
3) 7
4) 6

3 Which data set describes a situation that could be classified as qualitative?
1) the elevations of the five highest mountains in the world
2) the ages of presidents at the time of their inauguration
3) the opinions of students regarding school lunches
4) the shoe sizes of players on the basketball team

4 Based on the line of best fit drawn below, which value could be expected for the data in June 2015?



1) 230
2) 310
3) 480
4) 540

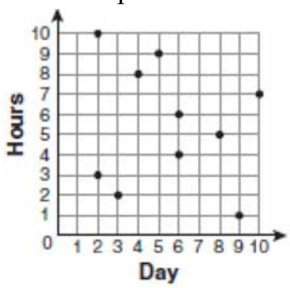
5 A school newspaper will survey students about the quality of the school's lunch program. Which method will create the *least* biased results?

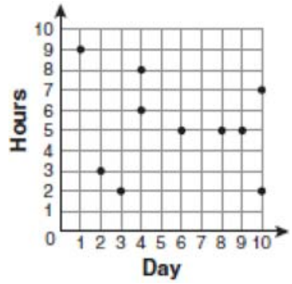
1) Twenty-five vegetarians are randomly surveyed.
2) Twenty-five students are randomly chosen from each grade level.
3) Students who dislike the school's lunch program are chosen to complete the survey.
4) A booth is set up in the cafeteria for the students to voluntarily complete the survey.

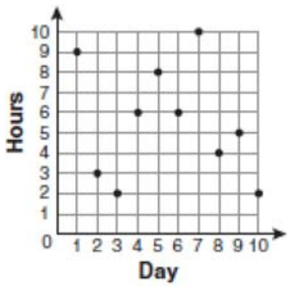
6 For 10 days, Romero kept a record of the number of hours he spent listening to music. The information is shown in the table below.

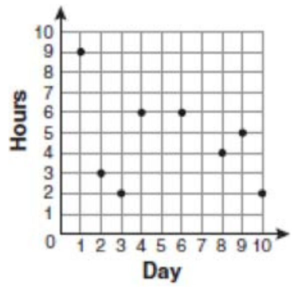
Day	1	2	3	4	5	6	7	8	9	10
Hours	9	3	2	6	8	6	10	4	5	2

Which scatter plot shows Romero's data graphically?

1) 

3) 

2) 

4) 

7 Which table does *not* show bivariate data?

1)

Height (inches)	Weight (pounds)
39	50
48	70
60	90

3)

Quiz Average	Frequency
70	12
80	15
90	6

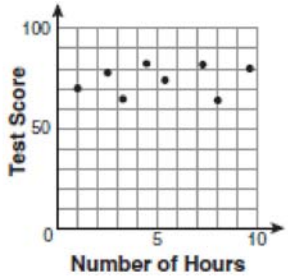
2)

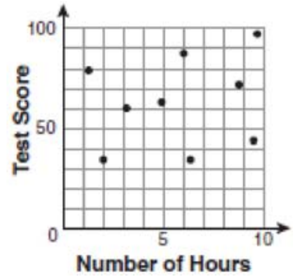
Gallons	Miles Driven
15	300
20	400
25	500

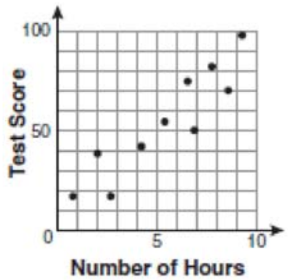
4)

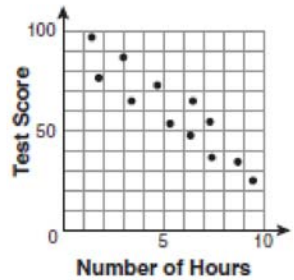
Speed (mph)	Distance (miles)
40	80
50	120
55	150

8 There is a negative correlation between the number of hours a student watches television and his or her social studies test score. Which scatter plot below displays this correlation?

1) 

3) 

2) 

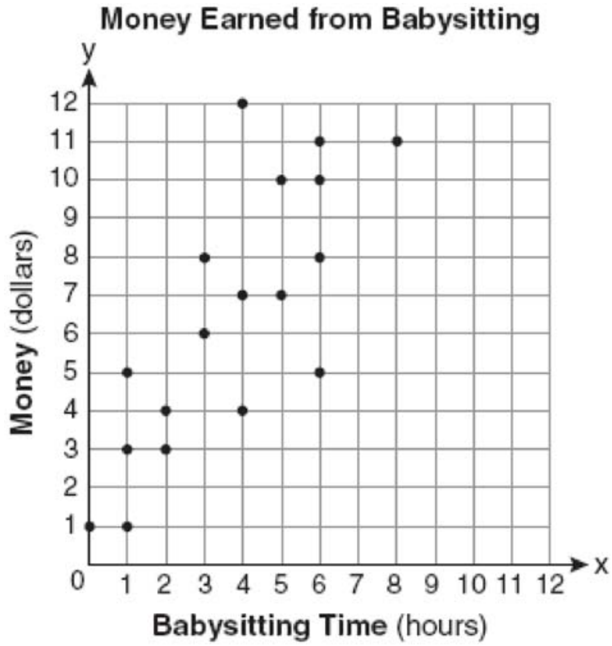
4) 

9 The values of 11 houses on Washington St. are shown in the table below.

Value per House	Number of Houses
$100,000	1
$175,000	5
$200,000	4
$700,000	1

Find the mean value of these houses in dollars. Find the median value of these houses in dollars. State which measure of central tendency, the mean or the median, *best* represents the values of these 11 houses. Justify your answer.

10 Which equation most closely represents the line of best fit for the scatter plot below?



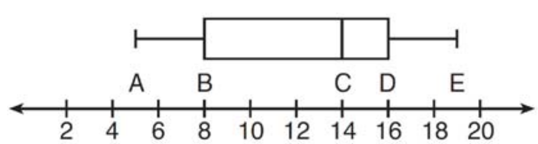
1) $y = x$

2) $y = \frac{2}{3}x + 1$

3) $y = \frac{3}{2}x + 4$

4) $y = \frac{3}{2}x + 1$

11 The box-and-whisker plot shown below represents the number of magazine subscriptions sold by members of a club.



Which statistical measures do points *B*, *D*, and *E* represent, respectively?

1) minimum, median, maximum

2) first quartile, median, third quartile

3) first quartile, third quartile, maximum

4) median, third quartile, maximum

12 Mr. Taylor raised all his students' scores on a recent test by five points. How were the mean and the range of the scores affected?

1) The mean increased by five and the range increased by five.
2) The mean increased by five and the range remained the same.
3) The mean remained the same and the range increased by five.
4) The mean remained the same and the range remained the same.

13 To calculate the volume of a small wooden cube, Ezra measured an edge of the cube as 2 cm. The actual length of the edge of Ezra's cube is 2.1 cm. What is the relative error in his volume calculation to the *nearest hundredth*?

1) 0.13
2) 0.14
3) 0.15
4) 0.16

Quadratics - 9% of the questions

Standard Form of a Quadratic Equation

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Axis of Symmetry

$$x = \frac{-b}{2a}$$

Discriminant

$$b^2 - 4ac$$

To Find the Vertex or Turning Point

Minima if a>0

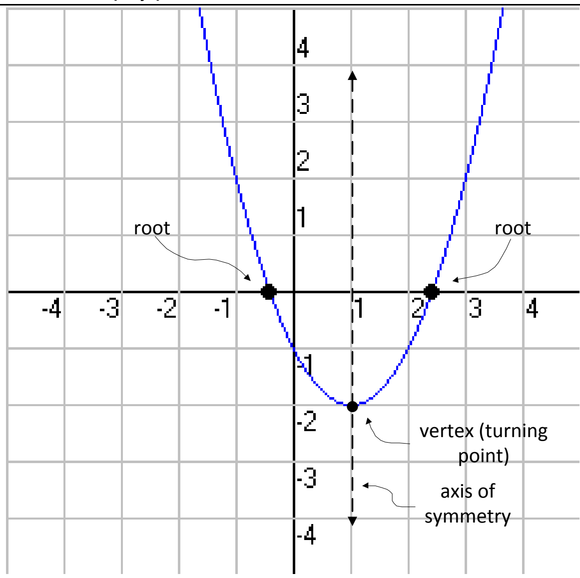
Maxima if a>0

1. Find the axis of symmetry.
2. Find the value of y for the axis of symmetry.
3. Express the vertex as an ordered pair in the form (*x*,*y*)

To Find the Roots, Solutions, or Zeros of a Quadratic Equation

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Know the Difference between Factors and Roots

- *A **factor** is: 1) a **divisor** of another number' or 2) an algebraic expression that is a **divisor** of another algebraic expression.*
- **A root** is a **solution** of the equation in the form $f(x) = 0$.

The roots of a quadratic equation can also be understood as the **x-axis intercepts** of the graph of the equation. This is because the coordinates of the x-axis intercepts, by definition, have y-values equal to zero.

A ROOT OR SOLUTION IS NOT A COORDINATE PAIR

(Roots and solutions are only values of x)

To Factor the Difference of Perfect Squares

$$\left(a^2 - b^2\right) = \left(a+b\right)\left(a-b\right)$$

Examples

$$x^2 - 4 = \left(x+2\right)\left(x-2\right)$$

$$x^4 - 9 = \left(x^2+3\right)\left(x^2-3\right)$$

Using the Multiplication Property of Zero

(to go from factors to roots)

$$x^2 + 2x - 24 = 0$$

$$\textit{factors} \quad \left(x+6\right)\left(x-4\right) = 0$$

$$\textit{Therefore}$$

$$x + 6 = 0 \quad \textit{and} \quad x - 4 = 0$$

$$x = -6 \ \textit{and} \qquad x = 4 \quad \textit{roots}$$

Factoring Trinomials

There are two general approaches to solving factoring problems.

- The first approach is to use a factoring algorithm, like the one below.
- The second approach, which works only with multiple choice problems, is to work backwards from the answer choices to find which choice yields the original problem.

1. Start with a factorable trinomial:

$$8x^2+22x+15$$

$b^2-4ac=22^2-4(8)(15)$

$b^2-4ac=4$ This can be factored.

$\sqrt{b^2-4ac}=2$

2. Identify the values of *a*, *b*, and *c*

a=8, b=22, c=15

3. Multiply *a* times *c*.

ac=120 $|ac|=120$

4. Find the factors of $|ac|$

1	120	6	20
2	60	$\boxed{8}$	$\boxed{15}$
3	40	10	12
4	30	12	10
5	24		

5. Box the set of factors in step 4 whose sum or difference equals $|b|$

6. Assign a positive or negative value to each factor. Write the signed factors below.

+ 8 + 15 = 22
b

7. Replace the middle term of the trinomial with two new terms.

$$8x^2+8x+15x+15$$

8. Group the new polynomial into two binomials using parentheses..

$$\left(8x^2+8x\right)+\left(15x+15\right)$$

9. Factor each binomial. (Note that the factors in parenthesis will always be identical.)

$$\left(8x^2+8x\right)+\left(15x+15\right)$$

$$8x(x+1)+15(x+1)$$

10. Extract the common factor and add the remaining terms as a second factor.

$$(x+1)$$

$$(x+1)(8x+15)$$

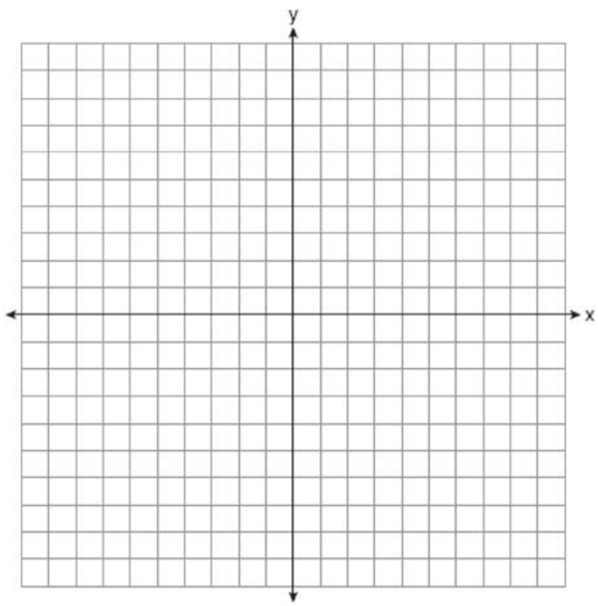
11. Check. Use the distributive property of multiplication to make sure that your binomials in Step 10 return you to the trinomial that you started with in Step 1. If so, put a check mark here.

ANSWERS - See page 76

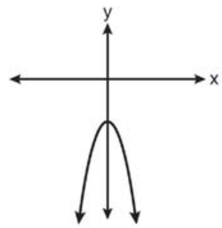
QUADRATICS

9% - QUADRATICS

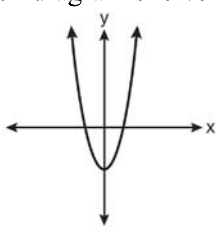
1 Graph the equation $y = x^2 - 2x - 3$ on the accompanying set of axes. Using the graph, determine the roots of the equation $x^2 - 2x - 3 = 0$.

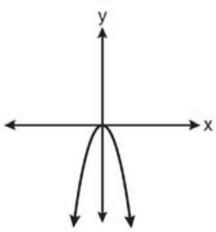


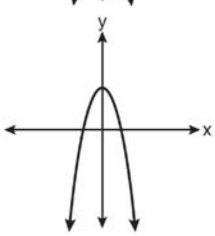
2 The diagram below shows the graph of $y = -x^2 - c$.

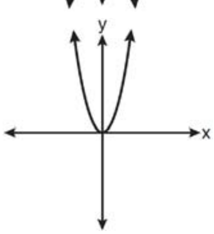


Which diagram shows the graph of $y = x^2 - c$?

1) 

2) 

3) 

4) 

3 What is the vertex of the parabola represented by the equation $y = -2x^2 + 24x - 100$?

1) $x = -6$	3) $(6, -28)$
2) $x = 6$	4) $(-6, -316)$

4 Which is the equation of a parabola that has the same vertex as the parabola represented by $y = x^2$, but is wider?

1) $y = x^2 + 2$	3) $y = 2x^2$
2) $y = x^2 - 2$	4) $y = \frac{1}{2}x^2$

5 What are the roots of the equation $x^2 - 7x + 6 = 0$?

1) 1 and 7	3) −1 and −6
2) −1 and 7	4) 1 and 6

6 The solution to the equation $x^2 - 6x = 0$ is

1) 0, only	3) 0 and 6
2) 6, only	4) $\pm\sqrt{6}$

7 A rectangle has an area of 24 square units. The width is 5 units less than the length. What is the length, in units, of the rectangle?

1) 6	3) 3
2) 8	4) 19

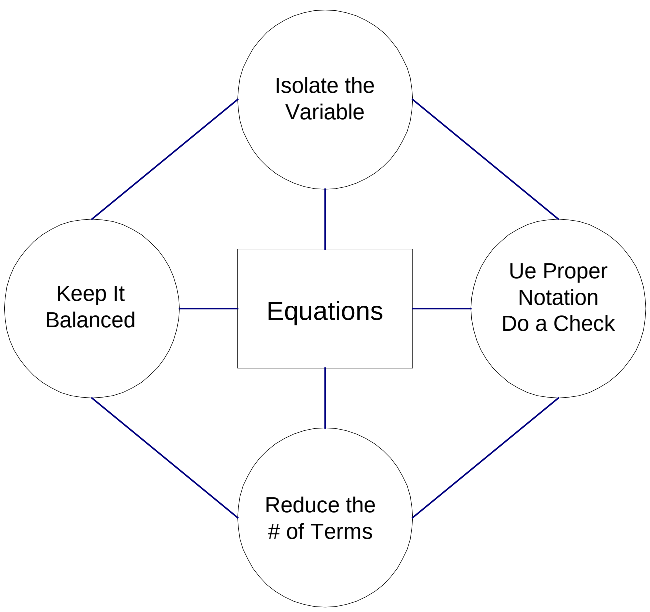
8 Factor completely: $5x^3 - 20x^2 - 60x$

9 Factor completely: $4x^3 - 36x$

Expressions and Equations – 8%

Terms	Expressions	Equations	Inequalities
A **term** is a number {1,2,3,…}, a variable {x,y,z,a,b,c…}, or the product of a number and a variable {2x, 3y, ½ a, etc.). Terms are separated by + or – signs in an expression, and the + or – signs are part of each term.	*Note*: Expressions never have equal signs in them. **Expression:** An expression is a mathematical phrase made up of terms, variables, and/or the product of terms and variables.	*Note*: Equations always have equal signs in them. **Equation:** An equation consists of two expressions with an equal sign between them. =	*Note*: Inequalities always have inequality signs in them. **Inequality:** An inequality consists of two expressions with an inequality sign between them. $\le$ $\ge$ $<$ $>$ $\ne$

Solving Equations



Solving Equations with Fractions

How do I get rid of fractions in an equation?

- One method that works every time is to multiply both sides of the equation by the denominator of the fraction that you want to eliminate.
  Example: To get rid of the fraction in the equation below, multiply both sides of the equation by 3, which is the denominator of the fraction.

$$\frac{2}{3}x+4=8$$

$$3\left(\frac{2}{3}x+4\right)=3(8)$$

$$\frac{\cancel{3}}{1}\left(\frac{2x}{\cancel{3}}\right)+3(4)=3(8)$$

$$2x+12=24$$

$$2x=12$$

$$x=6$$

Transforming Formulas

Isolate L in the given formula:

Given: $\quad \mathrm{P} = 2\mathrm{L} + 2\mathrm{W}$

Subtract$_{(2W)}$ $\quad -2\mathrm{W} \qquad -2\mathrm{W}$

$$\mathrm{P} - 2\mathrm{W} = 2\mathrm{L}$$

Divide$_{(2)}$ $\quad \dfrac{\mathrm{P} - 2\mathrm{W}}{2} = \dfrac{2\mathrm{L}}{2}$

$$\boxed{\frac{\mathrm{P} - 2\mathrm{W}}{2} = \mathrm{L}}$$

Solution Strategy: Isolate L variable using the same procedures for isolating a variable in any equation.

Modeling Equations

Addition +	**Subtraction -**
and gain increase of increased by more – more than plus raise sum total	decreased by difference fewer less – less than loss minus remainder take away
Multiplication ×	**Division ÷**
double fraction of multiplied by multiply percent of product of times triple twice	divide divided by divide equally over per quotient
Equal Sign =	**Inequality Signs < ≤ ≠ > ≥**
equals is exceeds by	is greater than/equal to is less than/equal to is not equal to
Parentheses ( ) quantity	

Examples:

Verbal Expression(s)	Operation/Property	Mathematical Expression
"five times x"	multiplication	$5x$
"x plus five" "five more than x" "the sum of x and 5"	addition	$x+5$
"y minus six" "six less than y" "the difference of y and 6"	subtraction	$y-6$
"x divided by seven" "one-seventh of x"	division	$\frac{x}{7}$
"two times the quantity of x minus five" "two times the difference of x and five"	distributive property	$2(x-5)$
"8 more than the quantity a plus b"	parentheses	$(a+b)+8$
"one-seventh of the quantity two times x plus three" "the quantity two times x plus three, divided by seven"	division	$\frac{2x+3}{7}$

ANSWERS - See page 78

EXPRESSIONS AND EQUATIONS

8% - EXPRESSIONS AND EQUATIONS

1 The value of the expression $-|a-b|$ when $a=7$ and $b=-3$ is

1) -10
2) 10
3) -4
4) 4

2 Chad complained to his friend that he had five equations to solve for homework. Are all of the homework problems equations? Justify your answer.

Math Homework

1. $3x^2 \cdot 2x^4$
2. $5-2x=3x$
3. $3(2x+7)$
4. $7x^2+2x-3x^2-9$
5. $\frac{2}{3}=\frac{x+2}{6}$

Name Chad

3 Which verbal expression is represented by $\frac{1}{2}(n-3)$?

1) one-half n decreased by 3
2) one-half n subtracted from 3
3) the difference of one-half n and 3
4) one-half the difference of n and 3

4 Which expression represents the number of hours in w weeks and d days?

1) $7w+12d$
2) $84w+24d$
3) $168w+24d$
4) $168w+60d$

5 If h represents a number, which equation is a correct translation of "Sixty more than 9 times a number is 375"?

1) $9h=375$
2) $9h+60=375$
3) $9h-60=375$
4) $60h+9=375$

6 The solution of the equation $5-2x=-4x-7$ is

1) 1
2) 2
3) -2
4) -6

7 If $3ax + b = c$, then x equals

1) $c - b + 3a$

2) $c + b - 3a$

3) $\frac{c - b}{3a}$

4) $\frac{b - c}{3a}$

8 If the formula for the perimeter of a rectangle is $P = 2l + 2w$, then w can be expressed as

1) $w = \frac{2l - P}{2}$

2) $w = \frac{P - 2l}{2}$

3) $w = \frac{P - l}{2}$

4) $w = \frac{P - 2w}{2l}$

Powers (Exponents) – 8%

Overview of Rules for Exponents

Rule: For any nonzero number a, $a^0 = 1$, and $a^{-n} = \frac{1}{a^n}$

Rule: For any nonzero number a and any rational numbers m and n, $a^m \cdot a^n = a^{m+n}$

Rule: For any nonzero number a and any rational numbers m and n, $(a^m)^n = a^{mn}$

Rule: For any nonzero numbers a and b and any rational number n $(ab)^n = a^n b^n$

Rule: For any nonzero number a and any rational numbers m and n, $\frac{a^m}{a^n} = a^{m-n}$

Rule: For any nonzero number *a*, $a^0 = 1$

Rule: For any nonzero number *a*, $a^{-n} = \frac{1}{a^n}$

Addition and Subtraction of Polynomials

The sum of

$4x^3 + 6x^2 + 2x - 3$ *and* $3x^3 + 3x^2 - 5x - 5$ is:

Solution:

$$\begin{array}{ll} & 4x^3 + 6x^2 + 2x - 3 \\ \text{Step 1} & \underline{3x^3 + 3x^2 - 5x - 5} \end{array}$$

$$\begin{array}{ll} & 4x^3 + 6x^2 + 2x - 3 \\ \text{Step 2} & \underline{3x^3 + 3x^2 - 5x - 5} \\ & 7x^3 + 9x^2 - 3x - 8 \end{array}$$

The answer is: $7x^3 + 9x^2 - 3x - 8$

Multiplication and Division of Polynomials

Multiplication of Polynomial Expressions

Multiplication of polynomial expressions combines the distributive property with the rules for multiplying exponential powers of the same base.

- **Distributive Property Multiplication**

$$a(b+c) = ab + ac$$
$$a(b-c) = ab - ac$$
$$(b+c)a = ba + ca$$
$$(b-c)a = ba - ca$$

- **Exponent Rule**: For any nonzero number *a* and any rational numbers *m* and *n*,

$$a^m \cdot a^n = a^{m+n}$$

To **divide** a polynomial by a monomial,

Step 1. **Factor** the denominator out of the numerator.

Step 2. Then, **cancel** the factor in both the numerator and denominator.

Scientific Notation

Rule: A number is in scientific notation if it is written in the form $a \times 10^n$, where n is an integer and $1 \le |a| < 10$

Exponential Functions

An **exponential function** is a function that contains a variable for an exponent.

Example: $y = 2^x$

ANSWERS - See page 79

POWERS

8% - POWERS

1 Which expression is equivalent to $-3x(x-4)-2x(x+3)$?

1) $-x^2-1$
2) $-x^2+18x$
3) $-5x^2-6x$
4) $-5x^2+6x$

2 What is the sum of $-3x^2-7x+9$ and $-5x^2+6x-4$?

1) $-8x^2-x+5$
2) $-8x^4-x+5$
3) $-8x^2-13x+13$
4) $-8x^4-13x^2+13$

3 What is $24x^2y^6-16x^6y^2+4xy^2$ divided by $4xy^2$?

1) $6xy^4-4x^5$
2) $6xy^4-4x^5+1$
3) $6x^2y^3-4x^6y$
4) $6x^2y^3-4x^6y+1$

4 Which expression represents $\dfrac{(2x^3)(8x^5)}{4x^6}$ in simplest form?

1) x^2
2) x^9
3) $4x^2$
4) $4x^9$

5 What is one-third of 3^6?

1) 1^2
2) 3^2
3) 3^5
4) 9^6

6 Daniel's Print Shop purchased a new printer for \$35,000. Each year it depreciates (loses value) at a rate of 5%. What will its approximate value be at the end of the fourth year?

1) \$33,250.00
2) \$30,008.13
3) \$28,507.72
4) \$27,082.33

7 What is the quotient of 8.05×10^6 and 3.5×10^2?

1) 2.3×10^3
2) 2.3×10^4
3) 2.3×10^8
4) 2.3×10^{12}

8 Which expression represents $\frac{25x - 125}{x^2 - 25}$ in simplest form?

1) $\frac{5}{x}$
2) $\frac{-5}{x}$
3) $\frac{25}{x-5}$
4) $\frac{25}{x+5}$

Systems of Equations – 8% of questions on the test

Solving Linear Systems

Strategy: Isolate the Same Variable in Both Equations to Solve a System

Example: Solve the system of equations
$$\begin{aligned} 3C+4M &= 12.50 \\ 3C+2M &= 8.50 \end{aligned}$$

STEP #1.

Isolate the same variable in both equations.

$Eq.\#1$
$$\begin{aligned} 3C+4M &= 12.50 \\ 3C &= 12.50-4M \\ C &= \frac{12.50-4M}{3} \end{aligned}$$

$Eq.\#2$
$$\begin{aligned} 3C+2M &= 8.50 \\ 3C &= 8.50-2M \\ C &= \frac{8.50-2M}{3} \end{aligned}$$

STEP #2.

Write a new equation using equivalent expressions. In this example, the two equivalent expressions are both equal to C.

$Eq.\#3$
$$\frac{12.50-4M}{3}=\frac{8.50-2M}{3}$$

STEP #3.

Solve the new equation for the first variable, which in this case is M.

$Eq.\#3$
$$\begin{aligned} \frac{12.50-4M}{3} &= \frac{8.50-2M}{3} \\ 3(12.50-4M) &= 3(8.50-2M) \\ 12.50-4M &= 8.50-2M \\ 12.50-8.50 &= -2M+4M \\ 4.00 &= 2M \\ \boxed{2.00 = M} \end{aligned}$$

STEP #4.

Substitute the value of the variable you found in the first equation and solve for the second variable.

$Eq.\#1$
$$\begin{aligned} 3C+4(\cancel{M}\,2.00) &= 12.50 \\ 3C+8 &= 12.50 \\ 3C &= 4.50 \\ \boxed{C=\frac{4.50}{3}=1.50} \end{aligned}$$

STEP #5

Always substitute the solutions into the original equations to see if the original equations balance. If they balance, your solutions are correct.

Writing Linear Systems

Systems of equations are often hidden in ordinary words.

- Define the variables involved in the problem.
- Write the two equations that are hidden in the problem.
  - Look for one equation in each sentence.
- Solve the system of equations.
- Check your work.

Quadratic Linear Systems

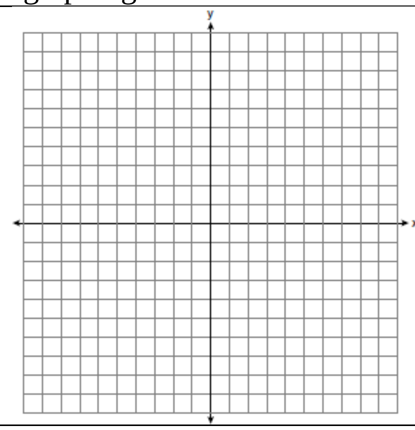
A **solution** to a system of linear and quadratic equations is any point where the graphs of the linear and quadratic equations intersect. A **solution set** for a system of linear and quadratic equations can contain zero, one, or two points.

- Solutions can often be identified by inspecting the graphs of the equations.
- If the graph of the equations are not provided, a graphing calculator can be used to create the graphs.

Exqample:

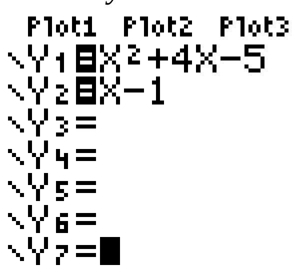
On the set of axes, solve the following system of equations graphically and state the coordinates of all points in the solution set.

$$y = x^2 + 4x - 5$$
$$y = x - 1$$

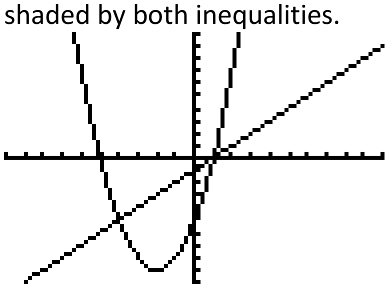


STEP #1.
Input both equations into the y-editor of the calculator.

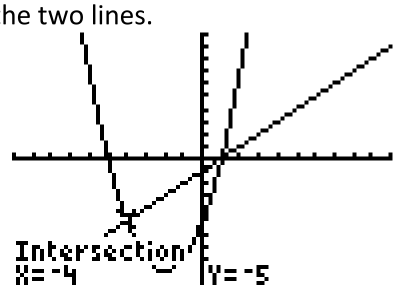
$$y = x^2 + 4x - 5$$
$$y = x - 1$$

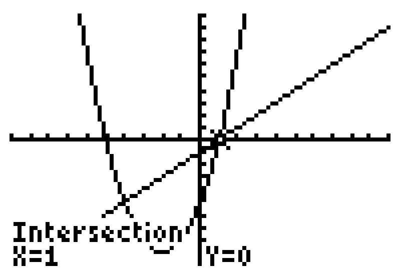


STEP #2.
Visually inspect the graph to estimate whether a given point falls in the area of the graph shaded by both inequalities.



STEP #3.
Use the $\boxed{\text{calculate}}$ $\boxed{\text{intersect}}$ function of the graphing calculator to find the exact intersection(s) of the two lines.





The two solutions are $(-4, -5)$ and $(1, 0)$.

STEP #4.
Check your solution by substituting both solutions into both equations. If both equations remain balanced, the solutions are good.

STEP #5
If you are answering an open response question on a Regents examination, copy the input screen, a table of values, the graph, and your solution check into your examination booklet.

Systems of Inequalities

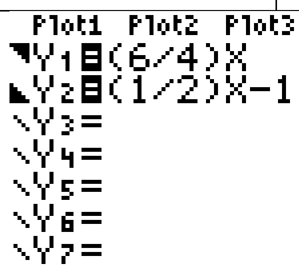
- Systems of inequalities are graphed in the same manner as systems of equations are graphed.
- The solution of the system of inequalities is the region of the coordinate plane that is shaded by both inequalities.

Example: Determine by graphing if the point (-4,-3) is in the solution set of the system of inequalities:

$$4y \ge 6x$$
$$-3x + 6y \le -6$$

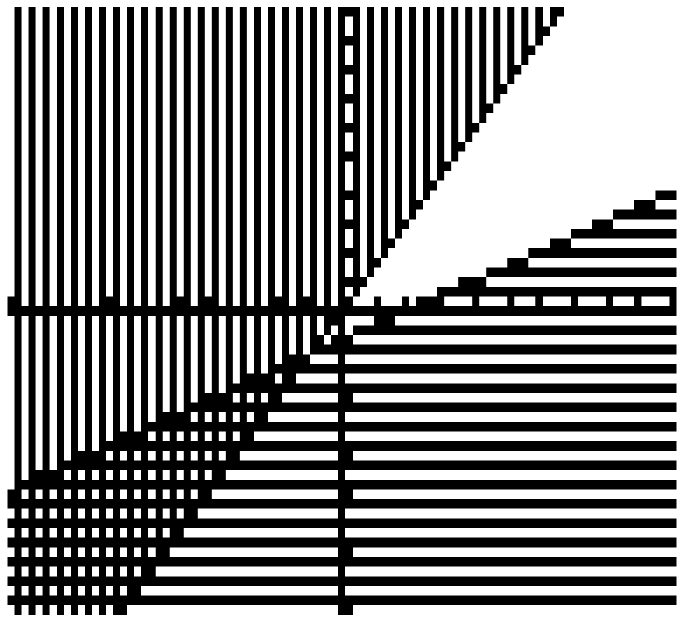
STEP #1.
Manually transform both inequalities into slope-intercept form ($y = mx + b$) and input both into the y-editor of the calculator.

Inequality #1	*Inequality* #2
$4y \ge 6x$ $y \ge \frac{6}{4}x$	$-3x+6y \le -6$ $6y \le 3x - 6$ $y \le \frac{3x-6}{6}$ $y \le \frac{1}{2}x - 1$



Note the greater than and less than icons at the far left of each inequality.

STEP #2.
Visually inspect the graph to estimate whether a given point falls in the area of the graph shaded by both inequalities.



STEP #3.
Verify that a point does or does not fall in the double shaded section of the graph by checking the point in each of the original inequalities. For example, the point $(-4,-3)$ appears to lie within the double shaded region of the graph. This needs to be verified:

Inequality #1

$$4(\cancel{y}-3) \ge 6(\cancel{x}-4)$$
$$-12 \ge -24 \ (true)$$

Inequality #2

$$-3(\cancel{x}-4) + 6(\cancel{y}-3) \le -6$$
$$12 - 18 \le -6$$
$$-6 \le -6 \quad (true)$$

STEP #4
If you are answering an open response question on a Regents examination, copy the input screen, a table of values, the graph, and your solution check into your examination booklet.

ANSWERS - See page 80

SYSTEMS of EQUATIONS and INEQUALITIES

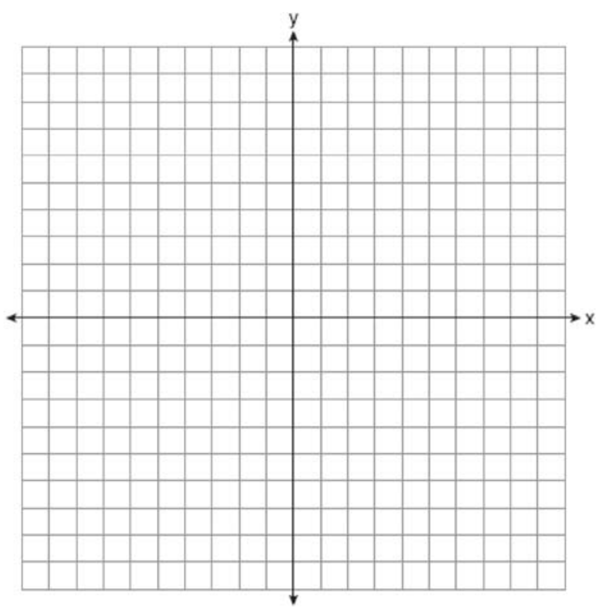
8% - SYSTEMS of EQUATIONS

1 Which ordered pair is a solution to the system of equations $y = x + 3$ and $y = x^2 - x$?

1) $(6,9)$
2) $(3,6)$
3) $(3,-1)$
4) $(2,5)$

2 On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution set.

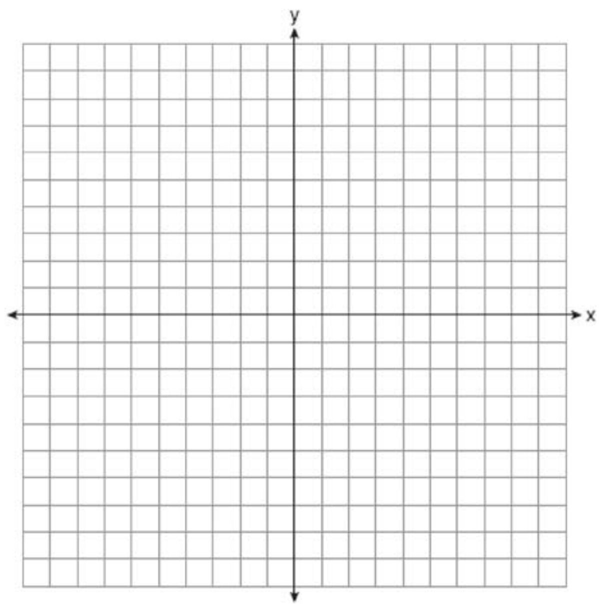
$$y = x^2 + 4x - 5$$
$$y = x - 1$$



3 On the set of axes below, solve the following system of equations graphically for all values of x and y. State the coordinates of all solutions.

$$y = x^2 + 4x - 5$$

$$y = 2x + 3$$



4 The equations $5x + 2y = 48$ and $3x + 2y = 32$ represent the money collected from school concert ticket sales during two class periods. If x represents the cost for each adult ticket and y represents the cost for each student ticket, what is the cost for each adult ticket?

1) \$20
2) \$10
3) \$8
4) \$4

5 Jack bought 3 slices of cheese pizza and 4 slices of mushroom pizza for a total cost of \$12.50. Grace bought 3 slices of cheese pizza and 2 slices of mushroom pizza for a total cost of \$8.50. What is the cost of one slice of mushroom pizza?

1) \$1.50
2) \$2.00
3) \$3.00
4) \$3.50

SYSTEMS of INEQUALITIES

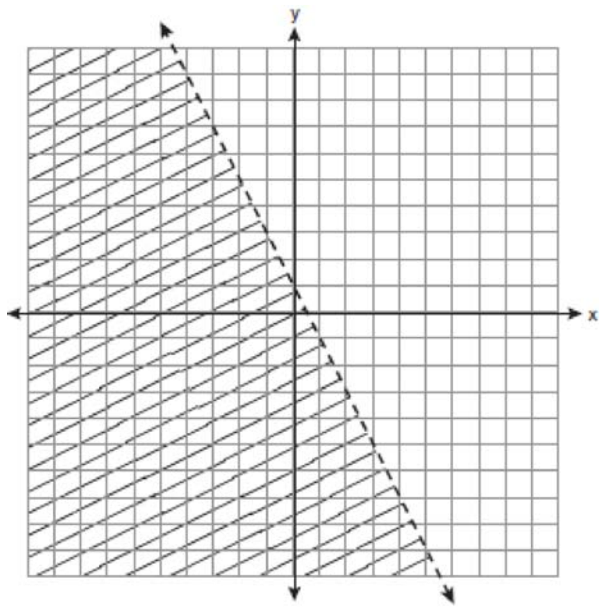
6 Which coordinates represent a point in the solution set of the system of inequalities shown below?

$$y \le \frac{1}{2}x + 13$$

$$4x + 2y > 3$$

1) $(-4,1)$
2) $(-2,2)$
3) $(1,-4)$
4) $(2,-2)$

7 Which inequality is represented by the graph below?



1) $y < 2x + 1$

2) $y < -2x + 1$

3) $y < \frac{1}{2}x + 1$

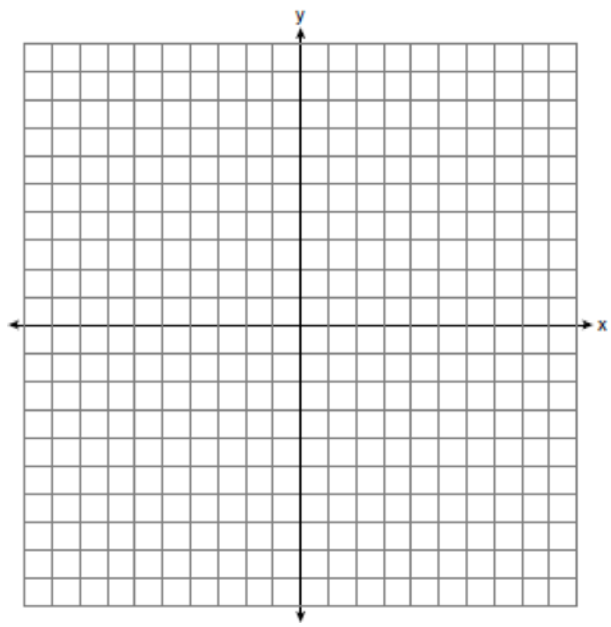
4) $y < -\frac{1}{2}x + 1$

8 On the set of axes below, solve the following system of inequalities graphically.

$$y < 2x + 1$$

$$y \geq -\frac{1}{3}x + 4$$

State the coordinates of a point in the solution set.



Rational Numbers – 8%

Rational Expressions

Simplifying rational polynomial expressions typically involves three steps:
Step 1. Factor the numerator and denominator.
Step 2. Use cancellation to eliminate or reduce terms.
Step 3. Rewrite the simplified expression.

<u>Examples</u>:

Given:	Simplify the fraction: $\dfrac{24}{18}$	Simplify the polynomial: $\dfrac{2x^2yz}{4xy}$
Step 1. Factor	$\dfrac{24}{18}=\dfrac{2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 3}$	$\dfrac{2x^2yz}{4xy}=\dfrac{2\cdot x\cdot x\cdot y\cdot z}{2\cdot 2\cdot x\cdot y}$
Step 2. Cancel	$\dfrac{{}^1\cancel{2}\cdot 2\cdot 2\cdot {}^1\cancel{3}}{{}^1\cancel{2}\cdot 3\cdot {}^1\cancel{3}}=\dfrac{1\cdot 2\cdot 2\cdot 1}{1\cdot 3\cdot 1}=\dfrac{4}{3}$	$\dfrac{{}^1\cancel{2}\cdot {}^1\cancel{x}\cdot x\cdot {}^1\cancel{y}\cdot z}{{}^1\cancel{2}\cdot 2\cdot {}^1\cancel{x}\cdot {}^1\cancel{y}}=\dfrac{1\cdot 1\cdot x\cdot 1\cdot z}{1\cdot 2\cdot 1\cdot 1}=\dfrac{xz}{2}$
Step 3. Rewrite	$\dfrac{1\cdot 2\cdot 2\cdot 1}{1\cdot 3\cdot 1}=\boxed{\dfrac{4}{3}}$	$\dfrac{1\cdot 1\cdot x\cdot 1\cdot z}{1\cdot 2\cdot 1\cdot 1}=\boxed{\dfrac{xz}{2}}$

Error Alert! $\dfrac{2x+3y}{2x}\neq\dfrac{{}^1\cancel{2x}+3y}{{}^1\cancel{2x}}=3y$

The $2x$ term in the denominator is not a factor of every term in the numerator. Hence, it cannot be cancelled.

Multiplication and Division of Rationals

$$\frac{a}{b}\otimes\frac{c}{d}=\frac{ac}{bd}$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$$

Addition and Subtraction of Rationals

$$\frac{a}{b}+\frac{c}{d}=\frac{(ad)+(bc)}{bd}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{(ad)-(bc)}{bd}$$

Undefined Rational Expressions

An algebraic fraction is undefined when the denominator equals zero.

<u>Division by zero is not allowed</u>!

How to find the value of a variable that makes an algebraic expression undefined.
Step 1. Write an equation where the expression in the denominator equals zero.
Step 2. Solve the equation for the variable.

ANSWERS - See page 82

RATIONALS

8% - RATIONALS

1 What is the sum of $\frac{d}{2}$ and $\frac{2d}{3}$ expressed in simplest form?

1) $\frac{3d}{5}$
2) $\frac{3d}{6}$
3) $\frac{7d}{5}$
4) $\frac{7d}{6}$

2 Express the product of $\frac{x+2}{2}$ and $\frac{4x+20}{x^2+6x+8}$ in simplest form.

3 Which value of x is the solution of $\frac{2x}{5}+\frac{1}{3}=\frac{7x-2}{15}$?

1) $\frac{3}{5}$
2) $\frac{31}{26}$
3) 3
4) 7

4 Solve for x: $\frac{x+1}{x}=\frac{-7}{x-12}$

5 For which value of x is $\frac{x-3}{x^2-4}$ undefined?

1) -2
2) 0
3) 3
4) 4

6 What is the value of x in the equation $\frac{2}{x} - 3 = \frac{26}{x}$?

1) -8
2) $-\frac{1}{8}$
3) $\frac{1}{8}$
4) 8

7 Which expression represents $\frac{2x^2-12x}{x-6}$ in simplest form?

1) 0
2) $2x$
3) $4x$
4) $2x+2$

8 Perform the indicated operation and simplify: $\frac{3x+6}{4x+12} \div \frac{x^2-4}{x+3}$

Probability – 7% of the Questions

Theoretical Probability

$$P_{(event)} = \frac{\text{number of time the event happens}}{\text{number of times the experiment is done}}$$

$$P_{\left(event\ A\ \ \underline{\text{and}}\ \ event\ B\right)} = P_{(event\ A)} \times P_{(event\ B)}$$

$$P_{\left(\underline{\text{either}}\ \ event\ A\ \ \underline{\text{or}}\ \ event\ B\right)} = P_{(event\ A)} + P_{(event\ B)}$$

Experimental Probability

$$\mathrm{P}_{(\text{event})} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Geometric Probability

$$\mathrm{P}_{(\text{event})} = \frac{\text{area (or volume) of favorable outcomes}}{\text{area (or volume) of possible outcomes}}$$

Conditional Probability

Conditional probability is computed based on the assumption that some specified event(s) has already occurred.

The **sum of all probabilities for mutually exclusive events** is always 1.

Multiplication Counting Principle

The **multiplication counting principle** states that,

- if event A has a possible outcomes, event B has b possible outcomes, and event C has c possible outcomes,
- then the number of possible outcomes for event A followed by event B followed by event C is determined by multiplying $a \times b \times c$.
  - This rule applies for two or more events.

A **counting box** is a visual representation of the number of possible outcomes for a particular event. **Counting boxes** are useful for determining the number of possible outcomes of a series of events. Counting boxes can also be used for solving permutation and combination problems.

Example: A certain car comes in three body styles with a choice of two engines, a choice of two transmissions, and a choice of six colors. What is the minimum number of cars a dealer must stock to have one car of every possible combination?

Body Styles		Engines		Trans-missions		Colors		
3	x	2	x	2	x	6	=	72

Permutations

A **permutation** of objects is an ordering of them. The order of the objects matters with permutations.
The number of permutations that can be created from *n* objects is ***n!***, which reads as **n-factorial**.

$$1! = 1$$
$$2! = 2 \times 1 = 2$$
$$3! = 3 \times 2 \times 1 = 6$$
$$4! = 4 \times 3 \times 2 \times 1 = 24$$
$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$
$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

${}_nP_r$ can be translated as **permutations of n things taken r at a time**. For example, if we have twenty people running a race, and we want to know how many different ways we can award first, second, and third place trophies, we would use ${}_nP_r$, where $n = 20$ and $r = 3$. We would write the problem as ${}_{20}P_3$, which can be translated as "permutations of 20 things taken 3 at a time."

Understanding ${}_nP_r$ using Counting Boxes. To count the permutations of n things taken r at a time:

- create a series of counting boxes with a total of r boxes, then
- fill in the boxes using the values of $n!$.

The total number of counting boxes
is equal to r

$${}_nP_r = \boxed{n} \times \boxed{n-1} \times \boxed{n-2} \times \boxed{n-3} =$$

The values of n! go inside the boxes.

Example #1:
If you have 20 runners in a race, **how many** ways can first, second, and third place trophies can be awarded?

The total number of counting boxes
is equal to r

$${}_{20}P_3 = \boxed{20} \times \boxed{19} \times \boxed{18} = 6{,}840$$

The values of n! go inside the boxes.

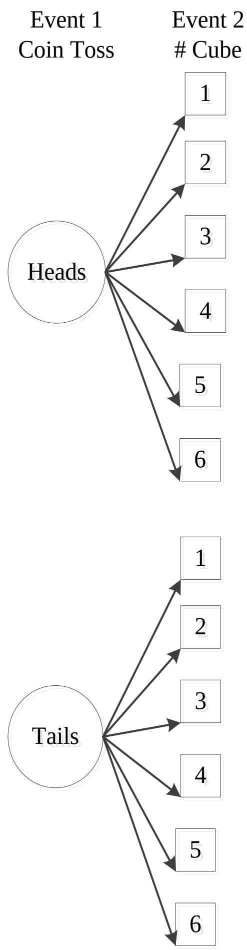
Testing Tip. The words "**how many**" on a Regents Math A examination almost always mean you are dealing with a permutations or a combinations problem.

Tree Diagrams and Sample Spaces

A **tree diagram** shows different possible outcomes from two or more events involving probability. The tree diagram shows the possible outcomes of tossing a coin followed by tossing a number cube. There are 12 possible outcomes, shown by the branches of this tree diagram.

TREE DIAGRAM

Possible Outcomes



SAMPLE SPACE

H1
H2
H3
H4
H5
H6
T1
T2
T3
T4
T5
T6

A **sample space** simply lists each of the possible outcomes from two or more events involving probability. The sample space shows the possible outcomes of tossing a coin followed by tossing a number cube. There are 12 possible outcomes, shown by a simple listing.

If you want to know the number of favorable outcomes from two events, you can use a tree diagram or sample space.

Example

The above tree diagram or sample space can be used to determine the probability of getting a tail on a coin toss followed by and even number. Simply count the number of outcomes with tails and even numbers (3), then count the total possible outcomes (12). Tails followed by an even number should occur 3 out of 12 times, or 1/4th of the time.

ANSWERS - See page 84

PROBABILITY

7% - PROBABILITY

1 Some books are laid on a desk. Two are English, three are mathematics, one is French, and four are social studies. Theresa selects an English book and Isabelle then selects a social studies book. Both girls take their selections to the library to read. If Truman then selects a book at random, what is the probability that he selects an English book?

2 The faces of a cube are numbered from 1 to 6. If the cube is rolled once, which outcome is *least* likely to occur?

1) rolling an odd number
2) rolling an even number
3) rolling a number less than 6
4) rolling a number greater than 4

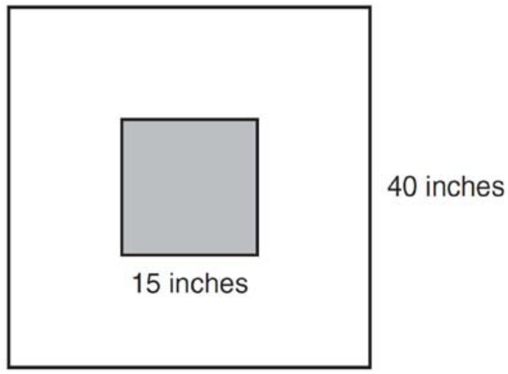
3 Two cubes with sides numbered 1 through 6 were rolled 20 times. Their sums are recorded in the table below.

4	9	8	9	2
9	4	6	12	10
8	7	9	11	10
8	7	9	3	5

What is the empirical probability of rolling a sum of 9?

1) $\frac{4}{20}$
2) $\frac{5}{20}$
3) $\frac{4}{36}$
4) $\frac{5}{36}$

4 The square dart board shown below has a side that measures 40 inches. The shaded portion in the center is a square whose side is 15 inches. A dart thrown at the board is equally likely to land on any point on the dartboard.



Find the probability that a dart hitting the board will *not* land in the shaded area.

5 A large company must chose between two types of passwords to log on to a computer. The first type is a four-letter password using any of the 26 letters of the alphabet, without repetition of letters. The second type is a six-digit password using the digits 0 through 9, with repetition of digits allowed. Determine the number of possible four-letter passwords. Determine the number of possible six-digit passwords. The company has 500,000 employees and needs a different password for each employee. State which type of password the company should choose. Explain your answer.

6 A restaurant sells kids' meals consisting of one main course, one side dish, and one drink, as shown in the table below.

Kids' Meal Choices

Main Course	Side Dish	Drink
hamburger	French fries	milk
chicken nuggets	applesauce	juice
turkey sandwich		soda

Draw a tree diagram or list the sample space showing all possible kids' meals. How many different kids' meals can a person order? Jose does not drink juice. Determine the number of different kids' meals that do *not* include juice. Jose's sister will eat *only* chicken nuggets for her main course. Determine the number of different kids' meals that include chicken nuggets.

7 The probability that it will snow on Sunday is $\frac{3}{5}$. The probability that it will snow on both Sunday and Monday is $\frac{3}{10}$. What is the probability that it will snow on Monday, if it snowed on Sunday?

1) $\frac{9}{50}$

2) 2

3) $\frac{1}{2}$

4) $\frac{9}{10}$

Numbers, Operations, and Properties – 7% of Questions

Set Theory

A <u>set</u> is a well-defined collection of items.
There are several different ways to describe sets.
<u>Listing the Elements.</u>
{a, b, c, d}   This is a set of four small case letters, each of which is named.
{1,2,3,4,5}   This is a set of five positive integers, each of which is named.
{….-3,-2,-1,0,1,2,3….}   This is the set of all integers. Note that the four periods at each end of the list is used to show that the pattern keeps going to infinity.
<u>Set Builder Notation</u>
Elements of a set can also be described using set builder notation.

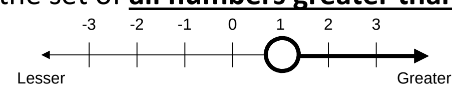
- The set {1,2,3,4,5} can be written in set builder notation as:

  $$\{x \mid 0 < x < 6, \textit{ where x is a whole number}\}$$

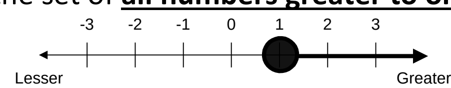
  This is read as "the set of all values of *x*, where 0 is less than *x* and *x* is less than 6, and where *x* is a whole number."

The set {1,2,3,4,5} can also be written in set builder notation as: $\{x \mid 1 \le x \le 5, \textit{ where x is a whole number}\}$

<u>Using Number Lines to Define Sets</u>
The number line representation of the set of **<u>all numbers greater than 1</u>** is as follows:



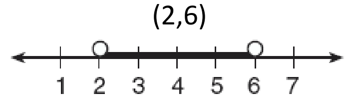
The number line representation of the set of **<u>all numbers greater to or equal to 1</u>** is as follows:



Note: If the circle is empty, that value *is not included* in the set. If the circle is filled in, that value *is included* in the set.

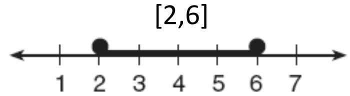
<u>Interval Notation</u>
Interval notation uses curved and squared parenthesis to show where an interval of numbers (as on a number line) begins and ends.
A **<u>curved</u>** parenthesis indicates that the number next to it **<u>is not</u>** included in the interval.

(2,6)



A **<u>squared</u>** parenthesis indicates that the number next to it **<u>is</u>** included in the interval.

[2,6]



The **<u>complement</u>** of a subset is the subset of elements that must be added to the first subset to yield the original set.

$$A \cap B$$

The **<u>intersection</u>** of two or more sets is the set of all elements that common to all of the given sets.

$$A \cup B$$

The **<u>union</u>** of two or more sets is the set of all elements contained in at least one of the sets.

Properties

Commutative Properties of Addition and Multiplication

For all real numbers a and b:

$$a+b=b+a \qquad a\cdot b=b\cdot a$$

Associative Properties of Addition and Multiplication

For all real numbers a, b, and c:

$$(a+b)+c=a+(b+c) \qquad (a\cdot b)\cdot c=a\cdot(b\cdot c)$$

Distributive Properties of Addition and Multiplication

$$a(b+c)=ab+ac \qquad a(b-c)=ab-ac$$

$$(b+c)a=ba+ca \qquad (b-c)a=ba-ca$$

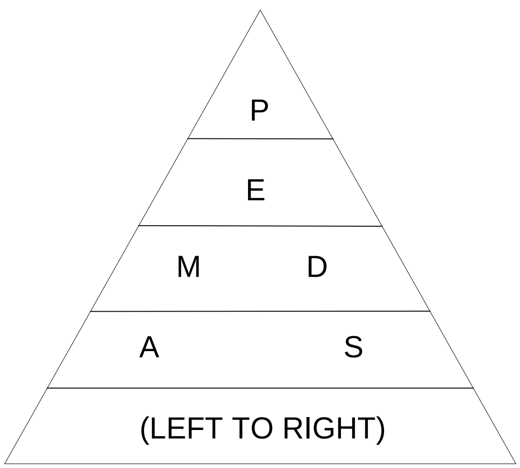
Identity Properties of Addition and Multiplication

$$a+0=a \text{ and } 0+a=a \qquad a\cdot 1=a \text{ and } 1\cdot a=a$$

Inverse Properties of Addition and Multiplication

$$a+(-a)=0 \text{ and } (-a)+a=0 \qquad a\cdot\frac{1}{a}=1 \text{ and } \frac{1}{a}\cdot a=1$$

Evaluating Expressions



When evaluating expressions, follow the order of operations as shown in the PEMDAS Pyramid. Start at the top of the triangle and:

- Level 1 - P: Perform any operations inside grouping symbols such as parentheses, brackets, and absolute value signs. Fraction bars should be treated as a grouping symbols like parentheses.
- Level 2 - E: Simplify any terms with exponents.
- Level 3 - M D: Perform any operations involving multiplication and division.
- Level 4 – A S: Perform any operations involving addition and subtraction.
- In levels 3 and 4, if there are two or more operations on the same level of the pyramid, start with the operation that is furthest left and proceed to the right.

Evaluating Expressions

An <u>expression</u> is a mathematical statement with no equal sign. An expression may have one or more <u>terms</u>. A <u>term</u> is a number, a variable, or the product of a number and a variable.

- To evaluate an expression, you must substitute a specified value for the variable(s) and simplify.
- Example: What is the value of the expression $(a^3+b^0)^2$ when a = -2 and b = 4?

$$\left(a^3+b^0\right)^2$$

$$\left(\left(\ \ \right)^3+\left(\ \ \right)^0\right)^2$$

$$\left((-2)^3+(4)^0\right)^2$$

$$(-8+1)^2$$

$$(-7)^2$$

$$\boxed{49}$$

ANSWERS - See page 85

NUMBERS, OPERATIONS AND PROPERTIES

NUMBERS, OPERATIONS, AND PROPERTIES - 7%

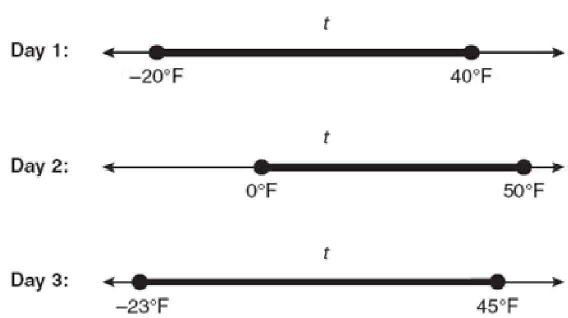
1 Which set-builder notation describes $\{-3,-2,-1,0,1,2\}$?

1) $\{x|-3 \le x < 2,$ where x is an integer$\}$
2) $\{x|-3 < x \le 2,$ where x is an integer$\}$
3) $\{x|-3 < x < 2,$ where x is an integer$\}$
4) $\{x|-3 \le x \le 2,$ where x is an integer$\}$

2 The set $\{11,12\}$ is equivalent to

1) $\{x|11 < x < 12,$ where x is an integer$\}$
2) $\{x|11 < x \le 12,$ where x is an integer$\}$
3) $\{x|10 \le x < 12,$ where x is an integer$\}$
4) $\{x|10 < x \le 12,$ where x is an integer$\}$

3 Maureen tracks the range of outdoor temperatures over three days. She records the following information.



Express the intersection of the three sets as an inequality in terms of temperature, t.

4 Which interval notation represents the set of all real numbers greater than 2 and less than or equal to 20?

1) $(2,20)$
2) $(2,20]$
3) $[2,20)$
4) $[2,20]$

5 Given: $U = \{x|0 < x < 10 \text{ and } x \text{ is an integer}\}$

$S = \{x|0 < x < 10 \text{ and } x \text{ is an odd integer}\}$

The complement of set S within the universal set U is

1) $\{0,2,4,6,8,10\}$
2) $\{2,4,6,8,10\}$
3) $\{0,2,4,6,8\}$
4) $\{2,4,6,8\}$

6 Consider the set of integers greater than -2 and less than 6. A subset of this set is the positive factors of 5. What is the complement of this subset?

1) $\{0,2,3,4\}$
2) $\{-1,0,2,3,4\}$
3) $\{-2,-1,0,2,3,4,6\}$
4) $\{-2,-1,0,1,2,3,4,5,6\}$

7 Which equation illustrates the associative property?

1) $x+y+z=x+y+z$
2) $x(y+z)=xy+xz$
3) $x+y+z=z+y+x$
4) $(x+y)+z=x+(y+z)$

Linear Equations – 6% of Questions

Slope Intercept Form	Slope Formula
$y = mx + b$ where *m = slope* and *b = y-intercept*	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Standard Form $Ax + By = C$	**Slope in Standard Form** $m = -\frac{A}{B}$

Writing Linear Equations Given a Point and Slope or Two Points

NOTE: If given two points, find slope first using slope formula.

Overview of Method: Start by writing $y = mx + b$ vertically in the first column and horizontally in the second and third columns.

	$y = mx + b$	$y = mx + b$
y= **m=** **x=** **b=**		

NOTE: Regents problems always provide three of the four variables in linear equations.

- Column 1 is a space for you to write what you know from the word problem.
- Column 2 is a space for you to solve for the fourth, or unknown, variable.
- Column 3 reminds you that you are not finished until you have written the final equation by substituting m and b into the slope-intercept form.

Example: The following problem appeared in a Regents exam.

Given	What is an equation of the line that passes through the point (4, −6) and has a slope of −3?	
Solution Using the $y = mx + b$ graphic organizer.		
y = -6 **m = -3** **x = 4** **b =**	$y = mx + b$ $-6 = (-3)(4) + b$ $-6 = -12 + b$ $6 = b$	$y = mx + b$ $\boxed{y = -3x + 6}$

Identifying Points on a Line

To determine whether a given point is on a line, given the equation of the line, substitute the x and y values of the given point into the equation of the line.

If the left expression equals the right expression after the substitution, the point is on the line.

If the two expressions are not equal, the point is not on the line.

You can also input the equation into a graphing calculator and check the table of values to see if it shows the x and y values of the given point into the equation of the line.

Parallel and Perpendicular Lines

Two lines are parallel if and only if they have the **same slope** and **different y-intercepts**.

- All **vertical lines are parallel** to each other.
- All **horizontal lines are parallel** to each other.

Example: $y=\frac{2}{3}x+4$ is parallel to $y=\frac{2}{3}x+5$

To determine if two lines are parallel, find the slope of both equations. It the slopes are identical, the lines are parallel.

ANSWERS - See page 86

LINEAR EQUATIONS

6% - LINEAR EQUATIONS

1 In a linear equation, the independent variable increases at a constant rate while the dependent variable decreases at a constant rate. The slope of this line is

1) zero
2) negative
3) positive
4) undefined

2 What is the slope of the line represented by the equation $4x + 3y = 12$?

1) $\frac{4}{3}$
2) $\frac{3}{4}$
3) $-\frac{3}{4}$
4) $-\frac{4}{3}$

3 Which equation represents a line parallel to the graph of $2x - 4y = 16$?

1) $y = \frac{1}{2}x - 5$
2) $y = -\frac{1}{2}x + 4$
3) $y = -2x + 6$
4) $y = 2x + 8$

4 Which point lies on the line whose equation is $2x - 3y = 9$?

1) $(-1,-3)$
2) $(-1,3)$
3) $(0,3)$
4) $(0,-3)$

5 What is the slope of the line that passes through the points $(3,5)$ and $(-2,2)$?

1) $\frac{1}{5}$
2) $\frac{3}{5}$
3) $\frac{5}{3}$
4) 5

6 What is an equation for the line that passes through the coordinates $(2,0)$ and $(0,3)$?

1) $y = -\frac{3}{2}x + 3$
2) $y = -\frac{3}{2}x - 3$
3) $y = -\frac{2}{3}x + 2$
4) $y = -\frac{2}{3}x - 2$

Inequalities – 6% of Questions

Solving Inequalities

<u>Inequality Symbols:</u>

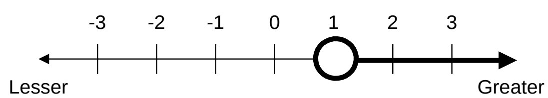
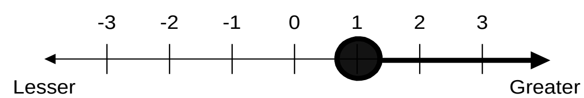
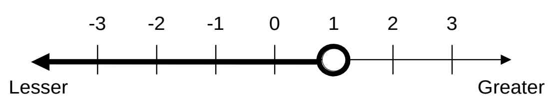
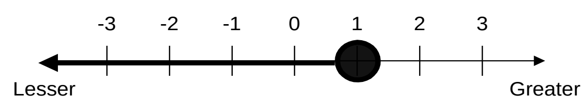
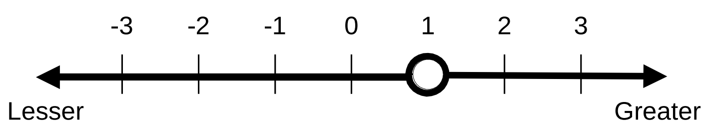
$<$ less than $>$ greater than

$\leq$ less than or equal to $\geq$ greater than or equal to

$\neq$ not equal to

The **<u>solution of an inequality</u>** includes any values that make the inequality true.
Solutions to inequalities can be graphed on a number line using open and closed dots.

<u>Open dots</u> vs. **<u>Closed dots</u>**

When the inequality sign does not contain an equality bar beneath it, the dot is open.	When the inequality sign contains includes an equality bar beneath it, the dot is closed, or shaded in.
Graph of $x > 1$ 	Graph of $x \geq 1$ 
Graph of $x < 1$ 	Graph of $x \leq 1$ 
Graph of $x \neq 1$ 	

<u>The Big Rule for Solving Inequalities</u>:
All the rules for solving equations apply to inequalties – plus one:

- **<u>When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.</u>**

Interpreting Solutions

There are two general approaches to determining if a given value is within the solution set of an equation or inequality.

- Substitute the value in the equation or inequality.
  - If the equation remains balanced, or the inequality remains true, the point is within the solution set.
  - If the equation is not balanced, or the inequality not true, the point is not within the solution set.
- Graph the inequality on a number line.
  - Visually inspect the graph to determine if a value is within the solution set.

Linear Inequalities

A linear inequality describes a region of the coordinate plane that has a **boundary line**.

- Every point within the region is a **solution of the inequality**.
  - Two or more linear inequalities together form a **system of linear inequalities**. Note: There will be two or more boundary lines in a system of linear inequalities.
  - A **solution of a system of linear inequalities** makes each inequality in the system true. The graph of a system shows all of its solutions.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

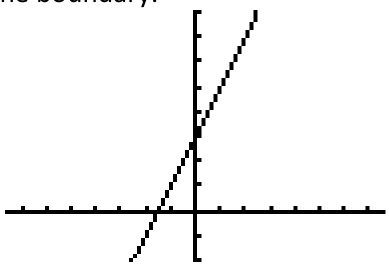
- When the inequality sign **contains** an equality bar beneath it $(\leq or \geq)$, use a **solid line** for the boundary.
- When the inequality sign **does not contain** an equality bar beneath it $(< or >)$, use a **dashed or dotted line** for the boundary

Step Two. Restore the inequality sign.

- If the inequality sign is $>$ or $\geq$, shade the area above the boundary line.
- If the inequality sign is $<$ or $\leq$, shade the area below the boundary line.
- If the boundary line is parallel to the y-axis, test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. If the boundary line runs through the point (0,0), select another point for testing.
  - If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.
  - If the test point makes the inequality true, shade the side of the boundary line that includes the test point.

Example Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$.
This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.



Next, test a point to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ is true, so the solution of the inequality is the region that contains the po

Therefore, we shade the side of the boundary line that contains the point (0,0)

Note: The TI-83+ graphing calculator does not have the ability to distinguish between solid and dashed lines on a graph of an inequality. The less than and greater than symbols are input using the far-left column of symbols that can be accessed through the $\boxed{\text{Y=}}$ feature.

ANSWERS - See page 87

INEQUALITIES

6% - INEQUALITIES

1 The statement $|-15| < x < |-20|$ is true when x is equal to

1) -16
2) -14
3) 17
4) 21

2 An electronics store sells DVD players and cordless telephones. The store makes a \$75 profit on the sale of each DVD player (d) and a \$30 profit on the sale of each cordless telephone (c). The store wants to make a profit of at least \$255.00 from its sales of DVD players and cordless phones. Which inequality describes this situation?

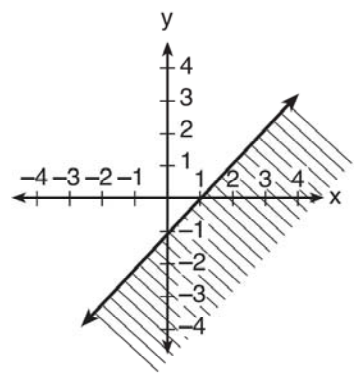
1) $75d + 30c < 255$
2) $75d + 30c \le 255$
3) $75d + 30c > 255$
4) $75d + 30c \ge 255$

3 A prom ticket at Smith High School is \$120. Tom is going to save money for the ticket by walking his neighbor's dog for \$15 per week. If Tom already has saved \$22, what is the minimum number of weeks Tom must walk the dog to earn enough to pay for the prom ticket?

4 What is the solution of the inequality $-6x - 17 \geq 8x + 25$?

1) $x \geq 3$
2) $x \leq 3$
3) $x \geq -3$
4) $x \leq -3$

5 The diagram below shows the graph of which inequality?



1) $y > x - 1$
2) $y \geq x - 1$
3) $y < x - 1$
4) $y \leq x - 1$

6 Mrs. Smith wrote "Eight less than three times a number is greater than fifteen" on the board. If x represents the number, which inequality is a correct translation of this statement?

1) $3x - 8 > 15$
2) $3x - 8 < 15$
3) $8 - 3x > 15$
4) $8 - 3x < 15$

Rate – 4% of Questions

Using Rate

A **ratio** is a comparison of two numbers by division.

- A **rate** is a special kind of ratio that compares two *different units of measurement* by division.
    - A **unit rate** is when the denominator of a rate is expressed as a single unit of measure.

      Examples: miles per gallon, $\left(\frac{x \text{ miles}}{1 \text{ gallon}}\right)$ words per minute, $\left(\frac{\text{x words}}{1 \text{ minute}}\right)$
    - **Unit rates** are useful for comparison purposes.

A **proportion** consists of two ratios with an equal sign between them.

Example:

$$\frac{60 \text{ miles}}{1 \text{ hour}} = \frac{\cancel{60} \text{ miles}}{\cancel{60} \text{ minutes}}$$

$$\frac{60 \text{ miles}}{1 \text{ hour}} = \frac{1 \text{ mile}}{1 \text{ minute}}$$

Speed

Speed is a rate. The measurement units of speed are $\frac{\text{distance}}{\text{time}}$.

A formula can be written $S = \frac{d}{t}$. By transforming this formula, we can get $t = \frac{d}{S}$ and $d = S \times t$

Conversions

Converting Fractions $\Rightarrow$ Decimals	Converting Decimals $\Rightarrow$ Fractions
A fraction can be converted to a decimal by dividing the numerator by the denominator. *Example:* $\frac{7}{13} = .\overline{538461}$	*A terminating decimal can be converted to a fraction by making it a fraction with a denominator of 1 and then moving the decimal point the required number of spaces to the right in both the numerator and denominator.* *Example:* $.123456789 = \frac{.123456789}{1} = \frac{123456789}{1,000,000,000}$ *Note: When a repeating decimal is truncated, information is lost forever and it will no longer be possible to convert it back to a fraction.*
Converting Decimals $\Rightarrow$ Percents	**Converting Percents $\Rightarrow$ Decimals**
A decimal can be converted to a percent by moving the decimal point two places to the right, which is the same as multiplying by 100. Examples: $.43 = 43\%$ $.431 = 43.1\%$	A percent can be converted to a decimal by moving the decimal point two places to the left, which is the same as dividing by 100. Examples: $57\% = .57$ $57.8\% = .578$

Converting Percents $\Rightarrow$ Fractions	Converting Fractions $\Rightarrow$ Percents
A percent can be converted to a fraction by placing it over a denominator of 100 and dropping the percent sign. Example: $34\% = \frac{34}{100}$ $47.2\% = \frac{47.2}{100} = \frac{472}{1,000}$	Option 1. Make a proportion using the fraction and $\frac{x}{100}$, where x equals the percent. Example: $\frac{3}{4} = \frac{x}{100}$ $3 \times 100 = 4x$ $300 = 4x$ $75 = x$ $\frac{3}{4} = 75\%$ Option 2. Convert the fraction to a decimal and then convert the decimal to a percent.

Percent of Change Formula

$$percent\ of\ change = \frac{amount\ of\ change}{original\ amount}$$

How to Solve Problems with Percents

Convert percents to decimals or fractions before multiplying or dividing them.

Percent "*of*" Something Wording. When a Regents problems uses "percent of" wording, it generally means multiplication, as in the following examples:

"…what percent of 22…"	means	x% times 22
"Ninety percent of the ninth grade students…"	means	.90 times (number of ninth grade students)
"…25% of the original price…"	means	.25 times (original price)

ANSWERS -See page 88

RATE

4% - RATE

1 If the speed of sound is 344 meters per second, what is the approximate speed of sound, in meters per hour?

60 seconds = 1 minute
60 minutes = 1 hour

1) 20,640
2) 41,280
3) 123,840
4) 1,238,400

2 Brianna's score on a national math assessment exceeded the scores of 95,000 of the 125,000 students who took the assessment. What was her percentile rank?

1) 6
2) 24
3) 31
4) 76

3 A hiker walked 12.8 miles from 9:00 a.m. to noon. He walked an additional 17.2 miles from 1:00 p.m. to 6:00 p.m. What is his average rate for the entire walk, in miles per hour?

1) 3.75
2) 3.86
3) 4.27
4) 7.71

4 Joseph typed a 1,200-word essay in 25 minutes. At this rate, determine how many words he can type in 45 minutes.

Measuring in the Plane and Space – 4% of Questions

Compositions of Polygons and Circles

- The **perimeter** is the distance around an enclosed geometric figure.
  - The perimeter of a polygon is found by adding together the lengths of all of its sides.
  - The perimeter of a circle is called the **circumference**.
    - **Circumference Formulas** for Circles
      - $C = \pi \cdot d$.
      - $C = 2\pi r$.
- The **area** of a geometric figure is the number of square units enclosed within the geometric figure. There are numerous formulas for finding area:

Area and Volume

Length is one dimensional and is measured in **units**.
Area is two dimensional and is measured in **square units**.
Volume is three dimensional and is measured in **cubic units**.

- An **area formula can be transformed into a volume formula** by adding a third dimension (*length*) as a multiple of the area formula.

Area Formulas – Two Dimensional	Volume Formulas – Three Dimensional
Triangles $A = \frac{1}{2}bh$	**Triangular Prisms** $V = \left(\frac{1}{2}bh\right)length$
Squares and Rectangles: $A = bh$	**Cubes and Rectangular Prisms** $V = (bh)length$
Trapezoids: $A = \frac{1}{2}(b_1 + b_2)h$	**Trapezoidal Prisms** $V = \left(\frac{1}{2}(b_1 + b_2)h\right)length$
Circles $A = \pi r^2$	**Cylinders** $V = (\pi r^2)length$

How to find the area or perimeter of a composition of polygons and circles:
Step 1. Divide the composition into its component parts.
Step 2. Solve each component part separately
Step 3. Add or subtract information from the component parts to solve the problem.

Surface Area

Surface Area Formulas

Rectangular Prism Surface Area

$$SA = 2lw + 2hw + 2lh$$

Cylinder Surface Area

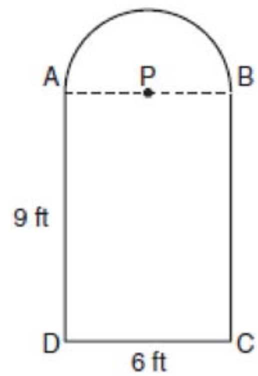
$$SA = 2\pi r^2 + 2\pi rh$$

NOTE: Both of these formulas appear on the formula page in every Integrated Algebra Regents Exam.

MEASURING IN THE PLANE AND SPACE

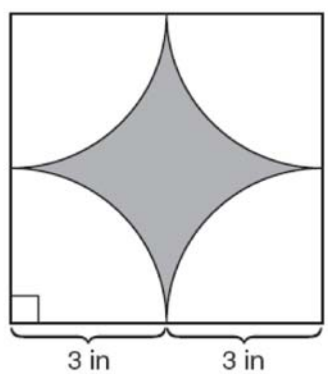
4% - MEASURING IN THE PLANE AND SPACE

1 Serena's garden is a rectangle joined with a semicircle, as shown in the diagram below. Line segment AB is the diameter of semicircle P. Serena wants to put a fence around her garden.



Calculate the length of fence Serena needs to the *nearest tenth of a foot*.

2 A designer created the logo shown below. The logo consists of a square and four quarter-circles of equal size.



Express, in terms of π, the exact area, in square inches, of the shaded region.

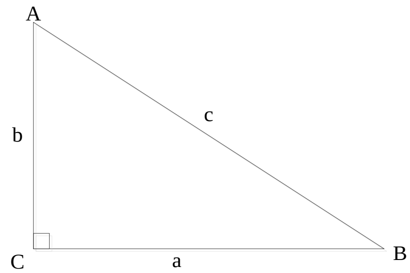
3 Lenny made a cube in technology class. Each edge measured 1.5 cm. What is the volume of the cube in cubic centimeters?

1) 2.25
2) 3.375
3) 9.0
4) 13.5

4 How many cubes with 5-inch sides will completely fill a cube that is 10 inches on a side?

1) 50
2) 25
3) 8
4) 4

Trigonometry – 4%



Basic trigonometry uses "**trig ratios**" to describe the relationships that exist between the two acute angles and the sides of a right triangle.

SOH-CAH-TOA is useful for remembering the trig ratios.

SOH	$s = \frac{o}{h}$	$\text{sin of angle} = \frac{opp}{hyp}$
CAH	$c = \frac{a}{h}$	$\text{cos of angle} = \frac{adj}{hyp}$
TOA	$t = \frac{o}{a}$	$\text{tan of angle} = \frac{opp}{adj}$

How to Find a Basic Trig Ratio:

- Step 1. Using a sketch or diagram of the right triangle, identify the angle associated with the trig ratio.
- Step 2. Write the formula for the trig ratio (*using SOH-CAH-TOA as a memory aid, if necessary*).
- Step 3. Identify the measures of the two sides that will be used in the trig ratio.
- Step 4. Substitute the measures into the formula.

Using SOH-CAH-TOA to Find a Side

- Step 1. Sketch a right triangle to represent the problem and determine:
  - the two parts that are known, and
  - the third part that needs to be found.

  NOTE: Every Regents problem provides two parts of a right triangle and asks you to find a third part.
- Step 2. Select the correct trig ratio
- Step 3. Substitute the values from the problem into the formula.
- Step 4. Use a scientific or graphing calculator to convert between trig ratios and angle measurements. *NOTE: Be sure to set graphing calculators to "degree" mode.*

Using SOH-CAH-TOA to Find an Angle

- Step 1. Sketch a right triangle to represent the problem and determine:
  - the two parts that are known, and
  - the third part that needs to be found.

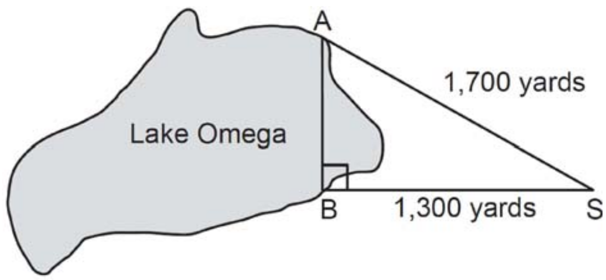
  NOTE: Every Regents problem provides two parts of a right triangle and asks you to find a third part.
- Step 2. Select the correct trig ratio
- Step 3. Substitute the values from the problem into the formula.
- Step 4. Use an **inverse trig function** in a scientific or graphing calculator to convert between trig ratios and angle measurements. *NOTE: Be sure to set graphing calculators to "degree" mode.*

ANSWERS -See page 90

TRIGONOMETRY

4% - TRIGONOMETRY

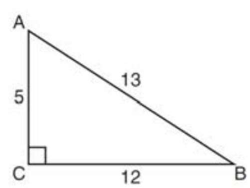
1 Campsite *A* and campsite *B* are located directly opposite each other on the shores of Lake Omega, as shown in the diagram below. The two campsites form a right triangle with Sam's position, *S*. The distance from campsite *B* to Sam's position is 1,300 yards, and campsite *A* is 1,700 yards from his position.



What is the distance from campsite *A* to campsite *B*, to the *nearest yard*?

1) 1,095
2) 1,096
3) 2,140
4) 2,141

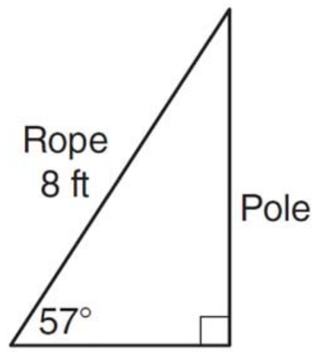
2 The diagram below shows right triangle *ABC*.



Which ratio represents the tangent of $\angle ABC$?

1) $\frac{5}{13}$
2) $\frac{5}{12}$
3) $\frac{12}{13}$
4) $\frac{12}{5}$

3 An 8-foot rope is tied from the top of a pole to a stake in the ground, as shown in the diagram below.



If the rope forms a 57° angle with the ground, what is the height of the pole, to the *nearest tenth of a foot*?

1) 4.4
2) 6.7
3) 9.5
4) 12.3

4 A man standing on level ground is 1000 feet away from the base of a 350-foot-tall building. Find, to the *nearest degree*, the measure of the angle of elevation to the top of the building from the point on the ground where the man is standing.

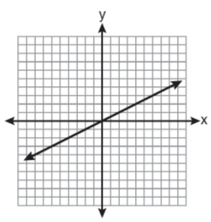
Functions – 3%

Defining Functions

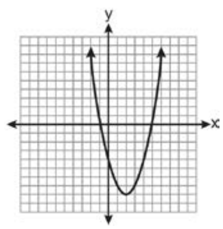
<u>Definition of a function</u>: a function takes the input value of an independent variable and pairs it with one and only one output value of a dependent variable.

- This means that **for every value of *x*, there is <u>one and only one</u> value of *y***.
    - **<u>Example</u>**:
      The set of points $\{(-1,6)(1,3)(2,5)(1,7)\}$ cannot be a function because:
        - when *x* has a value of 1, there are two different values of *y*, which are 3 and 7.
    - **<u>Vertical Line Test</u>**.
        - If you move a vertical line from left to right across the graph of a **<u>function</u>**, the vertical line will never intersect more than **<u>one</u>** point of the graph.
        - If you move a vertical line from left to right across the graph of a **<u>non-function</u>**, at certain values of x, the vertical line will intersect the graph at **<u>two</u>** or more points.

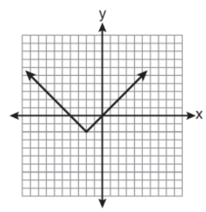
Families of Functions



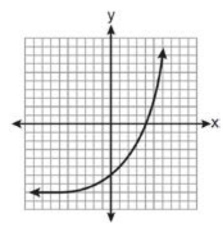
<u>Linear Functions</u>
...look like straight lines that are not vertical.



<u>Quadratic Functions</u>
...look like parabolas that open up or down.

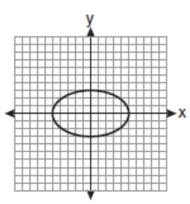


<u>Absolute Value Functions</u>
...look like v-shapes.

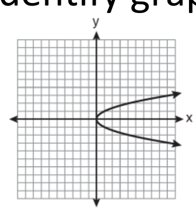


<u>Exponential Functions</u>
...look like one-sided curves.

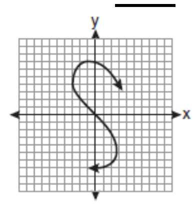
How to identify graphs that are **<u>not</u>** functions



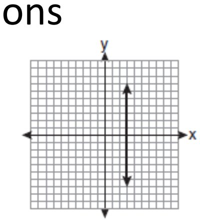
Circles and Ellipses
...are **<u>not functions</u>**.



Parabola-like graphs that open to the side
...are **<u>not functions</u>**.



S-Curves
...are **<u>not functions</u>**.



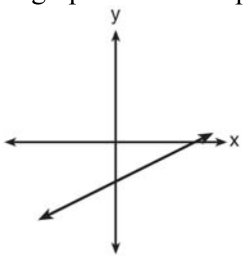
Vertical lines
...are **<u>not functions</u>**.

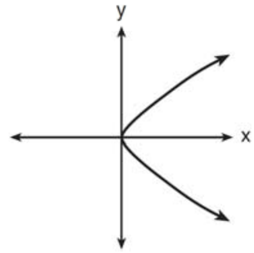
ANSWERS - See page 91

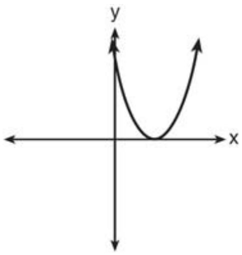
FUNCTIONS

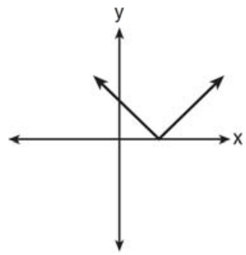
3% - FUNCTIONS - 3%

1 Which graph does *not* represent the graph of a function?

1) 

3) 

2) 

4) 

2 Which relation is a function?

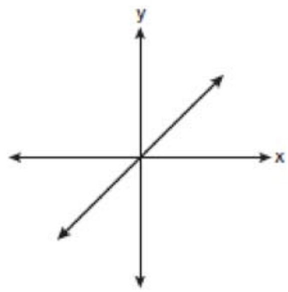
1) $\left\{\left(\frac{3}{4},0\right),(0,1),\left(\frac{3}{4},2\right)\right\}$

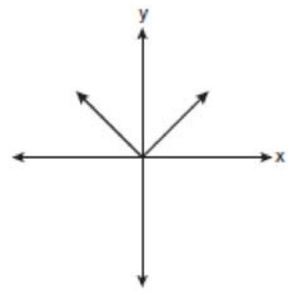
2) $\left\{(-2,2),\left(-\frac{1}{2},1\right),(-2,4)\right\}$

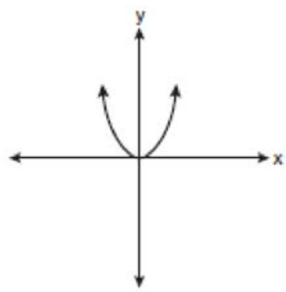
3) $\{(-1,4),(0,5),(0,4)\}$

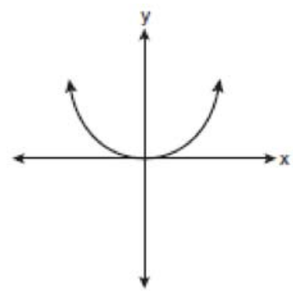
4) $\{(2,1),(4,3),(6,5)\}$

3 Which graph represents a linear function?

1) 

3) 

2) 

4) 

Radicals – 2% of Questions

Simplifying Radicals

A radical expression is in **simplest form** when the following 3 conditions are met:

- It has **no perfect square factors** other than 1. **Perfect squares** are the squares of whole numbers, as shown in the following tables:

Whole Numbers	Perfect Square Numbers
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

- It contains **no fractions** in the numerator or denominator, and
- It contains **no radical sign in the denominator** when expressed as a fraction.

Example: Express $3\sqrt{75}$ in simplest form.

Solution:

$3\sqrt{75}$

$3\sqrt{25}\sqrt{3}$

$3\times 5\sqrt{3}$

$15\sqrt{3}$

In simplest form, $3\sqrt{75} = 15\sqrt{3}$.

Check using graphing calculator:

$3\sqrt{75} = 25.98076211...$

$15\sqrt{3} = 25.98076211...$

Radicals can be added, subtracted, multiplied, and divided.

- A **radical<u></u>** is the number or value inside the radical sign.
  - **Like radicals** have **identical radicands** and can be combined through addition and subtraction.
  - **Unlike radicals** have **different radicands** and cannot be combined through addition and subtraction.

Addition of Radicals

To add radicals with identical radicands, think of them as like terms and combine their coefficients.

Examples:

$$3\sqrt{2} + 5\sqrt{2} = (3+5)\sqrt{2} = 8\sqrt{2}$$

$3\sqrt{2} + 5\sqrt{3}$ have different radicands and cannot be combined.

Subtraction of Radicals

To subtract radicals with identical radicands, think of them as like terms and combine their coefficients.

Example:

$$3\sqrt{2} - 5\sqrt{2} = (3-5)\sqrt{2} = -2\sqrt{2}$$

Multiplication of Radicals

For all whole numbers, $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

Example:

$$\sqrt{2}\times\sqrt{7}=\sqrt{14}$$

For all positive values of *a* and *b*, and all positive or negative values of *x* and *y*,

$$\left(x\sqrt{a}\right)\times\left(y\sqrt{b}\right)=xy\sqrt{ab}$$

Example:

$$\left(3\sqrt{2}\right)\times\left(5\sqrt{7}\right)=\left(3\times5\right)\times\left(\sqrt{2}\times\sqrt{7}\right)=15\sqrt{14}$$

Division of Radicals

For all whole numbers, $\dfrac{\sqrt{a}}{\sqrt{b}}=\sqrt{\dfrac{a}{b}}$

Example:

$$\frac{\sqrt{2}}{\sqrt{3}}=\sqrt{\frac{2}{3}}$$

For all positive values of *a* and *b*, and all positive or negative values of *x* and *y*,

$$\frac{x\sqrt{a}}{y\sqrt{b}}=\frac{x}{y}\times\frac{\sqrt{a}}{\sqrt{b}}=\frac{x}{y}\sqrt{\frac{a}{b}}$$

Example:

$$\frac{3\sqrt{2}}{5\sqrt{7}}=\frac{3}{5}\times\frac{\sqrt{2}}{\sqrt{7}}=\frac{3}{5}\sqrt{\frac{2}{7}}$$

Squaring of Radicals. When a radical is squared, the radical sign is eliminated and the radicand "comes out of the doghouse."

Example:

$$\left(\sqrt{3}\right)^2=\sqrt{3}\times\sqrt{3}=3$$

Rationalizing the Denominator

No rational expression should have a radical in the denominator.

- How to **rationalize the denominator**.
  Step 1. "Build a 1" using the radical in the denominator as both numerator and denominator.
  Step 2. Multiplying the original rational expression by 1 you built.

Examples:

- Given $\dfrac{1}{\sqrt{2}}$, build $\dfrac{\sqrt{2}}{\sqrt{2}}$, then multiply $\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right)=\dfrac{\sqrt{2}}{2}$
- Given $\dfrac{5}{\sqrt{3}}$, build $\dfrac{\sqrt{3}}{\sqrt{3}}$, then multiply $\dfrac{5}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right)=\dfrac{5\sqrt{3}}{3}$

ANSWERS - See page 92

RADICALS

4% - RADICALS

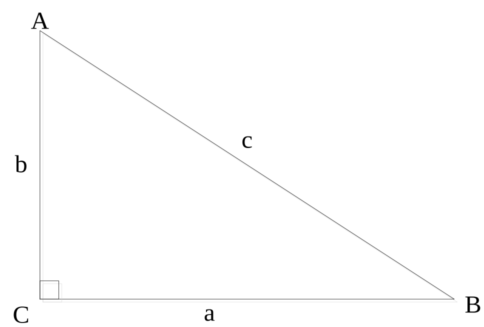
1 Express $\frac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.

2 Express $5\sqrt{72}$ in simplest radical form.

Triangles – 2%

Every right triangle has two legs and one hypotenuse.

- The **hypotenuse** of a right triangle is
  - always the longest side of a right triangle, and
  - always opposite the right angle.
- The **legs** of a right triangle are
  - the two sides of the triangle that form the right angle.



Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs of the triangle equals the square of the length of the hypotenuse.

Algebraically, the Pythagorean Theorem can be written as:

$$a^2 + b^2 = c^2$$

(where a and b are the lengths of the legs and c is the length of the hypotenuse)

By transforming the formula, you can use the length of the hypotenuse and the length of one side of a right triangle to find the length of the missing side, as follows:

$$a^2 = c^2 - b^2 \ \ and \ \ a = \sqrt{c^2 - b^2}$$

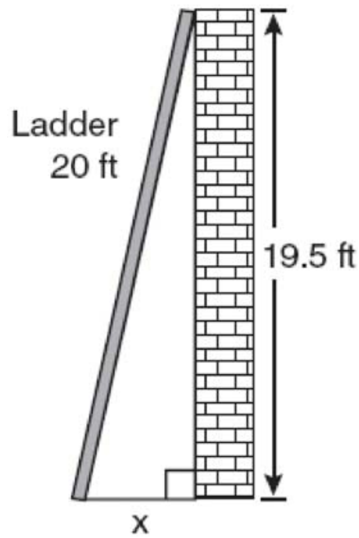
$$b^2 = c^2 - a^2 \ \ and \ \ b = \sqrt{c^2 - a^2}$$

ANSWERS - See page 93

TRIANGLES

2% - TRIANGLES

1 Don placed a ladder against the side of his house as shown in the diagram below.



Which equation could be used to find the distance, x, from the foot of the ladder to the base of the house?

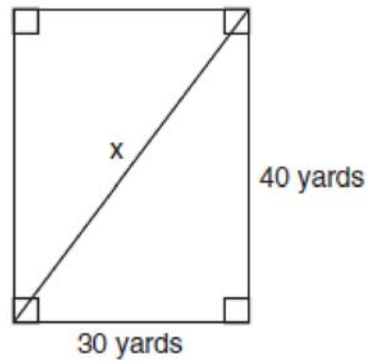
1) $x = 20 - 19.5$

2) $x = 20^2 - 19.5^2$

3) $x = \sqrt{20^2 - 19.5^2}$

4) $x = \sqrt{20^2 + 19.5^2}$

2 Tanya runs diagonally across a rectangular field that has a length of 40 yards and a width of 30 yards, as shown in the diagram below.



What is the length of the diagonal, in yards, that Tanya runs?

1) 50	3) 70
2) 60	4) 80

Bonus Problem – 150 Years of Regents Examinations

The New York Regents examination system was created by act of the New York State legislature during the U.S. Civil War, and the first Regents examination in Mathematics was administered in November, 1866. Since then, Regents examinations have been administered every year in public schools throughout the State of New York.

Questions from old Regents examinations in mathematics provide a unique window into the mathematics taught in New York's public schools during the last 150 years. The study of old questions can also help us to understand what everyday life was like in the past.

The bonus problem on the next page is from the year 1909, a time when very few things in New York moved faster than a horse. There were no cell phones, no graphing calculators, no computers, and no cars. As you solve the problem, think about how much things have changed since this Regents mathematics problem was administered to high school students in 1909.

You can find almost every Regents mathematics examination that has been administered in the State of New York at http://www.jmap.org/JMAP_REGENTS_EXAM_ARCHIVES.htm. If your parents, grand-parents, or great-grand-parents graduated from a New York public high school with a Regents diploma, you can probably find the Regents examination that they took when they were in high school.

ANSWER - See page 94

BONUS PROBLEM

0% - BONUS PROBLEM
- from a Regents Exam given to students in 1909

1

A man agreed to work for a farmer a year and to receive as wages $320 and a cow: at the end of nine months he was discharged and given $238 and the cow. Find the value of the cow. Give written analysis.

GRAPHS AND STATISTICS
Answer Section

1 ANS: 3 REF: 081017a TOP: Analysis of Data

2 ANS: 3
$25 - 18 = 7$

REF: 060822ia TOP: Frequency Histograms, Bar Graphs and Tables

3 ANS: 3
The other situations are quantitative.

REF: 060819ia TOP: Analysis of Data

4 ANS: 3 REF: 061303ia TOP: Scatter Plots

5 ANS: 2
To determine student opinion, survey the widest range of students.

REF: 011313ia TOP: Analysis of Data

6 ANS: 2 REF: fall0701ia TOP: Scatter Plots

7 ANS: 3 REF: 061011ia TOP: Analysis of Data

8 ANS: 4 REF: 060805ia TOP: Scatter Plots

9 ANS:
225000, 175000, the median better represents the value since it is closer to more values than the mean.

REF: fall0737ia TOP: Frequency Histograms, Bar Graphs and Tables

10 ANS: 4

REF: 080822ia TOP: Scatter Plots

11 ANS: 3 REF: 011408ia TOP: Box-and-Whisker Plots

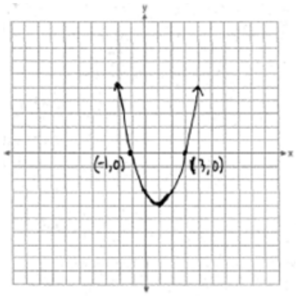
12 ANS: 2 REF: 081327ia TOP: Central Tendency

13 ANS: 2
The volume of the cube using Ezra's measurements is 8 (2^3). The actual volume is 9.261 (2.1^3). The relative error is $\left|\dfrac{9.261 - 8}{9.261}\right| \approx 0.14$.

REF: 060928ia TOP: Error

QUADRATICS
Answer Section

1 ANS:



REF: 060836ia TOP: Solving Quadratics by Graphing

2 ANS: 1 REF: 081015ia TOP: Graphing Quadratic Functions

3 ANS: 3

$x = \frac{-b}{2a} = \frac{-24}{2(-2)} = 6.$ $y = -2(6)^2 + 24(6) - 100 = -28$

REF: 061214ia TOP: Identifying the Vertex of a Quadratic Given Equation

4 ANS: 4 REF: 081322ia TOP: Identifying the Vertex of a Quadratic Given Graph

5 ANS: 4

$$x^2 - 7x + 6 = 0$$
$$(x-6)(x-1) = 0$$
$$x = 6 \quad x = 1$$

REF: 060902ia TOP: Roots of Quadratics

6 ANS: 3

$$x^2 - 6x = 0$$
$$x(x-6) = 0$$
$$x = 0 \quad x = 6$$

REF: 080921ia TOP: Solving Quadratics by Factoring

7 ANS: 2

$$l(l-5) = 24$$
$$l^2 - 5l - 24 = 0$$
$$(l-8)(l+3) = 0$$
$$l = 8$$

REF: 080817ia TOP: Geometric Applications of Quadratics

8 ANS:
$5x^3 - 20x^2 - 60x$

$5x(x^2 - 4x - 12)$

$5x(x + 2)(x - 6)$

REF: 011332ia TOP: Factoring Polynomials

9 ANS:
$4x(x + 3)(x - 3)$. $4x^3 - 36x = 4x(x^2 - 9) = 4x(x + 3)(x - 3)$

REF: 060932ia TOP: Factoring the Difference of Perfect Squares

EXPRESSIONS AND EQUATIONS
Answer Section

1 ANS: 1

$$-|a-b| = -\left|7-(-3)\right| = -|-10| = -10$$

REF: 011010ia TOP: Evaluating Expressions

2 ANS:

Not all of the homework problems are equations. The first problem is an expression.

REF: 080931ia TOP: Expressions

3 ANS: 4 REF: 061016ia TOP: Expressions

4 ANS: 3 REF: 061323ia TOP: Expressions

5 ANS: 2 REF: 080901ia TOP: Modeling Equations

6 ANS: 4

$$5-2x=-4x-7$$
$$2x=-12$$
$$x=-6$$

REF: 011305ia TOP: Solving Equations

7 ANS: 3

$$3ax+b=c$$
$$3ax=c-b$$
$$x=\frac{c-b}{3a}$$

REF: 080808ia TOP: Transforming Formulas

8 ANS: 2

$$P=2l+2w$$
$$P-2l=2w$$
$$\frac{P-2l}{2}=w$$

REF: 010911ia TOP: Transforming Formulas

POWERS
Answer Section

1 ANS: 4

$-3x(x-4)-2x(x+3)=-3x^2+12x-2x^2-6x=-5x^2+6x$

REF: 081114ia TOP: Addition and Subtraction of Monomials

2 ANS: 1 REF: 011213ia TOP: Addition and Subtraction of Polynomials

3 ANS: 2 REF: 011316ia TOP: Division of Polynomials

4 ANS: 3

$$\frac{(2x^3)(8x^5)}{4x^6}=\frac{16x^8}{4x^6}=4x^2$$

REF: fall0703ia TOP: Division of Powers

5 ANS: 3

$$\frac{3^6}{3^1}=3^5$$

REF: 061219ia TOP: Division of Powers

6 ANS: 3

$35000(1-0.05)^4\approx 28507.72$

REF: fall0719ia TOP: Exponential Functions

7 ANS: 2 REF: fall0725ia TOP: Operations with Scientific Notation

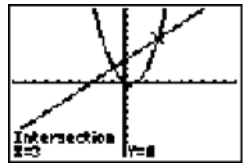
8 ANS: 4

$$\frac{25x-125}{x^2-25}=\frac{25(x-5)}{(x+5)(x-5)}=\frac{25}{x+5}$$

REF: 080821ia TOP: Rational Expressions

SYSTEMS of EQUATIONS and INEQUALITIES
Answer Section

1 ANS: 2



$x^2 - x = x + 3$. Since $y = x + 3$, the solutions are $(3, 6)$ and $(-1, 2)$.

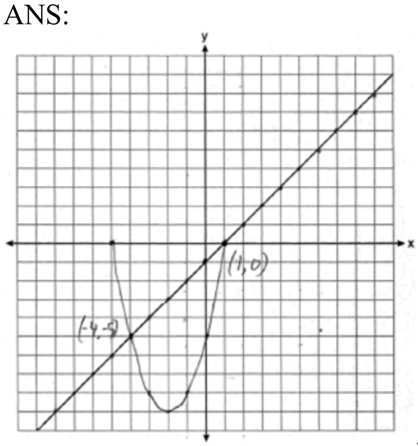
$x^2 - 2x - 3 = 0$

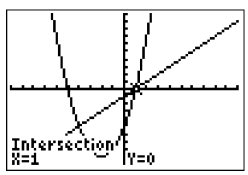
$(x - 3)(x + 1) = 0$

$x = 3 \text{ or } -1$

REF: 061118ia TOP: Quadratic-Linear Systems

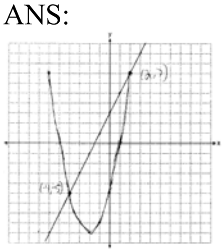
2 ANS:





REF: 080839ia TOP: Quadratic-Linear Systems

3 ANS:



REF: 011437ia TOP: Quadratic-Linear Systems

4 ANS: 3

$5x + 2y = 48$

$3x + 2y = 32$

$2x = 16$

$x = 8$

REF: fall0708ia TOP: Solving Linear Systems

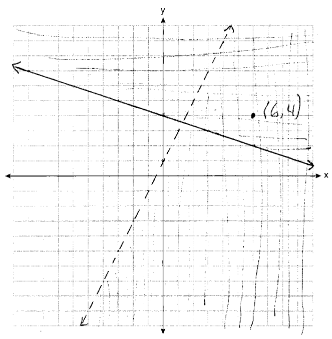
5 ANS: 2

$$3c + 4m = 12.50$$
$$3c + 2m = 8.50$$
$$2m = 4.00$$
$$m = 2.00$$

REF: 060806ia TOP: Writing Linear Systems
6 ANS: 4 REF: 061222ia TOP: Systems of Linear Inequalities
7 ANS: 2

The slope of the inequality is $-\frac{1}{2}$.

REF: fall0720ia TOP: Linear Inequalities
8 ANS:



REF: 081037ia TOP: Systems of Linear Inequalities

RATIONALS
Answer Section

1 ANS: 4

$$\frac{(d\times 3)+(2\times 2d)}{2\times 3}=\frac{3d+4d}{6}=\frac{7d}{6}$$

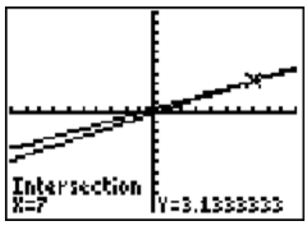
REF: fall0727ia TOP: Addition and Subtraction of Rationals

2 ANS:

$$\frac{x+2}{2}\times\frac{4(x+5)}{(x+4)(x+2)}=\frac{2(x+5)}{x+4}$$

REF: 081232ia TOP: Multiplication and Division of Rationals

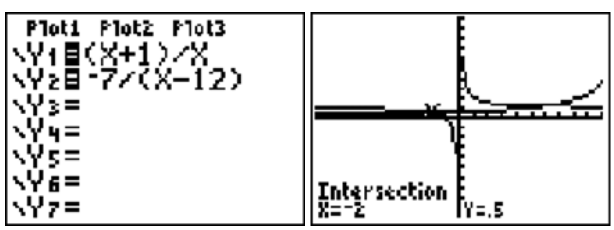
3 ANS: 4



$$\frac{2x}{5}+\frac{1}{3}=\frac{7x-2}{15}$$
$$\frac{(2x\times 3)+(5\times 1)}{5\times 3}=\frac{7x-2}{15}$$
$$\frac{6x+5}{15}=\frac{7x-2}{15}$$
$$6x+5=7x-2$$
$$x=7$$

REF: 080820ia TOP: Solving Equations with Fractional Expressions

4 ANS:



$6, -2.$ $\frac{x+1}{x}=\frac{-7}{x-12}$

$$(x+1)(x-12)=-7x$$
$$x^2-11x-12=-7x$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
$$x=6 \text{ or } -2$$

REF: fall0739ia TOP: Solving Rationals

5 ANS: 1 REF: fall0728ia TOP: Undefined Rationals

6 ANS: 1

$$\frac{2}{x}-3=\frac{26}{x}$$

$$-3=\frac{24}{x}$$

$$x=-8$$

REF: 010918ia TOP: Solving Rationals

7 ANS: 2

$$\frac{2x^2-12x}{x-6}=\frac{2x(x-6)}{x-6}=2x$$

REF: 060824ia TOP: Rational Expressions

8 ANS:

$$\frac{3}{4x-8}\cdot\frac{3x+6}{4x+12}\div\frac{x^2-4}{x+3}=\frac{3(x+2)}{4(x+3)}\cdot\frac{x+3}{(x+2)(x-2)}=\frac{3}{4(x-2)}$$

REF: 010935ia TOP: Multiplication and Division of Rationals

PROBABILITY
Answer Section

1 ANS:
$\frac{1}{8}$. After the English and social studies books are taken, 8 books are left and 1 is an English book.

REF: 060933ia TOP: Conditional Probability

2 ANS: 4
$P(O) = \frac{3}{6}, P(E) = \frac{3}{6}, P(< 6) = \frac{5}{6}, P(> 4) = \frac{2}{6}$

REF: 010903ia TOP: Theoretical Probability

3 ANS: 2 REF: 011415ia TOP: Experimental Probability

4 ANS:
$\frac{1375}{1600}$. $\frac{40^2 - 15^2}{40^2} = \frac{1375}{1600}$

REF: 011132ia TOP: Geometric Probability

5 ANS:
$26 \times 25 \times 24 \times 23 = 358,800$. $10^6 = 1,000,000$. Use the numeric password since there are over 500,000 employees

REF: 061239ia TOP: Permutations

6 ANS:
(H,F,M), (H,F,J), (H,F,S), (H,A,M), (H,A,J), (H,A,S), (C,F,M), (C,F,J), (C,F,S), (C,A,M), (C,A,J), (C,A,S), (T,F,M), (T,F,J), (T,F,S), (T,A,M), (T,A,J), (T,A,S). There are 18 different kids' meals, 12 do not include juice and 6 include chicken nuggets.

REF: 010939ia TOP: Sample Space

7 ANS: 3
$P(S) \cdot P(M) = P(S \text{ and } M)$

$\frac{3}{5} \cdot P(M) = \frac{3}{10}$

$P(M) = \frac{1}{2}$

REF: 081024ia TOP: Theoretical Probability

NUMBERS, OPERATIONS AND PROPERTIES
Answer Section

1 ANS: 4 REF: 081022ia TOP: Set Theory

2 ANS: 4 REF: 060930ia TOP: Set Theory

3 ANS:
$0 \le t \le 40$

REF: 060833ia TOP: Set Theory

4 ANS: 2 REF: 011119ia TOP: Set Theory

5 ANS: 4 REF: 011426ia TOP: Set Theory

6 ANS: 2
The set of integers greater than -2 and less than 6 is $\{-1,0,1,2,3,4,5\}$. The subset of this set that is the positive factors of 5 is $\{1,5\}$. The complement of this subset is $\{-1,0,2,3,4\}$.

REF: 060818ia TOP: Set Theory

7 ANS: 4 REF: 011114ia TOP: Properties of Reals

LINEAR EQUATIONS
Answer Section

1 ANS: 2 REF: 080823ia TOP: Slope

2 ANS: 4

$m = \frac{-A}{B} = \frac{-4}{3}$

REF: 061319ia TOP: Slope

3 ANS: 1

The slope of $2x - 4y = 16$ is $\frac{-A}{B} = \frac{-2}{-4} = \frac{1}{2}$

REF: 011026ia TOP: Parallel and Perpendicular Lines

4 ANS: 4

$$\begin{aligned} 2x - 3y &= 9 \\ 2(0) - 3(-3) &= 9 \\ 0 + 9 &= 9 \end{aligned}$$

REF: 081016ia TOP: Identifying Points on a Line

5 ANS: 2

$m = \frac{5-2}{3-(-2)} = \frac{3}{5}$

REF: 061004ia TOP: Slope

6 ANS: 1

$m = \frac{3-0}{0-2} = -\frac{3}{2}$. Using the given y-intercept $(0,3)$ to write the equation of the line $y = -\frac{3}{2}x + 3$.

REF: fall0713ia TOP: Writing Linear Equations

INEQUALITIES
Answer Section

1 ANS: 3 REF: 081317ia TOP: Interpreting Solutions
2 ANS: 4 REF: fall0715ia TOP: Modeling Inequalities
3 ANS:
7. $15x + 22 \geq 120$

$x \geq 6.5\overline{3}$

REF: fall0735ia TOP: Modeling Inequalities
4 ANS: 4
$-6x - 17 \geq 8x + 25$

$-42 \geq 14x$

$-3 \geq x$

REF: 081121ia TOP: Solving Inequalities
5 ANS: 4 REF: 061320ia TOP: Linear Inequalities
6 ANS: 1 REF: 080803ia TOP: Modeling Inequalities

RATE
Answer Section

1 ANS: 4

$$\frac{344\text{ m}}{\text{sec}} \times \frac{60\text{ sec}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} = 1,238,400\,\frac{\text{m}}{\text{hr}}$$

REF: 060911ia TOP: Conversions

2 ANS: 4

$$\frac{95000}{125000} = .76$$

REF: 061207ia TOP: Quartiles and Percentiles

3 ANS: 1

$$\frac{12.8+17.2}{3+5} = 3.75$$

REF: 061117ia TOP: Speed

4 ANS:

2,160 $\frac{1,200}{25} = \frac{x}{45}$

$$25x = 54,000$$

$$x = 2,160$$

REF: 081032ia TOP: Using Rate

MEASURING IN THE PLANE AND SPACE
Answer Section

1 ANS:
33.4. Serena needs 24 $(9+6+9)$ feet of fencing to surround the rectangular portion of the garden. The length of the fencing needed for the semicircular portion of the garden is $\frac{1}{2}\pi d = 3\pi \approx 9.4$ feet.

REF: fall0733ia TOP: Compositions of Polygons and Circles

2 ANS:
$36-9\pi$. 15.6. Area of square–area of 4 quarter circles. $(3+3)^2 - 3^2\pi = 36-9\pi$

REF: 060832ia TOP: Compositions of Polygons and Circles

3 ANS: 2
$1.5^3 = 3.375$

REF: 060809ia TOP: Volume

4 ANS: 3
$\frac{10^3}{5^3} = \frac{1000}{125} = 8$

REF: 011312ia TOP: Volume

TRIGONOMETRY
Answer Section

1 ANS: 1

$\sqrt{1700^2 - 1300^2} \approx 1095$

REF: 011221ia TOP: Pythagorean Theorem

2 ANS: 2

$\tan ABC = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$

REF: 081112ia TOP: Trigonometric Ratios

3 ANS: 2

$\sin 57 = \frac{x}{8}$

$x \approx 6.7$

REF: 061108ia TOP: Using Trigonometry to Find a Side

4 ANS:

$\tan x = \frac{350}{1000}$

$x \approx 19$

REF: 061335ia TOP: Using Trigonometry to Find an Angle

FUNCTIONS
Answer Section

1 ANS: 3 REF: 081308ia TOP: Defining Functions

2 ANS: 4
In (4), each element in the domain corresponds to a unique element in the range.

REF: 011105ia TOP: Defining Functions

3 ANS: 1 REF: 060801ia TOP: Families of Functions

RADICALS
Answer Section

1 ANS:
$$\frac{\sqrt{84}}{2\sqrt{3}} = \frac{\sqrt{4}\sqrt{21}}{2\sqrt{3}} = \sqrt{\frac{21}{3}} = \sqrt{7}$$

REF: 011431ia TOP: Operations with Radicals

2 ANS:
$30\sqrt{2}$. $5\sqrt{72} = 5\sqrt{36}\sqrt{2} = 30\sqrt{2}$

REF: fall0731ia TOP: Simplifying Radicals

TRIANGLES
Answer Section

1 ANS: 3 REF: 060825ia TOP: Pythagorean Theorem

2 ANS: 1

$30^2 + 40^2 = c^2$. 30, 40, 50 is a multiple of 3, 4, 5.

$2500 = c^2$

$50 = c$

REF: fall0711ia TOP: Pythagorean Theorem

BONUS PROBLEM
Answer Section

1 ANS:
$8.00.

9 months is $\frac{3}{4}$ of a year. The man would have earned $240 plus $\frac{3}{4}$ of the cow after working 9 months. Instead, he received the whole cow and his pay was reduced by $2. This means that the unearned $\frac{1}{4}$ of the cow's value equals $2. If $\frac{1}{4}$ of the cow was worth $2, the whole cow was worth four times as much, which is $8.