(e-book edition v.245)

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to additional resources at www.jmap.org.

JMAP
Study Guide
for
Algebra I (Common Core)

About this Study Guide

The resources contained in this study guide are suitable for use in any state with mathematics curricula aligned to the Common Core. This study guide contains every question (as of January 2016) used by the New York State Education Department (NYSED) to assess high school students on the Algebra I (Common Core) Regents mathematics curriculum.

Teachers and students are welcome to copy and use this study guide and other JMAP resources for individual and classroom use. Lesson plans associated with this study guide are available at no cost in manipulable docx format on the JMAP website. Because of the newness of the Common Core curriculum and the lack of test questions available for constructing assessment based lessons, it may be necessary to supplement this study guide with additional resources that are available at no cost at www.jmap.org.

New York State Regents examination problems are used with every lesson. Because of the newness of the Common Core curriculum and the lack of test questions available for constructing assessment based lessons, it may be necessary to supplement the problem sets in these lessons with additional materials if the standard has not been assessed frequently. The Common Core State Standard at beginning of each topic contains a hyperlink to additional resources. You can also Google any common core standard and the word JMAP for additional resources.

Example: A.REI.1 JMAP

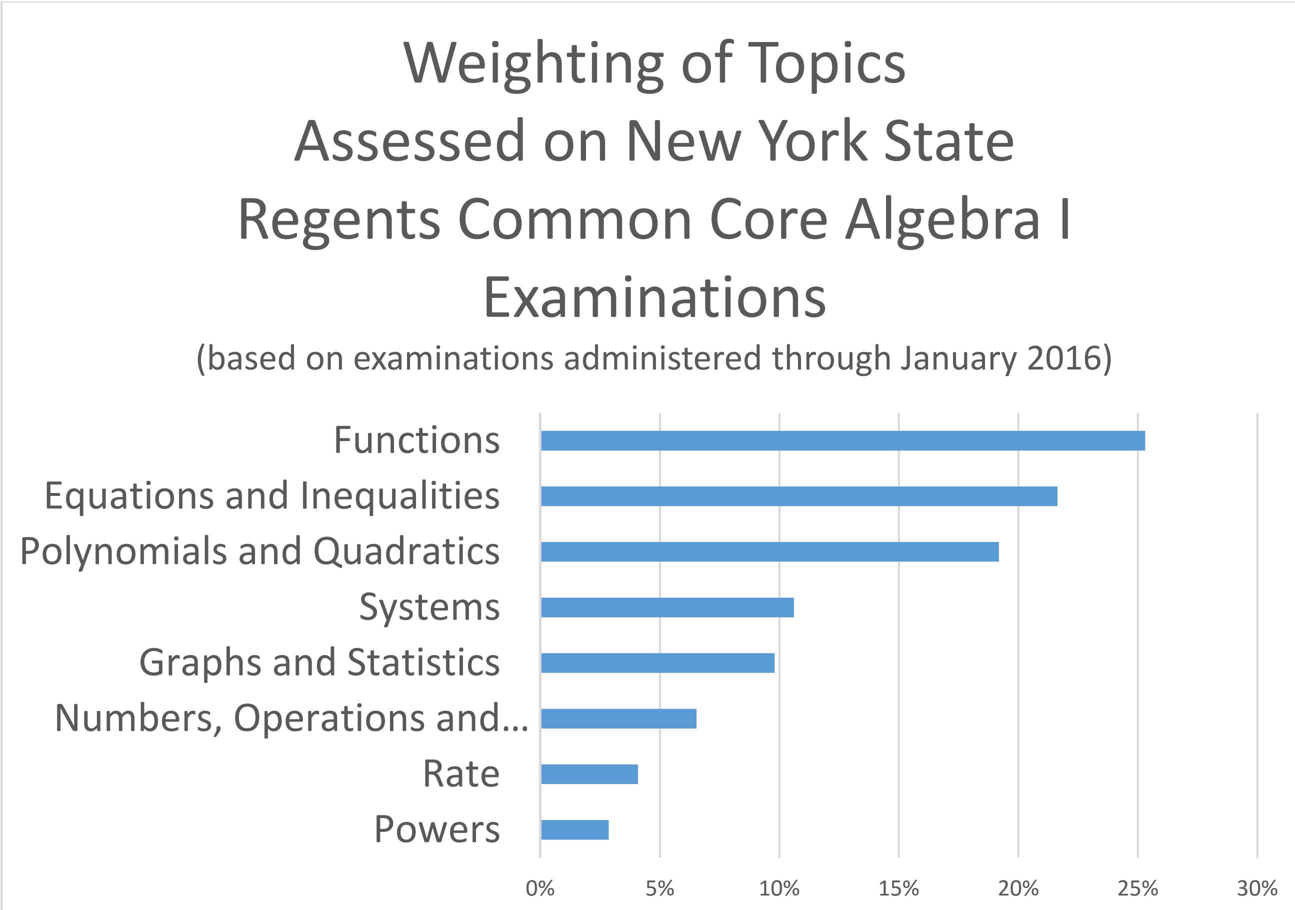
Each problem set focuses on problem solving skills using an interpretation of George Polya's universal algorithm for problem solving. A free graphic of Polya's algorithm, which is suitable for classroom display, can be downloaded from http://www.jmap.org/images/Polya.jpg. Strategies and step by step solutions are provided for all Regents questions. Other strategies may also be appropriate. Most solutions end with DIMS, an acronym for "Does It Make Sense?"

"Writing the Math" is a pedagogical approach believed to facilitate mathematical understanding and is recommended as a supplemental assignment on every topic. Basically, it involves copying a problem from a Regents examination, then creating and solving a new problem that assess the same mathematical understanding. A template for "Writing the Math" appears together with Polya's algorithm at the end of this study guide.

If you find errors in this text, or if you have a recommendation for improving these resources, please let us know. .

Steve and Steve
www.jmap.org
March, 2016

JMAP is a non-profit initiative working for the benefit of teachers and their students. JMAP provides free resources to New York teachers and receives no state or local government support. If you wish to support JMAP's efforts, please consider making a charitable donation through JMAP's website. While JMAP is not associated with NYSED or the New York City Department of Education (NYCDOE), Steve Sibol (Editor and Publisher) and Steve Watson (Principal and Cofounder) are Brooklyn public high school math teachers. Special appreciation goes to the many math teachers who have shared their ideas about how to improve JMAP.

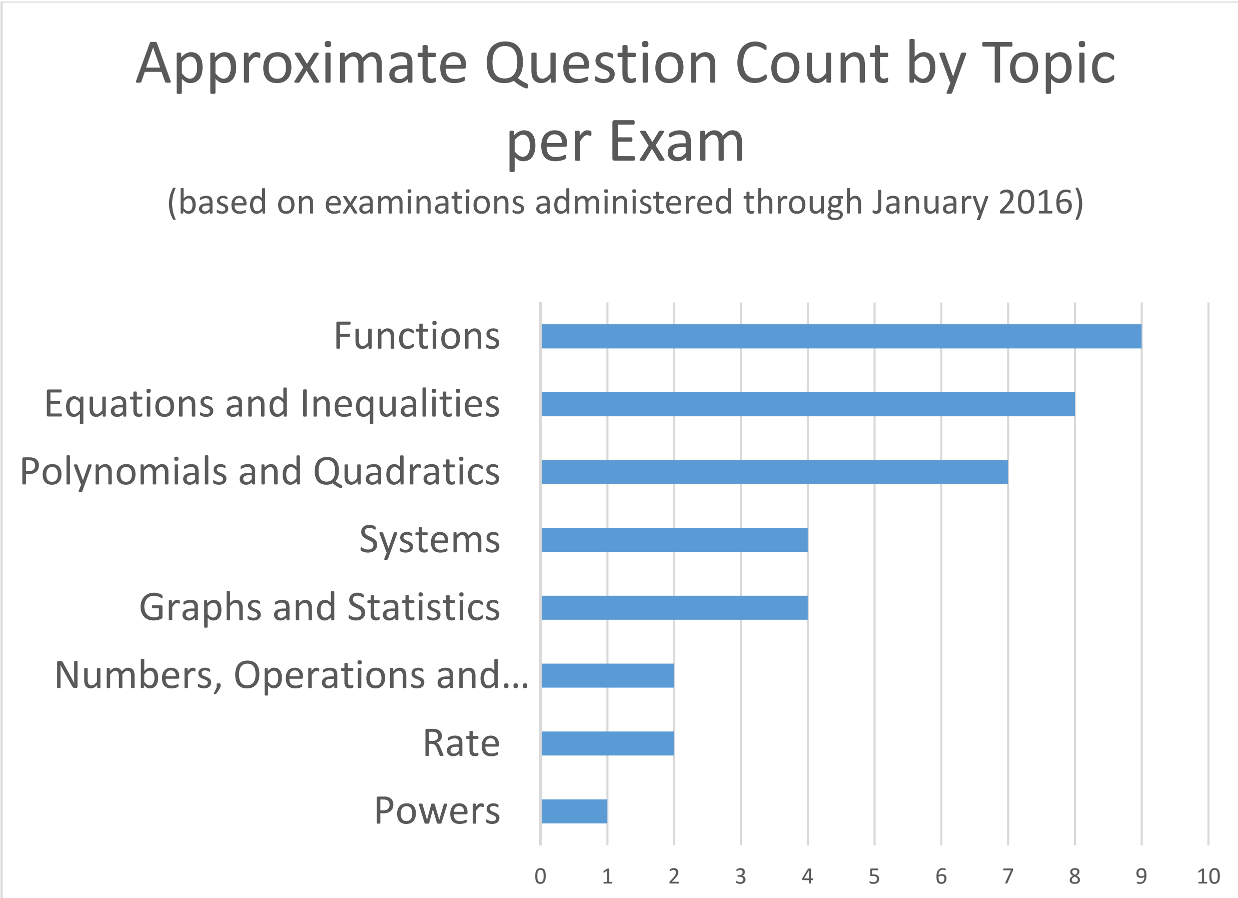


There have been seven Common Core Algebra I Regents examinations, each with 37 problems. Altogether, 245 problems have been used to assess New York's Common Core Algebra I standards.

This chart shows that approximately 25% of the first 245 problems assessed knowledge of functions. A little more than 20% of the problems assessed knowledge of equations and inequalities, and a little less than 20% assessed knowledge of polynomials and quadratics. About 10% of the 245 problems involved systems and another 10% involved graphs and statistics. The lowest three categories of problems combined represent about 15 % of the total.

Past and future assessment practices can reasonably be expected to remain consistent, and the distribution of topics on future examinations can reasonably be expected to reflect assessment norms established during the first seven examinations. If this be true, then the percentages shown in the above chart can be used to provide rough guesses about of the number of questions on specific topics that can be expected on future exams.

The percentages shown in the above chart are used below to estimate the number of problems that will appear on future exams in each category. There are 37 problems on each examination.



Meanings of Abbreviations Used by the Common Core State Standards

A.APR	Algebra – Polynomials and Rational Expressions
A.CED	Algebra - Creating Expressions that Describe
A.REI	Algebra – Reasoning with Equations and Inequalities
A.SSE	Algebra – Seeing Structure in Expressions
F.BF	Functions – Building Functions
F.IF	Functions – Interpreting Functions
F.LE	Functions – Linear, Quadratic and Exponential Models
N.Q	Numbers - Quantity
N.RN	Numbers – Real Numbers
S.ID	Statistics – Interpreting Data

TABLE OF CONTENTS

S.ID.5: Frequency Tables

Summarize, represent, and interpret data on two categorical and quantitative variables.
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S.ID.6a: Linear, Quadratic and Exponential Regression

Summarize, represent, and interpret data on two categorical and quantitative variables
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S.ID.8: Calculate Correlation Coefficients

Interpret linear models
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.6c: Use Residuals to Assess Fit of a Function

Summarize, represent, and interpret data on two categorical and quantitative variables
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
c. Informally assess the fit of a function by plotting and analyzing residuals.

S.ID.9: Correlation and Causation

Interpret linear models
9. Distinguish between correlation and causation.

RATE

F.IF.6: Calculate and Interpret Rate of Change

Interpret functions that arise in applications in terms of the context.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.[?3?]

N.Q.1: Use Units to Solve Problems.

Reason quantitatively and use units to solve problems.
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

POWERS

A.SSE.3c: Use Properties of Exponents to Transform Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.?3?

   a. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t}$?6˜ 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

F.IF.8b: Use Properties of Exponents to Interpret Expressions

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

EQUATIONS AND INEQUALITIES

A.SSE.1: Terms, Factors, & Coefficients of Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.1: Create Equations and Inequalities

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A.CED.2: Graph Equations with Labels and Scales

Create equations that describe numbers or relationships.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.REI.10: Interpret Graphs as Sets of Solutions

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF .4: Relate Graphs to Events

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

A.CED.3: Interpret Solutions

Create equations that describe numbers or relationships.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A.CED.4: Transform Formulas

Create equations that describe numbers or relationships.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

F.LE.5: Interpret Parts of an Expression or Equation

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

FUNCTIONS

F.IF.1: Define Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2: Use Function Notation

Understand the concept of a function and use function notation.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.9: Four Views of a Function

Analyze functions using different representations.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.IF.3: Define Sequences as Functions

Understand the concept of a function and use function notation.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ?5= 1.*

F.BF.l: Model Explicit and Recursive Processes

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.?3?
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.3: Build New Functions from Existing Functions.

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.LE.2: Construct a Function Rule from Other Views of a Function

Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.IF.5: Use Domain and Range

Interpret functions that arise in applications in terms of the context.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*?3?

F.LE.lb: Distinguish between Families of Functions

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.3: Compare Families of Functions

Construct and compare linear, quadratic and exponential models and solve problems.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.IF.7b: Graph Root, Piecewise, Step, & Absolute Value Functions

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.?3?
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

POLYNOMIALS and QUADRATICS

A.APR.1: Arithmetic Operations on Polynomials

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.SSE.2: Factor Polynomials

Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

A.APR.3: Find Zeros of Polynomials

Understand the relationship between zeros and factors of polynomials.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.SSE.3a: Factor Quadratic Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.?3?

a. Factor a quadratic expression to reveal the zeros of the function it defines.

A.REI.4: Use Appropriate Strategies to Solve Quadratics

Solve equations and inequalities in one variable.

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

F.IF.8a: Identify Characteristics of Quadratics by Completing the Square

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

SYSTEMS

A-REI.5: Solve Systems by Elimination

Solve systems of equations.
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6: Solve Linear Systems Algebraically and by Graphing

Solve systems of equations.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.7: Solve Quadratic-Linear Systems

Solve systems of equations.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

A.REI.11: Find and Explain Solutions of Systems

Represent and solve equations and inequalities graphically.
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.REI.12: Graph Systems of Inequalities

Represent and solve equations and inequalities graphically.
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

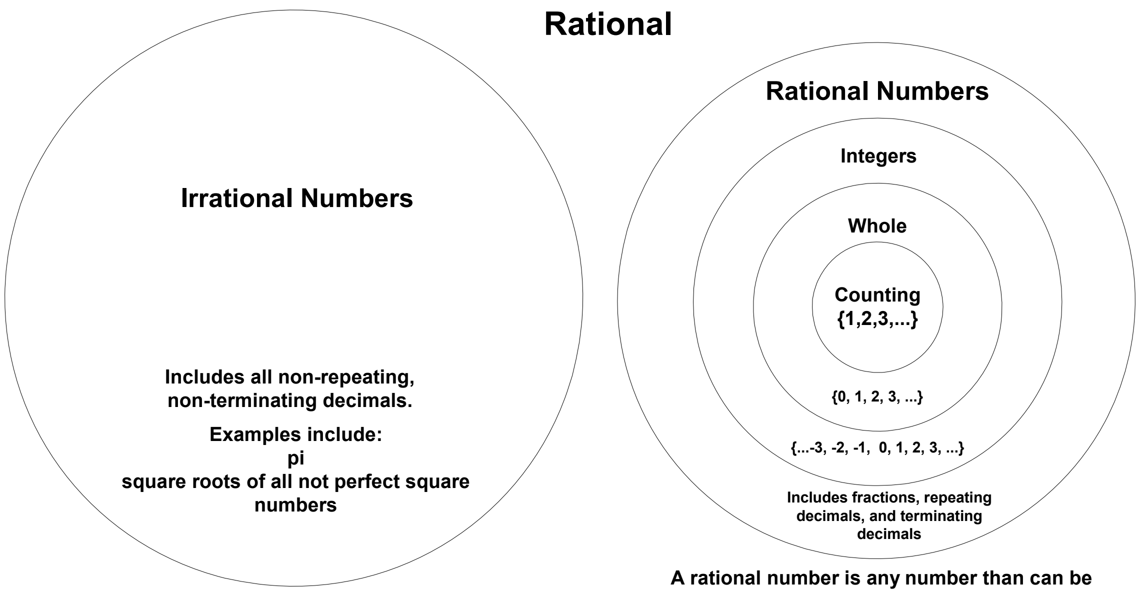
NUMBERS, OPERATIONS, AND PROPERTIES

N.RN.3: Use Properties of Rational and Irrational Numbers

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

The Set of Real Numbers
includes two major classifications of numbers
Irrational and
Rational



An irrational number is any number that cannot be expressed as the ratio of two integers.

A rational number is any number than can be expressed as the ratio of two integers.

<u>Is a Number Irrational or Rational?</u>

<u>Irrational Numbers</u>	<u>Rational Numbers</u>
If a decimal does not repeat or terminate, it is an irrational number. Numbers with names, such a π and e are irrational. They are given names because it is impossible to state their infinitely long values. The square roots of all numbers (that are not perfect squares) are irrational. If a term reduced to simplest form contains an irrational number, the term is irrational. .	If a number is an integer, it is rational, since it can be expressed as a ratio with the integer as the numerator and 1 as the denominator. If a decimal is a repeating decimal, it is a rational number. If a decimal terminates, it is a rational number.

Operations with Irrational and Rational Numbers

Addition and Subtraction:

When two rational numbers are added or subtracted, the result is rational.
When two irrational numbers are added or subtracted, the result is irrational.
When an irrational number and a rational number are added or subtracted, the sum is irrational.

Multiplication and Division:

When two rational numbers are multiplied or divided, the product is rational.
When an irrational number and a non-zero rational number are multiplied or divided, the product is irrational.
When two irrational numbers are multiplied or divided, the product is sometimes rational and sometimes irrational.

Example of Rational Product	Example of Irrational Product
$\sqrt{7}\times\sqrt{28}$ $\sqrt{7}\times\left(\sqrt{4}\times\sqrt{7}\right)$ $\left(\sqrt{7}\sqrt{7}\right)\sqrt{4}$ $7\times 2=14$ $\frac{14}{1}$	$\sqrt{7}\times\sqrt{3}$ $\sqrt{21}$ $4.582575695\ldots\ldots$ NOTE: Be careful using a calculator to decide if a number is irrational. The calculator stops when it runs out of room to display the numbers, and the whole number may continue beyond the calculator display.
Rational Quotient $\frac{\sqrt{20}}{\sqrt{5}}=\sqrt{\frac{20}{5}}=\sqrt{4}=2=\frac{2}{1}$	Irrational Quotient $\frac{\sqrt{10}}{\sqrt{5}}=\sqrt{\frac{10}{5}}=\sqrt{2}$

REGENTS PROBLEMS

1. For which value of P and W is $P+W$ a rational number?

a. $P=\frac{1}{\sqrt{3}}$ and $W=\frac{1}{\sqrt{6}}$

b. $P=\frac{1}{\sqrt{4}}$ and $W=\frac{1}{\sqrt{9}}$

c. $P=\frac{1}{\sqrt{6}}$ and $W=\frac{1}{\sqrt{10}}$

d. $P=\frac{1}{\sqrt{25}}$ and $W=\frac{1}{\sqrt{2}}$

2. Given: $L=\sqrt{2}$

   $M=3\sqrt{3}$

   $N=\sqrt{16}$

   $P=\sqrt{9}$

   Which expression results in a rational number?

   a. $L+M$
   b. $M+N$
   c. $N+P$
   d. $P+L$

3. Which statement is *not* always true?

   a. The product of two irrational numbers is irrational.
   b. The product of two rational numbers is rational.
   c. The sum of two rational numbers is rational.
   d. The sum of a rational number and an irrational number is irrational.

4. Which statement is *not* always true?

   a. The sum of two rational numbers is rational.
   b. The product of two irrational numbers is rational.
   c. The sum of a rational number and an irrational number is irrational.
   d. The product of a nonzero rational number and an irrational number is irrational.

5. Ms. Fox asked her class "Is the sum of 4.2 and $\sqrt{2}$ rational or irrational?" Patrick answered that the sum would be irrational. State whether Patrick is correct or incorrect. Justify your reasoning.

6. Given the following expressions:

   I. $-\frac{5}{8}+\frac{3}{5}$ III. $\left(\sqrt{5}\right)\cdot\left(\sqrt{5}\right)$

   II. $\frac{1}{2}+\sqrt{2}$ IV. $3\cdot\left(\sqrt{49}\right)$

   Which expression(s) result in an irrational number?

   a. II, only
   b. III, only
   c. I, III, IV
   d. II, III, IV

N.RN.3: Use properties of rational and irrational numbers.
Answer Section

1. ANS: B

$\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{9}}=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers. Reject any answer choice that does not contain two rational numbers.

Reject answer choice a because $\frac{1}{\sqrt{3}}$ is irrational.

Choose answer choice b because both $P=\frac{1}{\sqrt{4}}$ and $W=\frac{1}{\sqrt{9}}$ can be expressed as rational numbers, as shown above.

PTS: 2 REF: 081522ai NAT: N.RN.3 TOP: Classifying Numbers

2. ANS: C

$\sqrt{16}+\sqrt{9}=\frac{7}{1}$ may be expressed as the ratio of two integers.

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers.

STEP 1 Determine whether numbers L, M, N, and P are ratiional, then reject any answer choice that does not contain two rational numbers.

$$L=\sqrt{2}\text{ is irrational}$$
$$M=3\sqrt{3}\text{ is irrational}$$
$$N=\sqrt{16}=4\text{ and is rational}$$
$$P=\sqrt{9}\text{=3 and is rational}$$

STEP 2 Reject any answer choice that does not include $N+P$. Choose answer choice c.

PTS: 2 REF: 061413a1 NAT: N.RN.3 TOP: Classifying Numbers

3. ANS: A

Strategy: Find a counterexample to prove one of the answer choices is *not* always true.

Answer choice a is not always true because: $\sqrt{3}$ and $\sqrt{12}$ are both irrational numbers, but $\sqrt{3}\times\sqrt{12}=\sqrt{3\times 12}=\sqrt{36}=6$, and 6 is a rational number, so the product of two irrational numbers is not *always* irrational.

PTS: 2 REF: 081401a1 NAT: N.RN.3 TOP: Classifying Numbers

4. ANS: B
Strategy: Find a counterexample to prove one of the answer choices is *not* always true. This will usually involve the *product* or *quotient* of two irrational numbers since the outcomes of addition and subtraction of irrational numbers are more predictable.

Answer choice b is not always true because: $\sqrt{2}$ and $\sqrt{3}$ are both irrational numbers, but $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$, and $\sqrt{6}$ is an rational number, so the product of two irrational numbers is not *always* rational.

PTS: 2 REF: 061508AI NAT: N.RN.3 TOP: Classifying Numbers

5. ANS:
Patrick is correct. The sum of a rational and irrational is irrational.

Strategy: Determine whether 4.2 and $\sqrt{2}$ are rational or irrational numbers, then apply the rules of operations on rational and irrational numbers.

4.2 is rational because it can be expressed as $\frac{42}{10}$, which is the ratio of two integers.
$\sqrt{2}$ is irrational because it cannot be expressed as the ratio of two integers.

The rules of addition and subtraction of rational and irrational numbers are:
When two rational numbers are added or subtracted, the result is rational.
When two irrational numbers are added or subtracted, the result is irrational.
When an irrational number and a rational number are added or subtracted, the sum is irrational.

PTS: 2 REF: 011525a1 NAT: N.RN.3 TOP: Classifying Numbers

6. ANS: A
Strategy: Eliminate wrong answers.
Expression I results in a rational number because the set of rational numbers is closed under addition.
$-\frac{5}{8} + \frac{3}{5} = \frac{-25}{40} + \frac{24}{40} = \frac{-1}{40}$
Expression II is is correct because the additon of a rational number and an irrational number always results in an irrational number.
$\frac{1}{2} + \sqrt{2} = 0.5 + 1.414203562\ldots = 1.914203562\ldots$
Expression III results in a rational number because $\left(\sqrt{5}\right) \cdot \left(\sqrt{5}\right) = \sqrt{5 \cdot 5} = \sqrt{25} = 5 = \frac{5}{1}$, which is the ratio of two integers.
Expression IV results in a rational number because $3 \cdot \left(\sqrt{49}\right) = 3 \cdot 7 = 21 = \frac{21}{1}$, which the ratio of two integers.
Expression II is the only expression that results in an irrational number, so Choice (a) is the correct answer.

PTS: 2 REF: 011604ai NAT: N.RN.3 TOP: Classifying Numbers

A.REI.1: Understand Solving Equations as a Process of Reasoning and Explain the Reasoning

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

PROPERTIES

Commutative Properties of Addition and Multiplication

For all real numbers a and b:

$a+b=b+a \qquad a\cdot b=b\cdot a$

Associative Properties of Addition and Multiplication

For all real numbers a, b, and c:

$(a+b)+c=a+(b+c) \qquad (a\cdot b)\cdot c=a\cdot(b\cdot c)$

Distributive Properties of Addition and Multiplication

$a(b+c)=ab+ac \qquad a(b-c)=ab-ac$

$(b+c)a=ba+ca \qquad (b-c)a=ba-ca$

Addition Property of Equality

The addition of the same number or expression to both sides of an equation results is permitted.

Multiplication Property of Equality

The multiplication of both sides of an equation by the same number or expression is permitted.

IDENTITY ELEMENTS

Identity Element: The **identity element** is always associated with an *operation*. The **identity element** for a given *operation* is the element that preserves the identity of other elements under the given operation.

Addition

The **identity element** for addition is the number 0

$a+0=a \text{ and } 0+a=a$

The number 0 does not change the value of other numbers under addition.

Multiplication

The **identity element** for multiplication is the number 1

$a\cdot 1=a \text{ and } 1\cdot a=a$

The number 1 does not change the value of other numbers under multiplication.

<u>Inverse Properties of Addition and Multiplication</u>

Inverse: The **<u>inverse</u>** of a number or expression under a given *operation* will result in the **<u>identity element</u>** for that operation. Therefore, it is necessary to know what the **<u>identity element</u>** of an operation is before finding the **<u>inverse</u>** of a given number or expression.

Addition

The additive inverse of a number or expression results in 0 under addition.

$$a + (-a = 0) \text{ and } (-a) + a = 0$$

$$(x+y) + (-x-y) = 0 \; and \; (-x-y) + (x+y) = 0$$

Multiplication

The multiplicative inverse of a number or expression results in 1 under multiplication.

$$a \times \frac{1}{a} = 1 \text{ and } \frac{1}{a} \times a = 1 \qquad\qquad \frac{1}{a} \times a = 1$$

$$(x+y)\left(\frac{1}{(x+y)}\right) = 1 \text{ and } \left(\frac{1}{(x+y)}\right)(x+y) = 1$$

Regents Problems

1. When solving the equation $4(3x^2 + 2) - 9 = 8x^2 + 7$, Emily wrote $4(3x^2 + 2) = 8x^2 + 16$ as her first step. Which property justifies Emily's first step?
   a. addition property of equality
   b. commutative property of addition
   c. multiplication property of equality
   d. distributive property of multiplication over addition

2. Fred's teacher gave the class the quadratic function $f(x) = 4x^2 + 16x + 9$.

   a) State two different methods Fred could use to solve the equation $f(x) = 0$.

   b) Using one of the methods stated in part *a*, solve $f(x) = 0$ for *x*, to the *nearest tenth.*

A.REI. 1: Understand Solving Equations as a Process of Reasoning and Explain the Reasoning.
Answer Section

1. ANS: A
Strategy: Identify what changed during Emily's first step, then identify the property associated with what changed..

$$4(3x^2 + 2) - 9 = 8x^2 + 7$$

$$4(3x^2 + 2) = 8x^2 + 16$$

Emily moved the −9 term from the left expression of the equation to the right expression of the equation by adding +9 to both the left and right expressions.

Adding an equal amount to both sides of an equation is associated with the addition property of equality.

PTS: 2 REF: 061401a1 NAT: A.REI.1 TOP: Identifying Properties

2. ANS:
a) Quadratic formula and completing the square.

b) -0.7 and -3.3

Complete the Square Method	Quadratic Formula Method
$f(x) = 4x^2 + 16x + 9$	$f(x) = 4x^2 + 16x + 9$
$4x^2 + 16x + 9 = 0$	a=4, b=16, c=9
$4x^2 + 16x = -9$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\dfrac{4x^2}{4} + \dfrac{16x}{4} = \dfrac{-9}{4}$	$x = \dfrac{-16 \pm \sqrt{(16)^2 - 4(4)(9)}}{2(4)}$
$x^2 + 4x = -\dfrac{9}{4}$	$x = \dfrac{-16 \pm \sqrt{112}}{8}$
$x^2 + 4x + (2)^2 = -\dfrac{9}{4} + (2)^2$	$x = \dfrac{-16 + \sqrt{112}}{8} = \dfrac{-5.416}{8} = -.677 = -0.7$
$(x+2)^2 = -\dfrac{9}{4} + 4$	$x = \dfrac{-16 - \sqrt{112}}{8} = \dfrac{-26.583}{8} = -3.322 = -3.3$
$(x+2)^2 = -\dfrac{9}{4} + \dfrac{16}{4}$	
$(x+2)^2 = \dfrac{7}{4}$	
$x + 2 = \pm\sqrt{\dfrac{7}{4}}$	
$x + 2 = \pm\dfrac{\sqrt{7}}{2}$	
$x = -2 \pm \dfrac{\sqrt{7}}{2}$	
$x = -2 + \dfrac{\sqrt{7}}{2} = -0.677 = -0.7$	
$x = -2 - \dfrac{\sqrt{7}}{2} = -3.322 = -3.3$	

PTS: 1 REF: 011634ai NAT: A.REI.A

A.REI.3: Solving Linear Equations and Inequalitites in One Variable

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

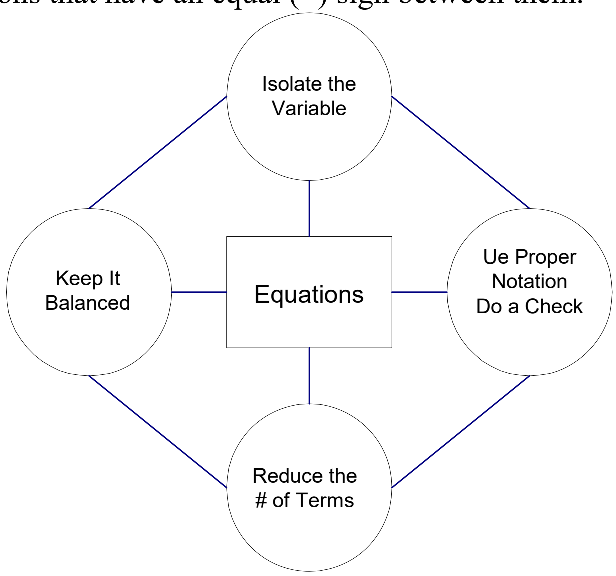
A **term** is a number {1,2,3,…}, a variable {x,y,z,a,b,c…}, or the product of a number and a variable {2x, 3y, ½ a, etc.). Terms are separated by + or – signs in an expression, and the + or – signs are part of each term. (Everything inside parenthesis is treated as one term until the parentheses are removed.)

A **variable** is a letter that represents an unknown value(s). When we are asked to solve an equation, it usually means that we must isolate the variable and find its value.

A **coefficient** is a number that comes in front of a variable. A coefficient can be an integer, a decimal, or a fraction. A coefficient multiplies the variable. Every variable has a coefficient. If a variable appears to have no coefficient, it's coefficient is an "invisible 1"

An **expression** is a mathematical statement consisting of one or more terms.

An **equation** is two expressions that have an equal (=) sign between them.



Four Column Strategy

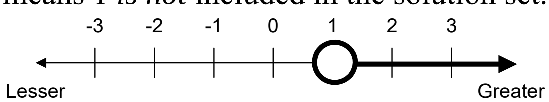
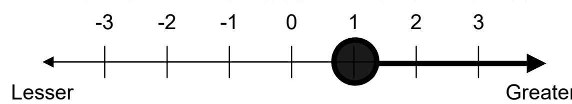
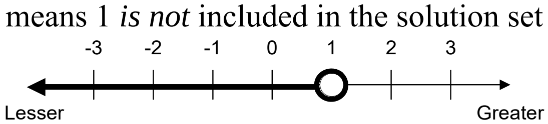
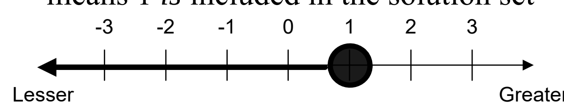
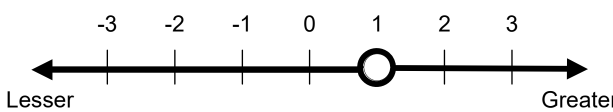
Notes	Left Hand Expression	Sign	Right Hand Expression
Given Add (6)	$2x-6$ $+6$	=	2 + 6
	$2x+0$	=	8
Divide (2)	$\frac{2x}{2}$	=	$\frac{8}{2}$
Answer	x	=	4
Check	$2(4)-6$ 8-6 2	= = =	2 2 2

Inequality Symbols:

$<$	less than	$>$	greater than
$\leq$	less than or equal to	$\geq$	greater than or equal to
$\neq$	not equal to		

The **solution of an inequality** includes any values that make the inequality true.
Solutions to inequalities can be graphed on a number line using open and closed dots.

Open Dots v Closed Dots
Square vs Curved Parentheses

<table>
<tr><td>When the inequality sign does not contain an equality bar beneath it, the dot is open.</td><td>When the inequality sign contains includes an equality bar beneath it, the dot is closed, or shaded in.</td></tr>
<tr><td>Graph of $x > 1$
or (1...
means 1 is not included in the solution set.
</td><td>Graph of $x \geq 1$
or [1...
means 1 is included in the solution set
</td></tr>
<tr><td>Graph of $x < 1$
or ...1)
means 1 is not included in the solution set
</td><td>Graph of $x \leq 1$
or ...1]
means 1 is included in the solution set
</td></tr>
<tr><td colspan="2">Graph of $x \neq 1$
</td></tr>
</table>

The Big Rule for Solving Inequalities:
All the rules for solving equations apply to inequalties – plus one:
When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.

Regents Problems

1. The inequality $7 - \frac{2}{3}x < x - 8$ is equivalent to

   a. $x > 9$
   b. $x > -\frac{3}{5}$
   c. $x < 9$
   d. $x < -\frac{3}{5}$

2. What is the value of x in the equation $\frac{x-2}{3} + \frac{1}{6} = \frac{5}{6}$?

   a. 4
   b. 6
   c. 8
   d. 11

3. Which value of x satisfies the equation $\frac{7}{3}\left(x + \frac{9}{28}\right) = 20$?

   a. 8.25
   b. 8.89
   c. 19.25
   d. 44.92

4. Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

5. Solve the inequality below to determine and state the smallest possible value for x in the solution set.

   $$3(x+3) \le 5x - 3$$

6. Solve for x algebraically: $7x - 3(4x - 8) \le 6x + 12 - 9x$
   If x is a number in the interval $[4,8]$, state all integers that satisfy the given inequality. Explain how you determined these values.

7. Determine the smallest integer that makes $-3x + 7 - 5x < 15$ true.

8. Given that $a > b$, solve for x in terms of a and b:

   $$b(x-3) \ge ax + 7b$$

A.REI.3: Solve Linear Equations and Inequalities in One Variable.
Answer Section

1. ANS: A
Strategy: Use the four column method for solving and documenting an equation or inequality.

Notes	Left Expression	Sign	Right Expression
Given:	$7-\frac{2}{3}x$	$<$	$x-8$
Add +8 to both expressions (Addition property of equality)	$15-\frac{2}{3}x$	$<$	x
Add $+\frac{2}{3}x$ to both expressions (Addition property of equality)	15	$<$	$x+\frac{2}{3}x$
Simplify	15	$<$	$\frac{5}{3}x$
Divide both expressions by $\frac{5}{3}$ (Division property of equality)	$\frac{\frac{15}{1}}{\frac{5}{3}}$	$<$	$\frac{\frac{5}{3}x}{\frac{5}{3}}$
Simplify	9	$<$	x
Rewrite	x	$>$	9

PTS: 2 REF: 011507a1 NAT: A.REI.3 TOP: Solving Linear Inequalities

2. ANS: A
Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given:	$\frac{x-2}{3}$	=	$\frac{4}{6}$
Multiply both expressions by 6 (Multiplication property of equality)	$\frac{6}{1}\left(\frac{x-2}{3}\right)$	=	$\frac{6}{1}\left(\frac{4}{6}\right)$
Cancel and Simplify	$\frac{2}{1}\left(\frac{x-2}{1}\right)$	=	$\frac{1}{1}\left(\frac{4}{1}\right)$
Simplify	$2x-4$	=	4
Add +4 to both expressions (Addition property of equality)	$2x$	=	8
Divide both expressions by 2 (Division property of equality)	x	=	4

PTS: 2 REF: 081420a1 NAT: A.REI.3 TOP: Solving Linear Equations
KEY: fractional expressions

3. ANS: A

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$\frac{7}{3}\left(x+\frac{9}{28}\right)$	=	20
Divide both expressions by $\frac{7}{3}$ (Division property of equality)	$\dfrac{\frac{7}{3}\left(x+\frac{9}{28}\right)}{\frac{7}{3}}$	=	$\dfrac{\frac{20}{1}}{\frac{7}{3}}$
Cancel and Simplify	$x+\frac{9}{28}$	=	$\frac{60}{7}$
Subtract $\frac{9}{28}$ from both expressions (Subtraction property of equality)	x	=	$\frac{60}{7}-\frac{9}{28}$
Simplify	x	=	$\frac{231}{28}$
Simplify	x	=	8.25

or

Notes	Left Expression	Sign	Right Expression
Given	$\frac{7}{3}\left(x+\frac{9}{28}\right)$	=	20
Distributive Property	$\frac{7}{3}x+\frac{7}{3}\left(\frac{9}{28}\right)$	=	20
Cancellation	$\frac{7}{3}x+\frac{1}{3}\left(\frac{9}{4}\right)$	=	20
Simplification	$\frac{7}{3}x+\frac{3}{4}$	=	20
Subtract $\frac{3}{4}$ from both expressions (Subtraction Property of Equality)	$\frac{7}{3}x$	=	$20-\frac{3}{4}$
Simplification	$\frac{7}{3}x$	=	$\frac{77}{4}$
Multiply both expressions by 12 (Multiplication property of equality)	$\frac{12}{1}\left(\frac{7x}{3}\right)$	=	$\frac{12}{1}\left(\frac{77}{4}\right)$
Cancel	$\frac{4}{1}\left(\frac{7x}{1}\right)$	=	$\frac{3}{1}\left(\frac{77}{1}\right)$
Siomplify	$28x$	=	231
Divide both expressions by 28 (Division property of equality)	$\frac{28x}{28}$	=	$\frac{231}{28}$
Simplify	x	=	8.25

PTS: 2 REF: 061405a1 NAT: A.REI.3 TOP: Solving Linear Equations
KEY: fractional expressions

4. ANS:
Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$2x + ax - 7$	$>$	-12
Substitute -1 for x	$2(-1) + a(-1) - 7$	$>$	-12
Simplify	$-2 - a - 7$	$>$	-12
Combine like terms	$-a - 9$	$>$	-12
Add +9 to both expressions (Addition property of equality)	$-a$	$>$	-3
Divide both expressions by -1 and reverse the sign	a	$<$	3

PTS: 2 REF: 061427a1 NAT: A.REI.3 TOP: Solving Linear Inequalities

5. ANS:
6 is the smallest possible value for x in the solution set.

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$3x + 9$	$\le$	$5x - 3$
Subtract 3x from both expressions (Subtraction property of equality)	9	$\le$	$2x - 3$
Add +3 to both expressions (Addition Property of equality)	12	$\le$	$2x$
Divide both expressions by 2 (Division property of equality)	6	$\le$	x
Rewrite	x	$\ge$	6

PTS: 2 REF: 081430a1 NAT: A.REI.3 TOP: Solving Linear Inequalities

6. ANS:
6, 7, 8 are the numbers greater than or equal to 6 in the interval.

Strategy: Use the four column method to solve the inequality, then interpret the solution.

STEP 1: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$7x - 3(4x - 8)$	$\leq$	$6x + 12 - 9x$
Clear parentheses (Distributive property)	$7x - 12x + 24$	$\leq$	$6x + 12 - 9x$
Simplify (Combine like terms)	$-5x + 24$	$\leq$	$-3x + 12$
Add 5x to both expressions (Addition property of equality)	24	$\leq$	$2x + 12$
Subtract 12 from both expressions (Subtraction property of equality)	12	$\leq$	2x
Divide both expressions by 2 (Division property of equality)	6	$\leq$	x
Rewrite	x	$\geq$	6

STEP 2: Interpret the solution set for the interval $[4,8]$.
The interval $[4,8]$ contains the integers 4, 5, 6, 7, and 8.
If $x \geq 6$, then the solution set of integers is $\{6,7,8\}$.

PTS: 4 REF: 081534ai NAT: A.REI.3 TOP: Solving Linear Inequalities

7. ANS:
0 is the smallest integer in the solution set.
.
Strategy: Use the four column method to solve the inequality, then interpret the solution.

STEP 1: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$-3x+7-5x$	$<$	15
Simplify (Combine like terms)	$-8x+7$	$<$	15
Add +8x to both expressions (Addition Property of Equality)	7	$<$	8x+15
Subtract 15 from both expressions (Subtraction Property of Equality)	-8	$<$	8x
Divide both expressions by +8 (Division property of equality)	-1	$<$	x
Rewrite	x	$>$	-1

STEP 2: Interpret the solution set for the smallest integer.
The smallest integer greater than -1 is 0.

PTS: 2 REF: 061530AI NAT: A.REI.3 TOP: Solving Linear Inequalities

8. ANS:
$x \le \dfrac{10b}{b-a}$

$$b(x-3) \ge ax+7b$$
$$bx-3b \ge ax+7b$$
$$bx-ax \ge 10b$$
$$x(b-a) \ge 10b$$

Since $a > b$, the expression $(b-a)$ must be negative, which means the inequality must be divided by a negative number to isolate x. When an inequality is divided or multiplied by a negative term, the direction of the inequality sign changes.

$$x \le \frac{10b}{b-a}$$

PTS: 2 REF: 011631ai NAT: A.REI.3 TOP: Solving Linear Inequalities

GRAPHS AND STATISTICS

S.ID.1: Dot Plots, Histograms and Box Plots

Summarize, represent, and interpret data on a single count or measurement variable
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

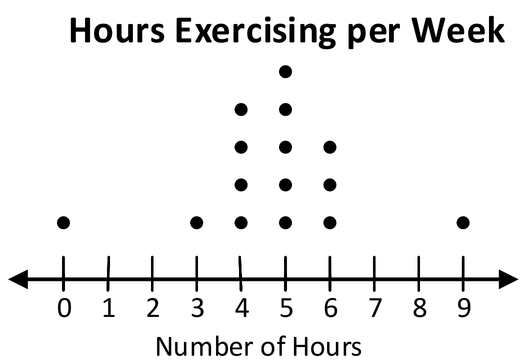
Vocabulary

univariate A set of data involving one variable.
multivariate A set of data involving more than one variable.

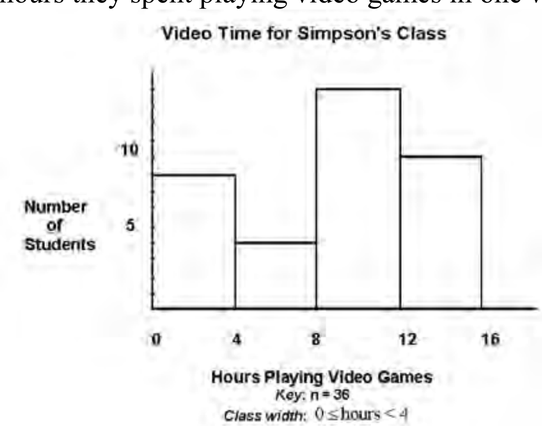
Big Ideas

A **dot plot** consists of data points plotted on a simple scale. **Dot plots** are used for continuous, quantitative, univariate data. Data points may be labelled if there are few of them. The horizontal axis is a number line that displays the data in *equal intervals*. The frequency of each bar is shown by the number of dots on the vertical axis.

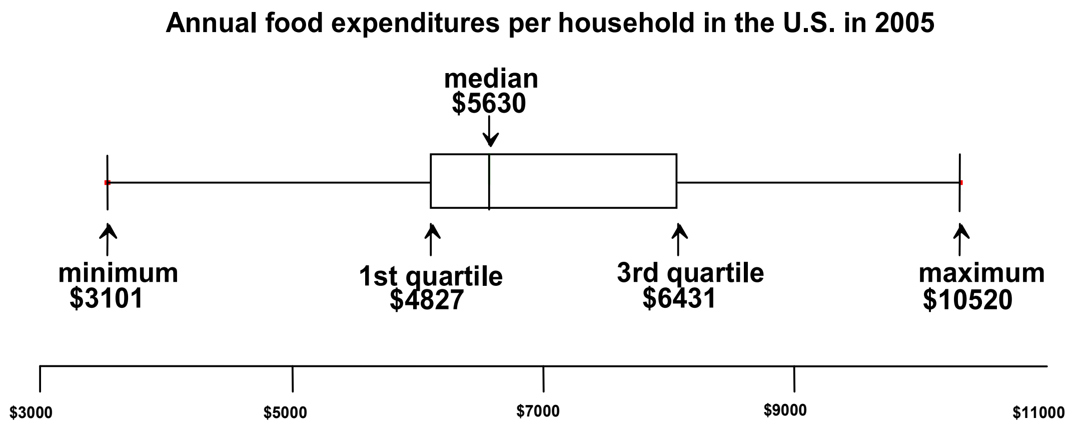
Example: This **dot plot** shows how many hours students excercise each week. Fifteen students were asked how many hours they exercise in one week.



A **histogram** is a frequency distribution for continuous, quantitative, univariate data. The horizontal axis is a number line that displays the data in equal intervals. The frequency of each bar is shown on the vertical axis.
Example: This histogram shows the number of students in Simpson's class that are in each interval. The students were asked how many hours they spent playing video games in one week.



A **box plot**, also known as a **box and whiskers chart**, is a visual display of a set of data showing the five number summary: minimum, first quartile, median, third quartile, and maximum. A **box plot** shows the range of scores within each quarter of the data. It is useful for examining the variation in a set of data and comparing the variation of more than one set of data.
Example:



REGENTS PROBLEM

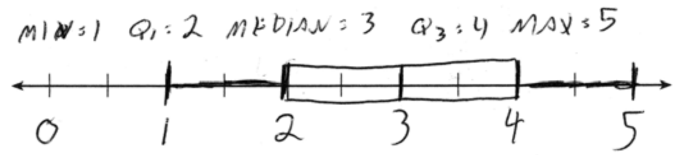
1. Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

Using an appropriate scale on the number line below, construct a box plot for the 15 values.

S.ID.1 Dot Plots, Histograms, and Box Plots
Answer Section

1. ANS:



Strategy: Follow these step-by-step procedures for creating a box and whiskers plot.

STEP 1. Organize the data set in ascending order, as follows. Be sure to include all the data.:
1, 1.5, 1.5, 2, 2, 2.5, 2.5, 3, 3, 3, 3.5, 4, 4, 4.5, 5

STEP 2. Plot a scale on the number line. In this case, the scale is 0 to five in equal intervals of .5 units.

STEP 3. Plot the minimum and maximum values: minimum = 1 and maximum = 2.

STEP 4. Identify the median. In this problem, there are fifteen numbers and the median is the middle number, which is 3. There are seven numbers to the left of 3 and seven numbers to the right of 3.

STEP 5. Plot and label the median = 3 (also known as Q2 or the second quartile).

STEP 6. Identify Q1, which is the *median of the bottom half* of the organized data set. The bottom half of the data includes all numbers below the median, which in this problem, includes the following numbers
1, 1.5, 1.5, 2, 2, 2.5, 2.5
The middle number in an organized list of seven numbers is the fourth number, which in this case is a 2.

STEP 7. Plot and label Q1 = 2.

STEP 8. Identify Q3, which is the *median of the top half* of the organized data set. The top half of the data includes all numbers above the median, which in this problem, includes the following numbers
3, 3, 3.5, 4, 4, 4.5, 5
Again, the middle number in an organized list of seven numbers is the fourth number, which in this case is a 4.

STEP 9. Plot and label Q3 = 4.

STEP 10. Finish the box plot by drawing boxes between the plotted points for Q1, Q2, and Q3.

PTS: 2 REF: 061432a1 NAT: S.ID.1 TOP: Box Plots

S.ID.2: Central Tendency and Dispersion

Summarize, represent, and interpret data on a single count or measurement variable
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Measures of Central Tendency

A **measure of central tendency** is a *summary statistic* that indicates the typical value or center of an organized data set. The three most common measures of central tendency are the *mean, median*, and *mode*.

Mean A measure of central tendency denoted by $\overline{x}$, read "x bar", that is calculated by adding the data values and then dividing the sum by the number of values. Also known as the arithmetic mean or arithmetic average. The algebraic formula for the mean is:

$$\text{Mean} = \frac{Sum\ of\ items}{Count} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Median A measure of central tendency that is, or indicates, the middle of a data set when the data values are arranged in ascending or descending order. ***If there is no middle number*, *the median is the average of the two middle numbers*.**

Examples:
The **median** of the set of numbers: {2, 4, 5, 6, 7, 10, 13} is 6
The **median** of the set of numbers: {6, 7, 9, 10, 11, 17} is 9.5

Mode A measure of central tendency that is given by the data value(s) that occur(s) most frequently in the data set.

Examples:
The **mode** of the set of numbers {5, 6, 8, 6, 5, 3, 5, 4} is 5.
The **modes** of the set of numbers {4, 6, 7, 4, 3, 7, 9, 1,10} are 4 and 7.
The **mode** of the set of numbers {0, 5, 7, 12, 15, 3} is none or there is no mode.

Measures of Spread

Interquartile Range: The difference between the first and third quartiles; a measure of variability resistant to outliers.

Standard Deviation: A measure of variability. **Standard deviation** measures the average distance of a data element from the mean. There are two types of **standard deviations**: population and sample.

Population Standard Deviation: If data is taken from the *entire population*, divide by n when averaging the squared deviations. The following is the formula for **population standard deviation**:

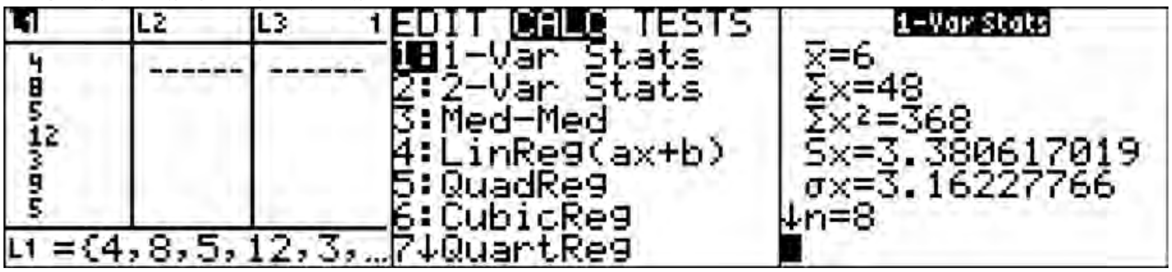
$$\sigma = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n}}$$

Sample Standard Deviation: If data is taken from a *sample* instead of the *entire population*, divide by $n-1$ when averaging the squared deviations. This results in a larger standard deviation. The following is the formula for **sample standard deviation**:

$$s = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}}$$

Tips for Computing Standard Deviations:
Use the STATS funtion of a graphing calculator to calculate standard deviation. Remember that the sample standard deviation (s) will be larger than the population standard deviation (σ).

1. Use STATS EDIT to input the data set.
2. Use STATS CALC 1-Var Stats to calculate standard deviations.

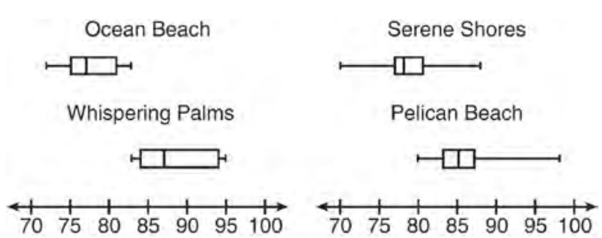


The outputs include:
$\overline{X}$, which is the mean (average),
$\sum x$, which is the sum of the data set.

$\sum x^2$, which is the sum of the squares of the data set.
Sx, which is the sample standard deviation.
σx, which is the population standard deviation.

REGENTS PROBLEMS

1. Corinne is planning a beach vacation in July and is analyzing the daily high temperatures for her potential destination. She would like to choose a destination with a high median temperature and a small interquartile range. She constructed box plots shown in the diagram below.



Which destination has a median temperature above 80 degrees and the smallest interquartile range?

a. Ocean Beach
b. Whispering Palms
c. Serene Shores
d. Pelican Beach

2. Christopher looked at his quiz scores shown below for the first and second semester of his Algebra class.
Semester 1: 78, 91, 88, 83, 94
Semester 2: 91, 96, 80, 77, 88, 85, 92
Which statement about Christopher's performance is correct?
   a. The interquartile range for semester 1 is greater than the interquartile range for semester 2.
   b. The median score for semester 1 is greater than the median score for semester 2.
   c. The mean score for semester 2 is greater than the mean score for semester 1.
   d. The third quartile for semester 2 is greater than the third quartile for semester 1.

3. Isaiah collects data from two different companies, each with four employees. The results of the study, based on each worker's age and salary, are listed in the tables below.

Company 1

Worker's Age in Years	Salary in Dollars
25	30,000
27	32,000
28	35,000
33	38,000

Company 2

Worker's Age in Years	Salary in Dollars
25	29,000
28	35,500
29	37,000
31	65,000

Which statement is true about these data?
   a. The median salaries in both companies are greater than $37,000.
   b. The mean salary in company 1 is greater than the mean salary in company 2.
   c. The salary range in company 2 is greater than the salary range in company 1.
   d. The mean age of workers at company 1 is greater than the mean age of workers at company 2.

4. The two sets of data below represent the number of runs scored by two different youth baseball teams over the course of a season.

Team A: 4, 8, 5, 12, 3, 9, 5, 2

Team B: 5, 9, 11, 4, 6, 11, 2, 7

Which set of statements about the mean and standard deviation is true?

a. mean $A <$ mean B
standard deviation $A >$ standard deviation B

b. mean $A >$ mean B
standard deviation $A <$ standard deviation B

c. mean $A <$ mean B
standard deviation $A <$ standard deviation B

d. mean $A >$ mean B
standard deviation $A >$ standard deviation B

S.ID.2 Central Tendency and Dispersion
Answer Section

1. ANS: D
Strategy: Eliminate wrong answers based on daily high temperatures, then eliminate wrong answers based on size of interquartile ranges.

Ocean Breeze and Serene Shores can be eliminated because they *do not* have median high temperatures above 80 degrees. Whispering Palms and Pelican Beach *do* have median high temperatures above 80 degrees, so the correct answer must be either Whispering Palms or Pelican Beach.

The interquartile range is defined as the differfence between the first and third quartiles. Pelican Beach has a much smaller interquartile range than Whispering Palms, so Pelican Beach is the correct choice.

PTS: 2 REF: 011514a1 NAT: S.ID.2 TOP: Central Tendency and Dispersion

2. ANS: C
Strategy: Compute the mean, Q1, Q2, Q3, and interquartile range for each semester, then choose the correct answer based on the data.

	Mean	Q1	Median (Q2)	Q3	IQR
Semester 1	86.8	80.5	88	92.5	12
Semester 2	87	80	88	92	12

PTS: 2 REF: 061419a1 NAT: S.ID.2 TOP: Central Tendency and Dispersion

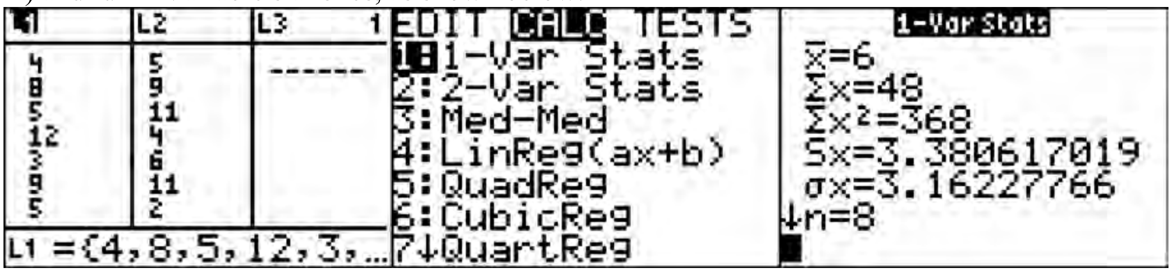
3. ANS: C
Strategy: Compute the median salary, mean salary, salary range, and mean age of employees for both companies, then select the correct answer.

		Company 1	Company 2
1	median salary	33,500	36,250
2	mean salary	33,750	44,125
3	salary range	8,000	36,000
4	mean age	28.25	28.25

PTS: 2 REF: 081404a1 NAT: S.ID.2 TOP: Central Tendency and Dispersion

4. ANS: A
Strategy: Compute the mean and standard deviations for both teams, then select the correct answer.

STEP 1. Enter the two sets of data into the STAT function of a graphing calculator, then select the first list (Team A) and run 1-Variable statistics, as shown below:



STEP 2. Repeat STEP 1 for the second list (Team B).

STEP 3. Use the data from the graphing calculator to choose the correct answer.
Choice a: mean A < mean B

$6 < 6.875$

standard deviation A > standard deviation B

$3.16227766 > 3.059309563$

Both statements in choice A are true.
$A: \overline{x} = 6;\ \sigma_x = 3.16 \quad B: \overline{x} = 6.875;\ \sigma_x = 3.06$

PTS: 2 REF: 081519ai NAT: S.ID.2 TOP: Central Tendency and Dispersion

S.ID.3 Outliers/Extreme Data Points

Interpreting Categorical & Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Vocabulary

Outlier An observation point that is distant from other observations.

Big Idea

An **outlier** can significantly influence the measures of central tendency and/or spread in a data set.

Measures of Central Tendency

Mean $\text{Mean} = \dfrac{Sum\ of\ items}{Count} = \dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

Median Arrange data set in ascending order, find the middle number.

Quartiles These are the three numbers that divide the data set into four parts, or quarters.

To find quartiles, first find the median, which separates the data set into two halves.

Q1 The first quartile is the median of the lower half of the data set, below the median..

Q2 The second quartile is the median of the entire data set.

Q3 The third quartile is the median of the upper half of the data set, above the median..

Mode The most common number(s) in a data set.

Measures of Spread (Dispersion)

Interquartile Range: The difference between the first and third quartiles; a measure of variability resistant to outliers.

Standard Deviation: A measure of variability. **Standard deviation** measures the average distance of a data element from the mean.

For additional information, see also S.ID.2 Central Tendency and Dispersion

Regents Problem

1. The table below shows the annual salaries for the 24 members of a professional sports team in terms of millions of dollars.

0.5	0.5	0.6	0.7	0.75	0.8
1.0	1.0	1.1	1.25	1.3	1.4
1.4	1.8	2.5	3.7	3.8	4
4.2	4.6	5.1	6	6.3	7.2

The team signs an additional player to a contract worth 10 million dollars per year. Which statement about the median and mean is true?

a. Both will increase.
b. Only the median will increase.
c. Only the mean will increase.
d. Neither will change.

S.ID.3 Outliers/Extreme Data Points
Answer Section

1. ANS: C
Median remains at 1.4.

Strategy:
Compare the current median and mean to the new median and mean:

STEP 1. Compare the medians:
The data are already in ascending order, so the median is the middle number. In this case, the data set contains 24 elements - an even number of elements. This means there are two middle numbers, both of which are 1.4. When the data set contains an even number of elements, the median is the average of the two middle numbers, which in this case is $\frac{1.4+1.4}{2} = 1.4$

The new data set will contain 10 as an additional element, which brings the total number of elements to 25. The new median will be the 13th element, which is 1.4.

The current median and the new median are the same, so we can eliminate answer choices a and b.

STEP 2. Compare the means:
The mean will increase because the additional element (10) is bigger than any current element. It is not necessary to do the calculations. We can eliminate answer choice d.

DIMS? Does it make sense that the answer is choice c?
Yes. The median will stay and 1.4 and only the mean will increase.

PTS: 2 REF: 061520AI NAT: S.ID.3 TOP: Central Tendency and Dispersion

S.ID.5: Frequency Tables

S.ID.5: Frequency Tables

Summarize, represent, and interpret data on two categorical and quantitative variables.
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

frequency table A table that shows how often each item, number, or range of numbers occurs in a set of data.

Example: The data {5 ,7, 6 , 8 , 9 , 5 , 13 , 2 , 1 , 6 , 5 , 14 , 10 , 5 , 9} can be displayed as a frequency distribution in a table..

Interval	Frequency
1-5	6
6-10	7
11-15	2

NOTES: It is sometimes easier to arrange the data in ascending or descending order when making a frequency table. Here is the data set that is summarized in the preceding table in both original and ascending orders.

{5 ,7, 6 , 8, 9, 5, 13, 2, 1, 6, 5, 14, 10, 5, 9}
{1, 2, 5, 5, 5, 5, 7, 6 , 6, 8 , 9 , 9, 10, 13 , 14}

When rearranging data sets and/or building frequency tables, it is a good practice to count the data elements to make sure that all elements have been included.

REGENTS PROBLEM

1. The school newspaper surveyed the student body for an article about club membership. The table below shows the number of students in each grade level who belong to one or more clubs.

	1 Club	2 Clubs	3 or More Clubs
9^{th}	90	33	12
10^{th}	125	12	15
11^{th}	87	22	18
12^{th}	75	27	23

If there are 180 students in ninth grade, what percentage of the ninth grade students belong to more than one club?

2. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	Drama
Male	70	35
Female	48	42

Based on the sample, predict how many of the school's 351 males would prefer comedy. Justify your answer.

S.ID.5: Frequency Tables
Answer Section

1. ANS:
25%
Strategy: Use data from the table and information from the problem to calculate a percentage.
STEP 1. Determine the total number of students in the ninth grade who are in 2 or more clubs (33+12).
STEP 2. Divide by the total number of students in the ninth grade (180).
STEP 3. Convert the decimal to a percentage

$$\frac{33+12}{180} = \frac{45}{180} = .25$$

$$.25 = 25\%$$

PTS: 2 REF: 011526a1 NAT: S.ID.5
TOP: Frequency Histograms, Bar Graphs and Tables

2. ANS:
234 of the school's 351 males prefer comedy based on the sample.
The school has 351 males.
$70 + 35 = 105$ males were surveyed.
Based on the sample, $\frac{70}{105} = \frac{2}{3}$ of the males preferred comedy.
$\frac{2}{3}(351) = \frac{702}{3} = 234.$

PTS: 2 REF: 011630ai NAT: S.ID.5 TOP: Frequency Tables

S.ID.6a: Linear, Quadratic and Exponential Regression

Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

 a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

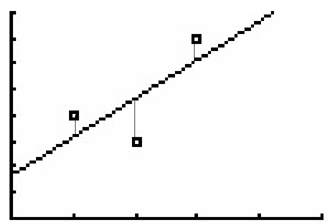
Regression Model: A function (e.g., linear, exponential, power, logarithmic) that fits a set of paired data. T*he model may enable other values of the dependent variable to be predicted.*

Big Ideas

The **individual data points** in a **scatterplot** form **data clouds** with shapes that suggest relationships between dependent and independent variables.

A **line of best fit** divides the data cloud into two equal parts with about the same number of data points on each side of the line.

Sum of the Squares: Mathematically speaking, a line of best fit is the line that produces the smallest sum of the squares. In the following diagram, the distance between each point and the line is measured vertically and then squared.



The line that produces the smallest sum of the squares is the line of best fit.

In linear regression, the line of best fit will always go through the point $\left(\bar{x}, \bar{y}\right)$, where $\bar{x}$ is the mean of all values of x, and $\bar{y}$ is the mean of all values of y. For example, the line of best fit for a scatterplot with points (2,5), (4,7) and (8,11) must include the point $\left(x = \frac{14}{3}, y = \frac{23}{3}\right)$, because these x and y values are the averages of all the x-values and all the y-values.

Calculating Regression Equations. Technology is almost always used to calcualte regression equations.
.

- **STEP 1**. Use STATS EDIT to Input the data into a graphing calculator.
- **STEP 2**. Use 2nd STAT PLOT to turn on a data set, then ZOOM 9 to inspect the graph of the data and determine which regression strategy will best fit the data.
- **STEP 3**. Use STAT CALC and the appropriate regression type to obtain the regression equation.
- **STEP 4**. Ask the question, "Does it Make Sense (DIMS)?"

DIFFERENT TYPES OF REGRESSION

The graphing calculator can calculate numerous types of regression equations, but it must be told which type to calculate. All of the calculator procedures described above can be used with various types of regression. The following screenshots show some of the many regressions that can be calculated on the TI-83/84 family of graphing calculators.

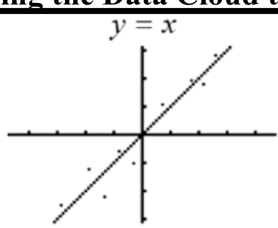
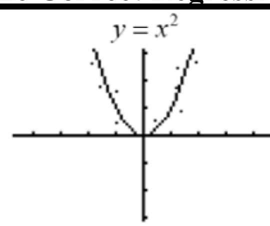
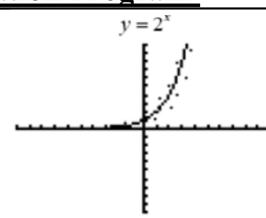
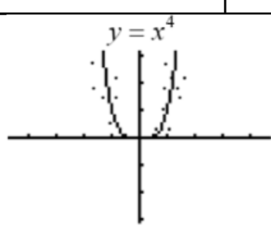
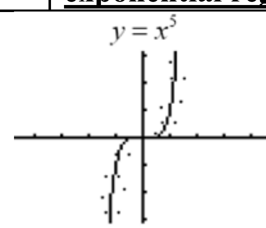
The general purpose of linear regression is to make predictions based on a line of best fit.

Choosing the Correct Type of Regression to Calculate

There are two general approaches to determining the type of regression to calculate:

- The decision of which type of regression to calculate can be made based on visual examination of the data cloud, or.
- On Regents examinations, the wording of the problem often specifies a particular type of regression to be used.

Using the Data Cloud to Select the Correct Regression Calculation Program

<table>
<tr>
<td>If the data cloud takes the general form of a straight line, use linear regression.</td>
<td>If the data cloud takes the general form of a parabola, use quadratic regression.</td>
<td>If the data cloud takes the general form of an exponential curve, use exponential regression.</td>
</tr>
<tr>
<td colspan="2">NOTE: All equations in the form of $y = ax^n$, where $a \neq 0$ and $n > 1$ and n is an odd number, take the form of parabolas. The larger the value of n, the wider the flat part at the bottom/top. Use quadratic or power regression.</td>
<td>NOTE: All equations in the form of $y = ax^n$, where $a \neq 0$ and $n > 1$ and n is an even number, take the form of hyperbolas. The larger the value of n, the wider the flat part in the middle. Use cubic or power regression.</td>
</tr>
<tr>
<td colspan="3">Note: In all previous Regents problems requiring the calculation of power regression, the wording of the problem specifically called for power regression.</td>
</tr>
</table>

REGENTS PROBLEMS

1. The table below shows the number of grams of carbohydrates, x, and the number of Calories, y, of six different foods.

Carbohydrates (x)	Calories (y)
8	120
9.5	138
10	147
6	88
7	108
4	62

Which equation best represents the line of best fit for this set of data?

a. $y = 15x$
b. $y = 0.07x$
c. $y = 0.1x - 0.4$
d. $y = 14.1x + 5.8$

2. An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.

Number of Weeks	1	2	3	4
Number of Downloads	120	180	270	405

Write an exponential equation that models these data. Use this model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the nearest download. Would it be reasonable to use this model to predict the number of downloads past one year? Explain your reasoning.

3. Each day Toni records the height of a plant for her science lab. Her data are shown in the table below.

Day (n)	1	2	3	4	5
Height (cm)	3.0	4.5	6.0	7.5	9.0

The plant continues to grow at a constant daily rate. Write an equation to represent $h(n)$, the height of the plant on the nth day.

4. Emma recently purchased a new car. She decided to keep track of how many gallons of gas she used on five of her business trips. The results are shown in the table below.

Miles Driven	Number of Gallons Used
150	7
200	10
400	19
600	29
1000	51

Write the linear regression equation for these data where miles driven is the independent variable. (Round all values to the *nearest hundredth*.)

5. About a year ago, Joey watched an online video of a band and noticed that it had been viewed only 843 times. One month later, Joey noticed that the band's video had 1708 views. Joey made the table below to keep track of the cumulative number of views the video was getting online.

Months Since First Viewing	Total Views
0	843
1	1708
2	forgot to record
3	7124
4	14,684
5	29,787
6	62,381

Write a regression equation that best models these data. Round all values to the *nearest hundredth*. Justify your choice of regression equation. As shown in the table, Joey forgot to record the number of views after the second month. Use the equation from part *a* to estimate the number of full views of the online video that Joey forgot to record.

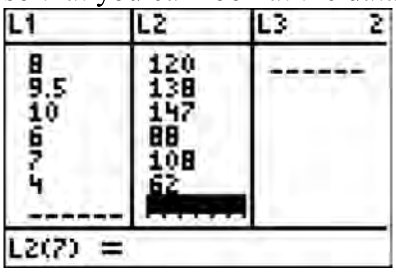
6. Rachel and Marc were given the information shown below about the bacteria growing in a Petri dish in their biology class.

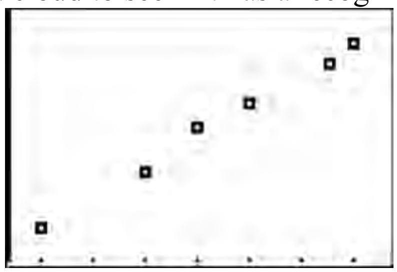
Number of Hours, x	1	2	3	4	5	6	7	8	9	10
Number of Bacteria, $B(x)$	220	280	350	440	550	690	860	1070	1340	1680

Rachel wants to model this information with a linear function. Marc wants to use an exponential function. Which model is the better choice? Explain why you chose this model.

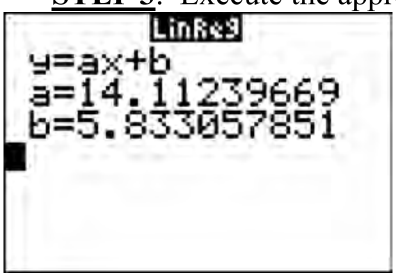
S.ID.6a: Linear, Quadratic and Exponential Regression
Answer Section

1. ANS: D
Strategy: Input the data into a graphing calculator, inspect the data cloud, and find a regression equation to model the data table, input the regression equation into the y-editor, predict the missing value.
   - **STEP 1**. Input the data into a graphing calculator or plot the data cloud on a graph, if necessary, so that you can look at the data cloud to see if it has a recognizable shape.





   - **STEP 2**. Determine which regression strategy will best fit the data. The graph looks like the graph of an linear function, so choose linear regression.
   - **STEP 3**. Execute the appropriate regression strategy in the graphing calculator.



Write the regression equation in a format that can be compared to the answer choices: $y = 14.11x + 5.83$
   - **STEP 4**. Compare the answer choices to the regression equation and select choice d.

PTS: 2 REF: 081421a1 NAT: S.ID.6a TOP: Regression

2. ANS:

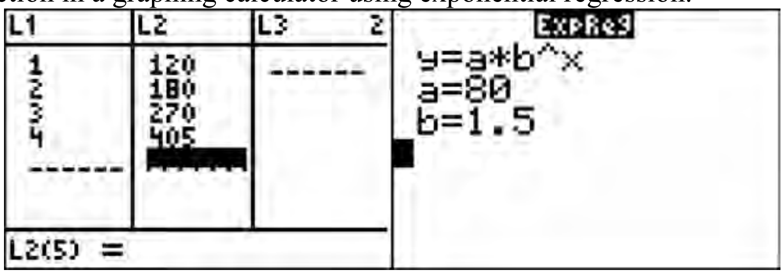
a) $y = 80(1.5)^x$

b) $80(1.5)^{26} \approx 3{,}030{,}140.$

c) No, because the prediction at $x = 52$ is already too large.

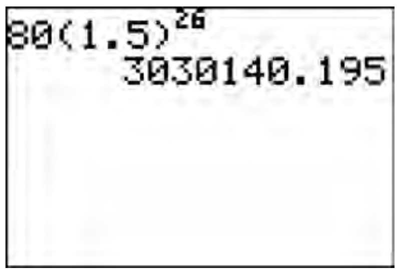
Strategy: Use data from the table and exponential regression in a graphing calculator.

STEP 1: Model the function in a graphing calculator using exponential regression.



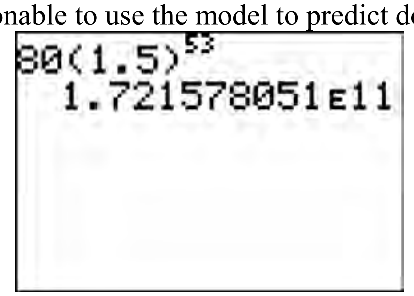
The exponential regression equation is $y = 80(1.5)^x$

STEP 2. Use the equation to predict the number of downloads when $x = 26$.



Rounded to the nearest download, the answer is 3,030,140.

STEP 3. Determine if it would be reasonable to use the model to predict downloads past one year.



It would not be reasonable to use this model to make predictions past one year. The number of predicted downloads is more 170 billion downloads, which is more than 20 downloads in one week for every person in the world.

DIMS? Does It Make Sense? For near term predicitons, yes. For long term predictions, no.

PTS: 4 REF: 061536AI NAT: S.ID.6A (JMAP) A.CED.2 (NY)
TOP: Regression

3. ANS:
$y = 1.5x + 1.5$

Strategy 1: The problem states that the plant grows at a constant daily rate, so the rate of change is constant. Use the slope-intercept form of a line, $y = mx + b$, and data from the table to identify the slope and y-intercept.

STEP 1: Extend the table to show the y-intercept, as follows:

Day (n)	0	1	2	3	4	5
Height (cm)	1.5	3	4.5	6	7.5	9

The y-intercept is 1.5, so we can write $y = mx + 1.5$.

STEP 2. Use the slope formula and any two pairs of data to find the slope. In the following calculation, the points (1,3) and (5,9) are used.

$$y = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{5-1} = \frac{6}{4} = \frac{3}{2} = 1.5$$

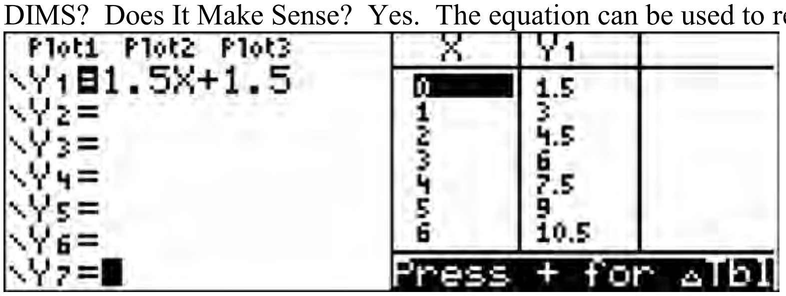
The slope is 1.5, so we can write $y = 1.5x + 1.5$

DIMS?: See below.

Strategy 2: Use linear regression.

The equation is $y = 1.5x + 1.5$

DIMS? Does It Make Sense? Yes. The equation can be used to reproduce the table view, as follows:

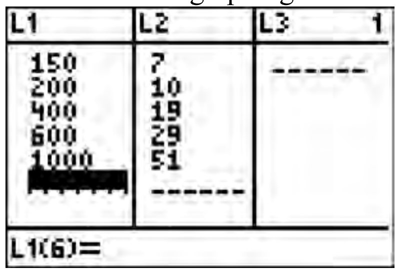


PTS: 2 REF: 081525ai NAT: A.CED.1 TOP: Modeling Linear Functions

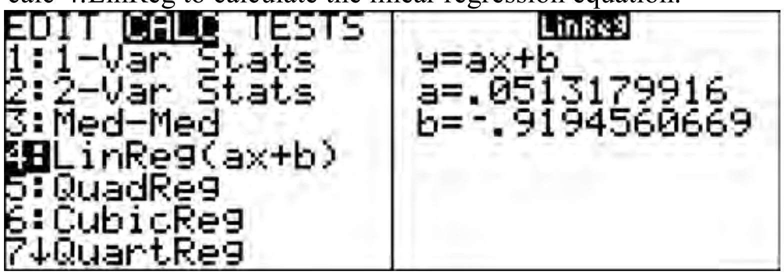
4. ANS:
$y = 0.05x - 0.92$

Strategy: Input the data into a graphing calculator.

- **STEP 1**. Use stats-edit to Input the data into a graphing calculator.



- **STEP 2**. Use stats-calc-4:LinReg to calculate the linear regression equation.



- **STEP 3**. Round all values to the nearest hundredth.

PTS: 2 REF: fall1307a1 NAT: S.ID.6a TOP: Regression

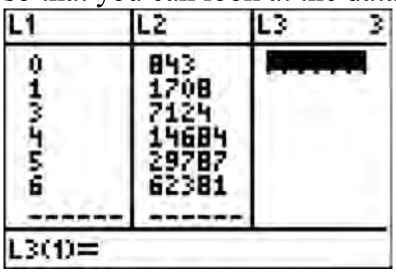
5. ANS:

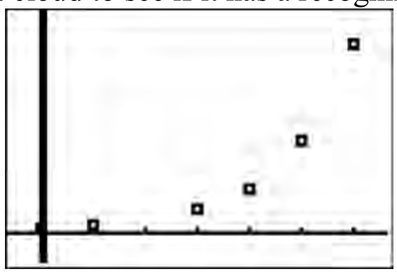
Part a: $f(x) = 836.47(2.05)^x$ The data appear to grow at an exponential rate.

Part b: $f(2) = 836.47(2.05)^2 = 3515$

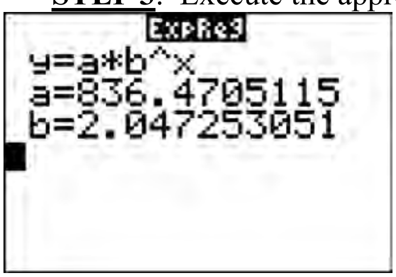
Strategy: Input the data into a graphing calculator, inspect the data cloud, and find a regression equation to model the data table, input the regression equation into the y-editor, predict the missing value.

- **STEP 1**. Input the data into a graphing calculator or plot the data cloud on a graph, if necessary, so that you can look at the data cloud to see if it has a recognizable shape.



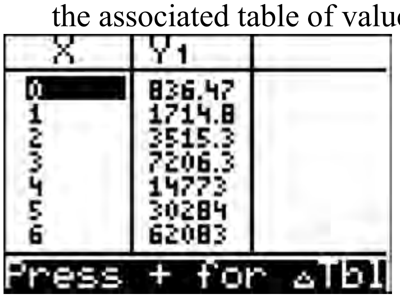


- **STEP 2**. Determine which regression strategy will best fit the data. The graph looks like the graph of an exponential function, so choose exponential regression.
- **STEP 3**. Execute the appropriate regression strategy in the graphing calculator.



Round all values to the nearest hundredth: $y = 836.47(2.05)^x$

- **STEP 4**. Input the regression equation into the y-editor feature of the graphing calculator and view the associated table of values to find the value of y when x equals 2.



Round 3515.3 to 3515.

- **STEP 4**. In

Ask the question, “Does it Make Sense (DIMS)?” that the missing total number of views in month 2 would be around 3515 views?

PTS: 4 REF: fall1313a1 NAT: S.ID.6a TOP: Regression

6. ANS:
Exponential, because the function does not grow at a constant rate.

Strategy 1.
Compare the rates of change for different pairs of data using the slope formula.

Rate of change between (1, 220) and (5, 550): $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{550 - 220}{5 - 1} = \dfrac{330}{4} = 82.5$

Rate of change between (6, 690) and (10, 1680): $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1680 - 690}{10 - 6} = \dfrac{990}{4} = 247.5$

Strategy 2: Use stat plots in a graphing calculator to create a scatterplot view of the multivariate data.

The graph view of the data clearly shows that the data is not linear.

PTS: 2 REF: 081527ai NAT: S.ID.6
TOP: Comparing Linear and Exponential Functions

S.ID.8: Calculate Correlation Coefficients

Interpret linear models

8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

Vocabulary

correlation A statistical measure that quantifies how pairs of variables are related; a linear relationship between two variables.

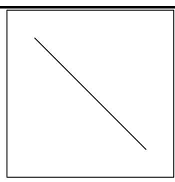
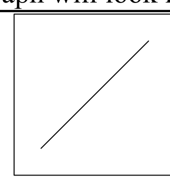
correlation coefficient A number between -1 and 1 that indicates the strength and direction of the linear relationship between two sets of numbers. The letter “r” is used to represent correlation coefficients. In all cases, $-1 \le r \le 1$.

Interpreting a Correlation Coefficient - What It Means

Every correlation coefficient has two pieces of information:

1. The **sign of the correlation**. A correlation is either positive or negative.
2. The **absolute value of the correlation**.
   a. The closer the absolute value of the correlation is to 1, the stronger the correlation between the variables.
   b. The closer the absolute value of the correlation is to zero, the weaker the correlation between the variables.

The **sign of the correlation** tells you what the graph will look like and

 **Negative Correlation** In general, one set of data decreases as the other set increases. An example of a negative correlation between two variables would be the relationship between absentiism from school and class grades. As one variable increases, the other would be expected to decrease.	**No Correlation** Sometimes data sets are not related and there is no general trend. A correlation of zero does not always mean that there is no relationship between the variables. It could mean that the relationship is not **linear**. For example, the correlation between points on a circle or a regular polygon would be zero or very close to zero, but the points are very predictably related.	 **Positive Correlation** In general, both sets of data increase together. An example of a positive correlation between two variables would be the relationship between studying for an examination and class grades. As one variable increases, the other would also be expected to increase.

The **absolute value of the correlation** tells you the strength of the correlation.

0	⟷	1
Weak Correlation	Absolute value of r	Strong Correlation

In a perfect correlation, when $r = \pm 1$, all data points balance the equations and also lie on the graph of the equation.

<u>How to Calculate a Correlation Coefficient Using a Graphing Calculator</u>:

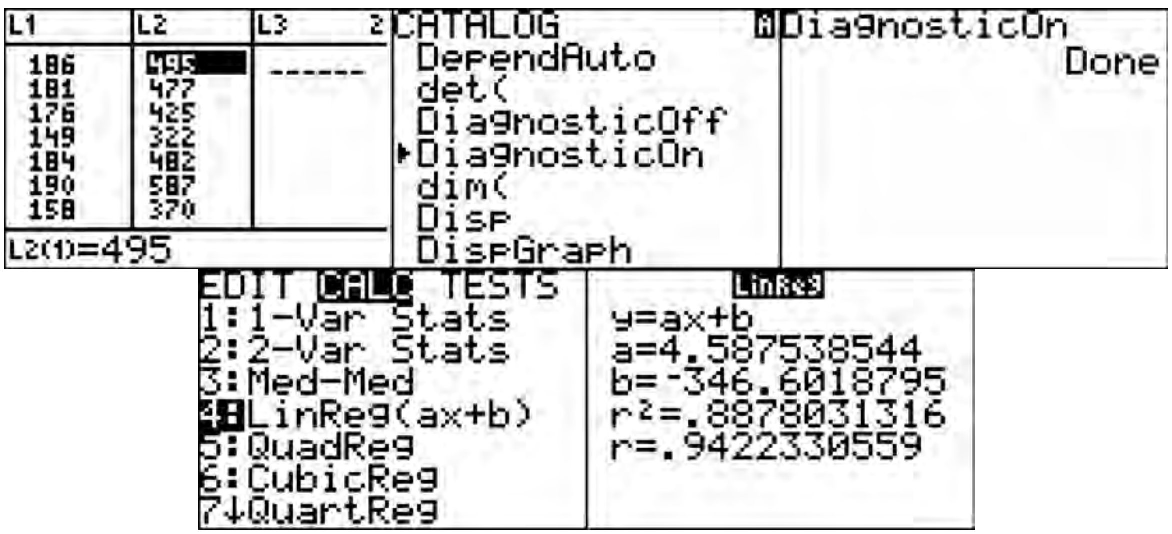
STEP 1. Press $\boxed{\text{STAT}}$ $\boxed{\text{EDIT}}$ $\boxed{\text{1:Edit}}$.

STEP 2. Enter bivariate data in the L1 and L2 columns. All the x-values go into L1 column and all the Y values go into L2 column. Press $\boxed{\text{ENTER}}$ after every data entry.

STEP 3. Turn the diagnostics on by pressing $\boxed{\text{2ND}}\boxed{\text{CATALOG}}$ and scrolling down to $\boxed{\text{DiagnosticsOn}}$. Then, press $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$. The screen should respond with the message $\boxed{\text{Done}}$. NOTE: If Diagnostics are turned *off,* the correlation coefficient will not appear beneath the regression equation.

Step 4. Press $\boxed{\text{STAT}}$ $\boxed{\text{CALC}}$ $\boxed{\text{4:4-LinReg (ax+b)}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$

Step 5. The *r* value that appears at the bottom of the screen is the correlation coefficient.



REGENTS PROBLEMS

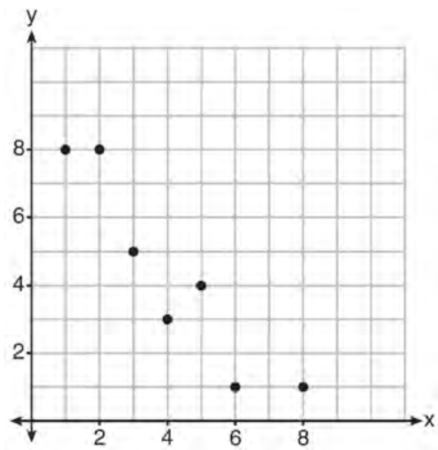
1. The table below shows the attendance at a museum in select years from 2007 to 2013.

Attendance at Museum

Year	2007	2008	2009	2011	2013
Attendance (millions)	8.3	8.5	8.5	8.8	9.3

State the linear regression equation represented by the data table when $x = 0$ is used to represent the year 2007 and y is used to represent the attendance. Round all values to the *nearest hundredth*. State the correlation coefficient to the *nearest hundredth* and determine whether the data suggest a strong or weak association.

2. What is the correlation coefficient of the linear fit of the data shown below, to the *nearest hundredth*?



a. 1.00
b. 0.93
c. −0.93
d. −1.00

3. Erica, the manager at Stellarbeans, collected data on the daily high temperature and revenue from coffee sales. Data from nine days this past fall are shown in the table below.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
High Temperature, t	54	50	62	67	70	58	52	46	48
Coffee Sales, f(t)	\$2900	\$3080	\$2500	\$2380	\$2200	\$2700	\$3000	\$3620	\$3720

State the linear regression function, $f(t)$, that estimates the day's coffee sales with a high temperature of t. Round all values to the *nearest integer*.

State the correlation coefficient, r, of the data to the *nearest hundredth*. Does r indicate a strong linear relationship between the variables? Explain your reasoning.

4. A nutritionist collected information about different brands of beef hot dogs. She made a table showing the number of Calories and the amount of sodium in each hot dog.

Calories per Beef Hot Dog	Milligrams of Sodium per Beef Hot Dog
186	495
181	477
176	425
149	322
184	482
190	587
158	370
139	322

a) Write the correlation coefficient for the line of best fit. Round your answer to the *nearest hundredth.*
b) Explain what the correlation coefficient suggests in the context of this problem.

S.ID.8: Calculate Correlation Coefficients
Answer Section

1. ANS:
$y = 0.16x + 8.27$ $r = 0.97$, which suggests a strong association.

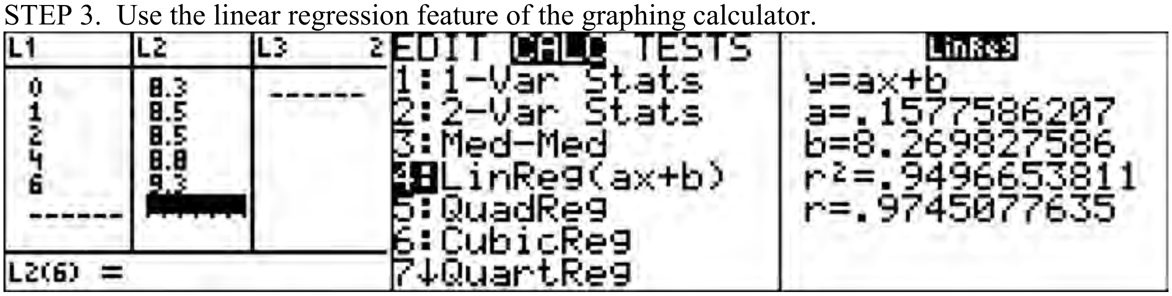
Strategy: Convert the table to data that can be input into a graphing calculator, then use the linear regression feature of the graphing calculator to respond to the question.

STEP 1. Convert the table for input into the calculator.

Attendance at Museum					
Year (L1)	0	1	2	4	6
Attendance (L2)	8.3	8.5	8.5	8.8	9.3

STEP 2. Make sure that STAT DIAGNOSTICS is set to "On" in the mode feature of the graphing calculator. Setting STAT DIAGNOSTICS to on causes the correlation coefficient (r) to appear with the linear regression output.

STEP 3. Use the linear regression feature of the graphing calculator.



NOTE: Round the graphing calculator output to the *nearest hundredth* as required in the problem.

STEP 4. Record your solution.

PTS: 4 REF: 081536ai NAT: S.ID.8 (JMAP) S.ID.6a (NYSED)
TOP: Regression KEY: linear

2. ANS: C
Strategy #1: This problem can be answered by looking at the scatterplot.

The slope of the data cloud is negative, so answer choices *a* and *b* can be eliminated because both are positive.

The data cloud suggests a linear relationship, put the dots are not in a perfect line. A perfect correlation of ±1 would show all the dots in a perfect line. Therefore, we can eliminate asnswer choice d.

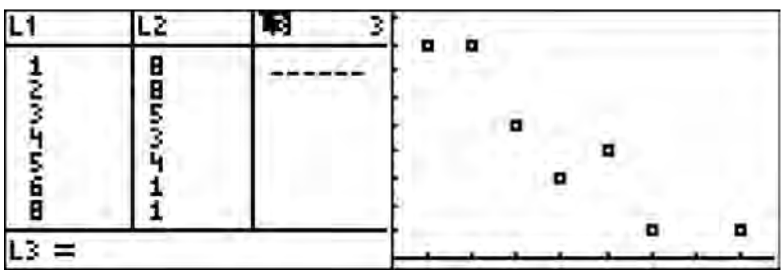
The correct answer is choice c.

DIMS: Does it make sense? Yes. The data cloud shows a negative correlation that is strong, but not perfect. Choice c is the best answer.

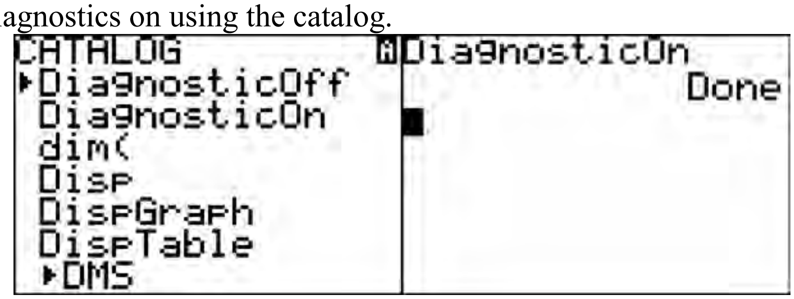
Strategy #2: Input the data from the chart in a graphing calculator and calculate the correlation coeffient using linear regression and the diagnostics on feature.

STEP 1. Creat a table of values from the graphing view of the function and input it into the graphing calculator.

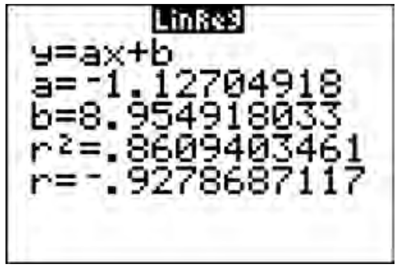
x	y
1	8
2	8
3	5
4	3
5	4
6	1
8	1



- **STEP 2**. Turn diagnostics on using the catalog.



- **STEP 3**. Determine which regression strategy will best fit the data. The graph looks like the graph of an linear function, so choose linear regression.
- **STEP 4**. Execute the appropriate regression strategy with diagnositcs on in the graphing calculator.



Round the correlation coefficient to the nearest hundredth: $r = -.93$

- **STEP 4**. Select answer choice c.

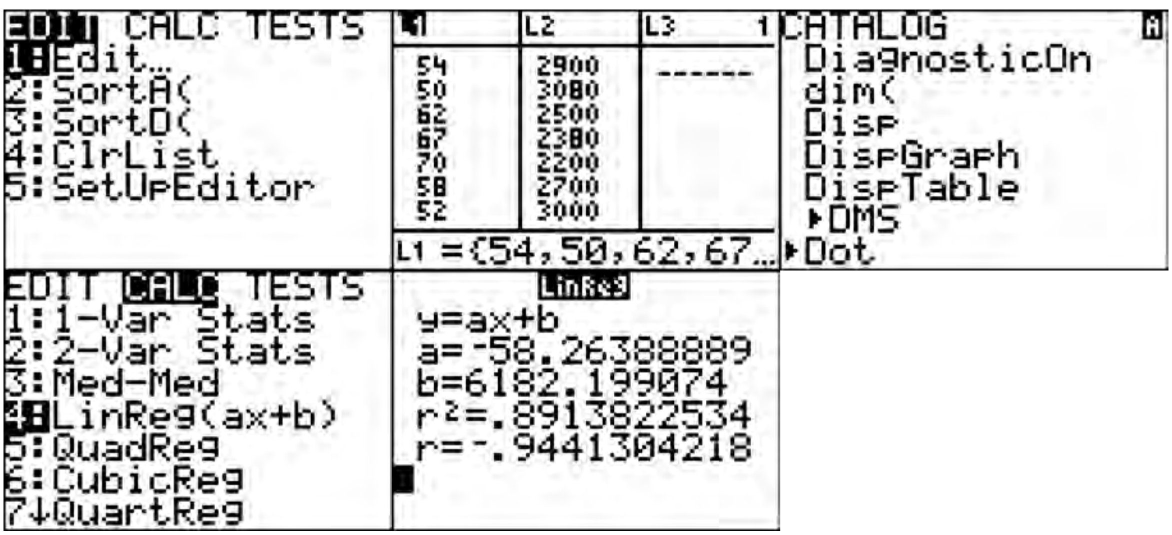
DIMS: Does it make sense? Yes. The data cloud shows a negative correlation that is strong, but not perfect. Choice c is the best answer.

PTS: 2 REF: 061411a1 NAT: S.ID.8 TOP: Correlation Coefficient and Residuals

3. ANS:
$f(t) = -58t + 6182$
$r = -.94$

The correlation coefficient indicates a strong linear relationship because the absolute value of r is close to 1.

Strategy: Input the table of values into the stats-editor of a graphing calculator, then use the stats-calc-linear regression with "diagnostics on" to obtain both the linear regression equation and the correlation coeffient (r). The following screenshots illustrate the solution using a TI-84 family graphing calculator.

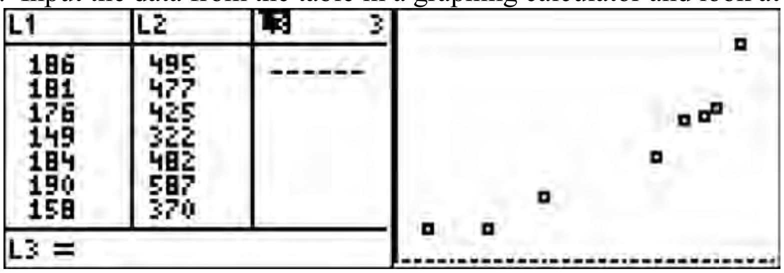


PTS: 4 REF: 011635ai NAT: S,ID.8 JMAP S.ID.6 NYSED
TOP: Regression KEY: linear

4. ANS:
$r \approx 0.94$. The correlation coefficient suggests that as calories increase, so does sodium.

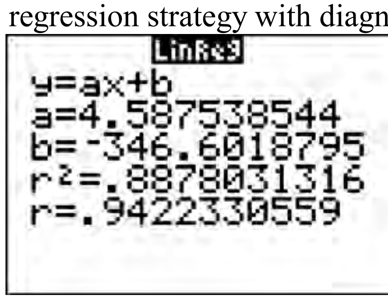
Strategy: Use data from the table and a graphing calculator to find both the regression equation and its correlation coefficient.

STEP 1. Input the data from the table in a graphing calculator and look at the data cloud.



- **STEP 2**. Turn diagnostics on using the catalog.

- **STEP 3**. Determine which regression strategy will best fit the data. The graph looks like the graph of an linear function, so choose linear regression.
- **STEP 4**. Execute the appropriate regression strategy with diagnositcs on in the graphing calculator.



Round the correlation coefficient to the nearest hundredth: $r = .94$

DIMS: Does it make sense? Yes. The data cloud and the table show a positive correlation that is strong, but not perfect. A correlation coefficient of .94 is positive, but not a perfectly straight line.

PTS: 4 REF: 011535a1 NAT: S.ID.8 TOP: Correlation Coefficient and Residuals

S.IDS.6c: Use Residuals to Assess Fit of a Function

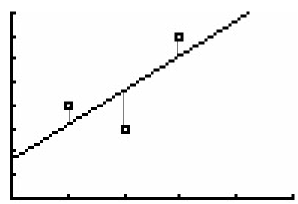
Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   c. Informally assess the fit of a function by plotting and analyzing residuals.

Vocabulary

A **residual** is the vertical distance between where a regression equation predicts a point will appear on a graph and the actual location of the point on the graph (scatterplot). If there is no difference between where a regression equation places a point and the actual position of the point, the **residual** is zero. A **residual** can also be understood as the difference in predicted and actual y-values (dependent variable values) for a given value of x (the independent variable).

$$residual = \left(actual\ \ y\ \ value\right)\left(predicted\ \ y\ \ value\right)$$



A **residual plot** is a scatter plot that shows the residuals as points on a vertical axis (y-axis) above corresponding (paired) values of the independent variable on the horizontal axis (x-axis).

Any *pattern* in a residual plot suggests that the regression equation is *not appropriate* for the data.

Big Ideas

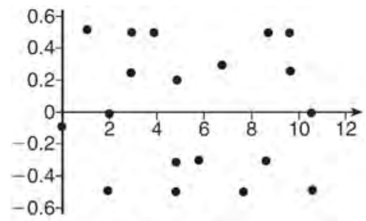
Patterns in residual plots are bad.
Residual plots with patterns indicate the regression equation is not a good fit.
Residual plots without patterns indicate the regression equation is a good fit.

A **residual plot** without a *pattern* and with a near equal distribution of points above and below the x-axis suggests that the regression equation is a *good fit* for the data.

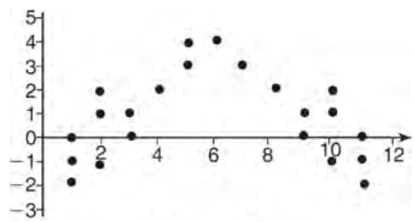
Residuals are automatically stored in graphing calculators when regression equations are calculated. To view a residuals scatterplot in the graphing calculator, you must use 2nd LIST to set the Y list variable to RESID, then use Zoom 9 to plot the residuals.

REGENTS PROBLEMS

1. The residual plots from two different sets of bivariate data are graphed below.



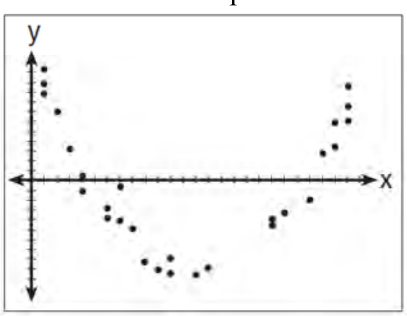
Graph A

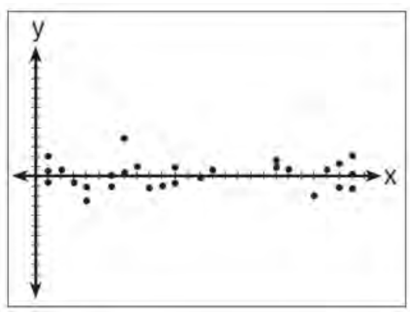


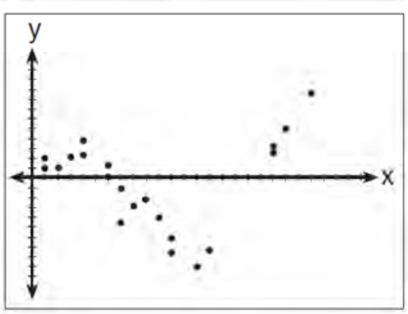
Graph B

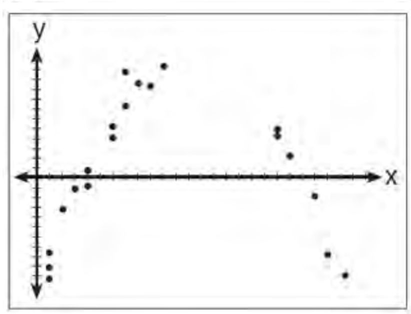
Explain, using evidence from graph *A* and graph *B*, which graph indicates that the model for the data is a good fit.

2. After performing analyses on a set of data, Jackie examined the scatter plot of the residual values for each analysis. Which scatter plot indicates the best linear fit for the data?

a. 

c. 

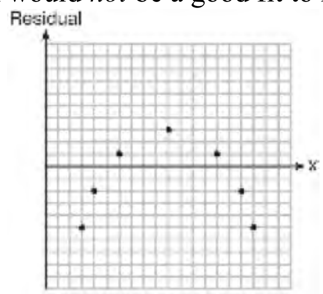
b. 

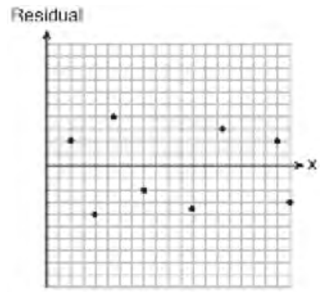
d. 

3. Which statistic would indicate that a linear function would *not* be a good fit to model a data set?

a. $r = -0.93$

b. $r = 1$

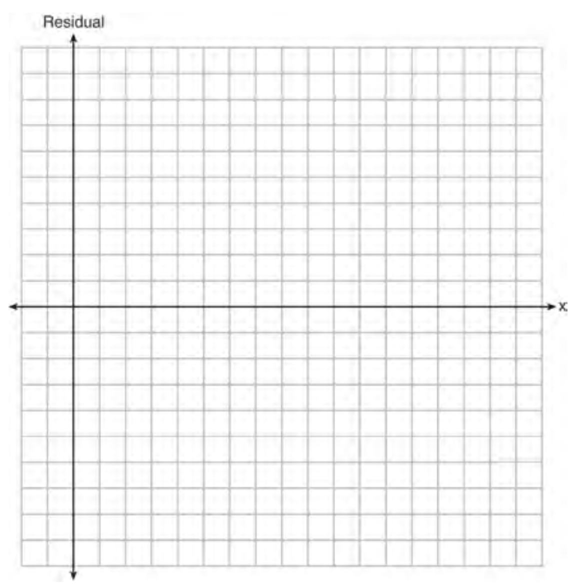
c. 

d. 

4. The table below represents the residuals for a line of best fit.

x	2	3	3	4	6	7	8	9	9	10
Residual	2	1	–1	–2	–3	–2	–1	2	0	3

Plot these residuals on the set of axes below.



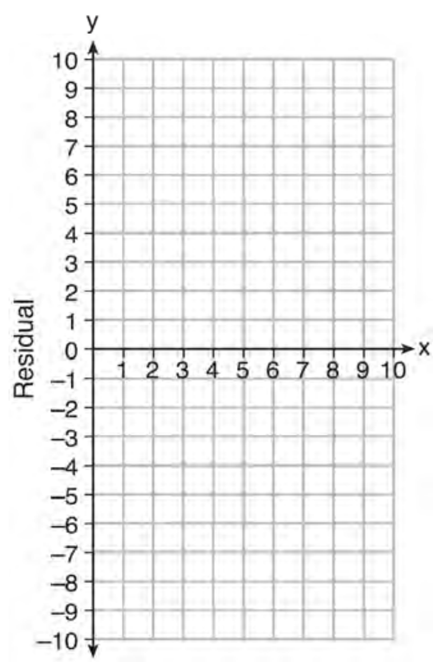
Using the plot, assess the fit of the line for these residuals and justify your answer.

5. Use the data below to write the regression equation ($y = ax + b$) for the raw test score based on the hours tutored. Round all values to the *nearest hundredth*.

Tutor Hours, x	Raw Test Score	Residual (Actual – Predicted)
1	30	1.3
2	37	1.9
3	35	–6.4
4	47	–0.7
5	56	2.0
6	67	6.6
7	62	–4.7

Equation: ___________________________

Create a residual plot on the axes below, using the residual scores in the table above.



Based on the residual plot, state whether the equation is a good fit for the data. Justify your answer.

S.ID.6c: Use Residuals to Assess Fit of a Function
Answer Section

1. ANS:
Graph A is a good fit because it does not have a clear pattern, whereas Graph B does have a clear pattern..

PTS: 2 REF: 061531AI NAT: S.ID.6b TOP: Correlation Coefficient and Residuals

2. ANS: C
For a residual plot, there should be no observable pattern and about the same number of dots above and below the x axis. Any pattern in a residual plot means that line is **not** a good fit for the data.

PTS: 2 REF: 011624ai NAT: S.ID.6 TOP: Correlation Coefficient and Residuals

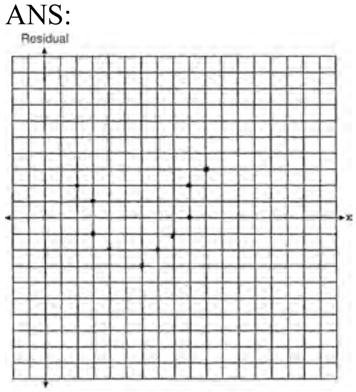
3. ANS: C
Strategy: Use knowledge of correlation coefficients and residual plots to determine which answer choice is **not** a good fit to model a data set.

STEP 1. A correlation coefficient close to -1 or 1 indicates a good fit, so answer choices a and b can be eliminated. Both suggest a good fit.

STEP 2. For a residual plot, there should be no observable pattern and a similar distribution of residuals above and below the *x*-axis. The residual plot in answer choice d shows a good fit, so answer choice d can be eliminated, leaving answer choice c as the correct answer.

DIMS? Does it make sense? Yes. The clear pattern in answer choice c tells us that the linear function is **not** a good fit to model the data set.

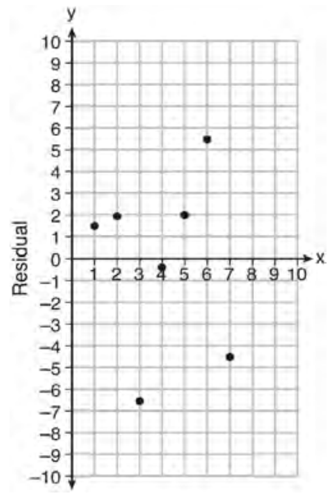
PTS: 2 REF: fall1303a1 NAT: S.ID.6c TOP: Correlation Coefficient and Residuals

4. ANS:


The line is a poor fit because the residuals form a pattern.

PTS: 2 REF: 081431a1 NAT: S.ID.6c TOP: Correlation Coefficient and Residuals

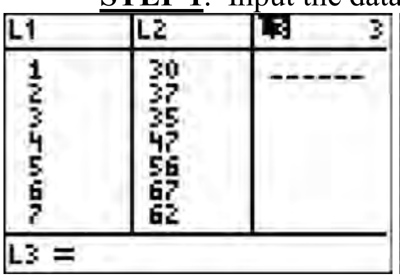
5. ANS:
$y = 6.32x + 22.43$



Based on the residual plot, the equation is a good fit for the data because the residual values are scattered without a pattern and are fairly evenly distributed above and below the x-axis.

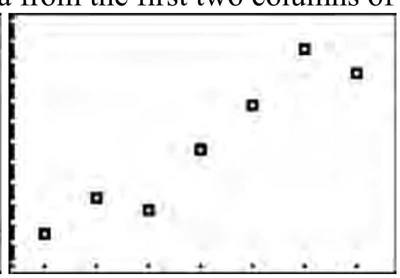
Strategies:
Use linear regression to find a regression equation that fits the first two columns of the table, then create a residuals plot using the first and third columns of the table to see if there is a pattern in the residuals.

- **STEP 1**. Input the data from the first two columns of the table into a graphing calculator.

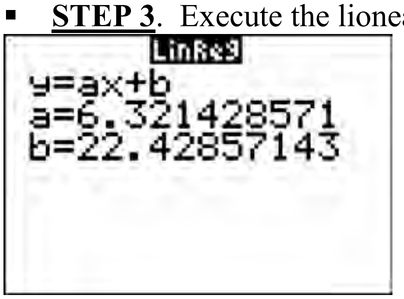


L1	L2	L3
1	30	------
2	37	
3	35	
4	47	
5	56	
6	67	
7	62	

L3 =



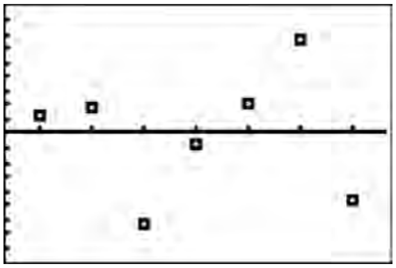
- **STEP 2**. Determine which regression strategy will best fit the data. The problem states that the regression equation should be in the form ($y = ax + b$), which means linear reression. The scatterplot produced by the graphing calculator also suggests linear regression.
- **STEP 3**. Execute the lionear regression strategy in the graphing calculator.



Round all values to the nearest hundredth: $y = 6.32x + 22.43$

- **STEP 4**. Plot the residual values on the graph provided using data from the first and third columns of the table. The graph shows a near equal number of points above the line and below the line, and the graph shows no pattern. The regression equation appears to be a good fit.

NOTE: The graphing calculator will also produce a residuals plot.



DIMS: Ask the question, “Does It Make Sense (DIMS)?” Yes. The regression equation produces the same residuals as shown in the table.

PTS: 4 REF: fall1314a1 NAT: S.ID.6c TOP: Correlation Coefficient and Residuals

S.ID.9: Correlation and Causation

Interpret linear models

9. Distinguish between correlation and causation.

Vocabulary

Correlation occurs when a change in one quantity is associated with, but *does not cause*, a change in another quantity,

Examples of Correlation:

The clock reads twelve noon and the sun is high in the sky.

The amount of ice cream sold is associated with the amount of hot chocolate sold.

Causation occurs when a change in one quantity *causes* a change in another quantity.

Examples of Causation:

The amount of flour used is associated with the number of loaves of bread baked.

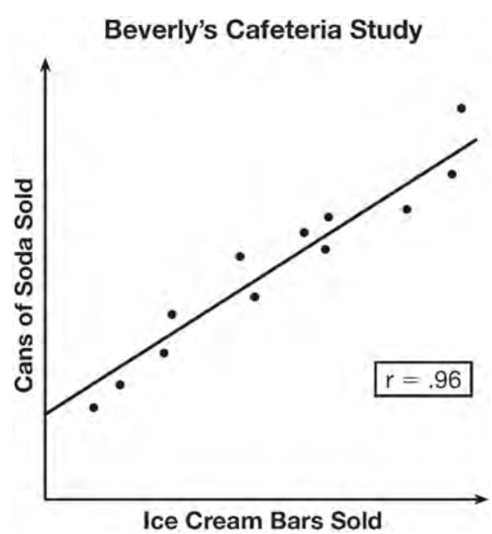
The number of calories burned and the amount of time spent exercising.

Big Idea

Correlation does not always mean causation.

REGENTS PROBLEMS

1. Beverly did a study this past spring using data she collected from a cafeteria. She recorded data weekly for ice cream sales and soda sales. Beverly found the line of best fit and the correlation coefficient, as shown in the diagram below.



Given this information, which statement(s) can correctly be concluded?

I. Eating more ice cream causes a person to become thirsty.

II. Drinking more soda causes a person to become hungry.

III. There is a strong correlation between ice cream sales and soda sales.

a. I, only
b. III, only
c. I and III
d. II and III

S.ID.9: Correlation and Causation
Answer Section

1. ANS: B
   Strategy: Determine the truth value of each statement, then determine which of the four answer choices best matches the truth values of the three statements.

   STEP 1. Determine the truth values of each statement:

   Statement I is **false**. Eating more ice cream **does not necessarily cause** a person to become thirsty.

   Statement II is **false**. Drinking more soda **does not necessarily cause** a person to become hungry.

   Statement III is **true**. **There is a strong correlation** between ice cream sales and soda sales.

   STEP 2. Use knowledge of correlation and causation to select the correct answer.

   Statement III is the only statement than can be correctly concluded. The answer is choice b.

   PTS: 2 REF: 061516AI NAT: S.ID.9 TOP: Analysis of Data

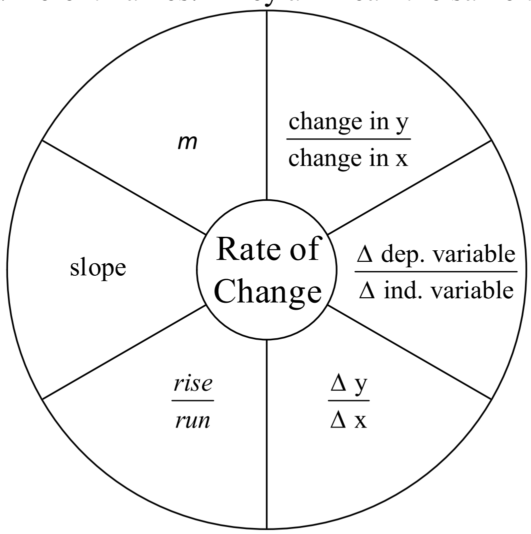
RATE

F.IF.6: Calculate and Interpret Average Rate of Change

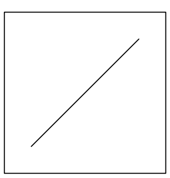
Interpret functions that arise in applications in terms of the context.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

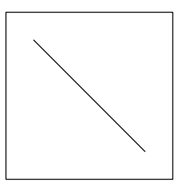
Rate of Change goes by many different names. They all mean the same thing,



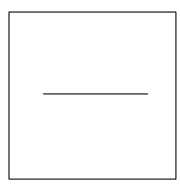
$$\text{Rate of Change} = \text{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{ dep. variable}}{\Delta \text{ ind. vaiable}} = \frac{\text{change in y}}{\text{change in x}}$$



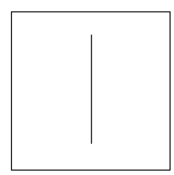
Positive Slope
Goes up from left to right.



Negative Slope.
Goes down from left to right.



Zero Slope.
A horizontal line has a slope of zero.



Undefined Slope.
A vertical line has an undefined slope.

Two ways to measure slope.

- Use the slope formula.
- Make a right triangle and measure the legs.

Slope Formula:

$$slope = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Measuring the Legs of Right Triangles:
You can use right triangles to measure or calculate the slope of any straight line.

1. Identify the coordinates of any two points on a line.
2. Determine if the slope of the line is positive or negative.3. Make a right triangle using the two given end-points as vertices on either end of the hypotenuse. (One leg will be parallel to the x-axis and the other leg will be parallel to the y-axis.)
4. Calculate or measure the height and the base of the right triangle.
5. Record the height and base of the triangle as a fraction in the form of
$$\frac{height}{base} = \frac{rise}{run} = m = slope.$$
6. When you combine your fraction with the sign of the slope of the line, you have the "algebraic" slope of the line.

Finding Slope from a Table of Values
A table of values shows paired values of independent and dependent variables, which makes it easy to use the slope formula to find the rate of change over specified intervals.
Example:

x values Independent Variable	y values Dependent Variable
2	7
3	9
5	13
9	21

The rate of change in this table is constant, meaning this table represents a linear function. Pick any two rows and calculate $\frac{\Delta y}{\Delta x}$. The result will always be $\frac{2}{1}$, or simply 2. It does not matter if you pick two adjacent rows or the top and bottom row. You can select the interval for which you wish to measure the rate of change.

Two ways to graph a line

- **Using two points**. Simply plot both points and sketch the line passing through them.
- **Using one point a slope**. Plot the given point first. Then, use slope and right triangles to find a second point. Then, sketch the line passing between them.
  - **Slope Intercept Form of a Line**. The equation $y = mx + b$ uses the point and slope method. The point used with this equation is always on the y-axis.

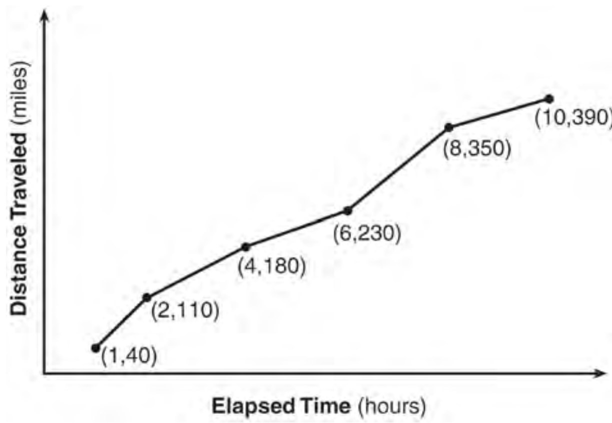
Direct Variation occurs when any of the following three conditions are met:

- The graph of the line passes through the origin (0,0),
- The table of values for a linear equation contains the ordered pair (0,0), or
- $b = 0$ or b is missing in the slope-intercept form of the equation of the line, as in $y = mx + 0$ or $y = mx$

Direct variation can be thought of as the set of all lines passing through the origin.
The **constant of variation is equal to the slope** of a line of direct variation.

REGENTS PROBLEMS

1. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.



During which interval was their average speed the greatest?

a. the first hour to the second hour	c. the sixth hour to the eighth hour
b. the second hour to the fourth hour	d. the eighth hour to the tenth hour

2. An astronaut drops a rock off the edge of a cliff on the Moon. The distance, $d(t)$, in meters, the rock travels after t seconds can be modeled by the function $d(t) = 0.8t^2$. What is the average speed, in meters per second, of the rock between 5 and 10 seconds after it was dropped?

a. 12	c. 60
b. 20	d. 80

3. Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?

a. 4.3	c. 5.4
b. 4.5	d. 6.0

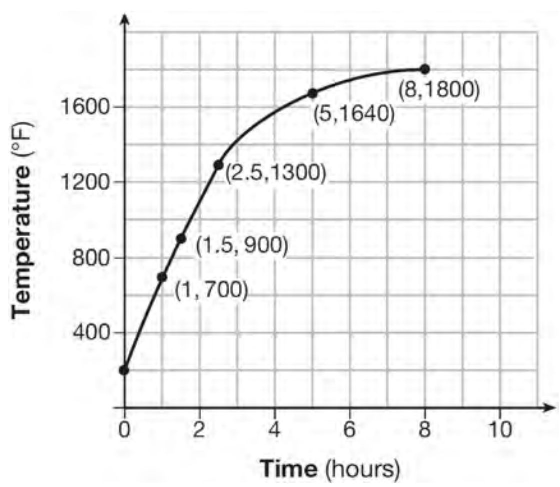
4. The table below shows the average diameter of a pupil in a person's eye as he or she grows older.

Age (years)	**Average Pupil Diameter** (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?

a. 2.4
b. 0.04
c. –2.4
d. –0.04

5. Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200ºF.



During which time interval did the temperature in the kiln show the greatest average rate of change?

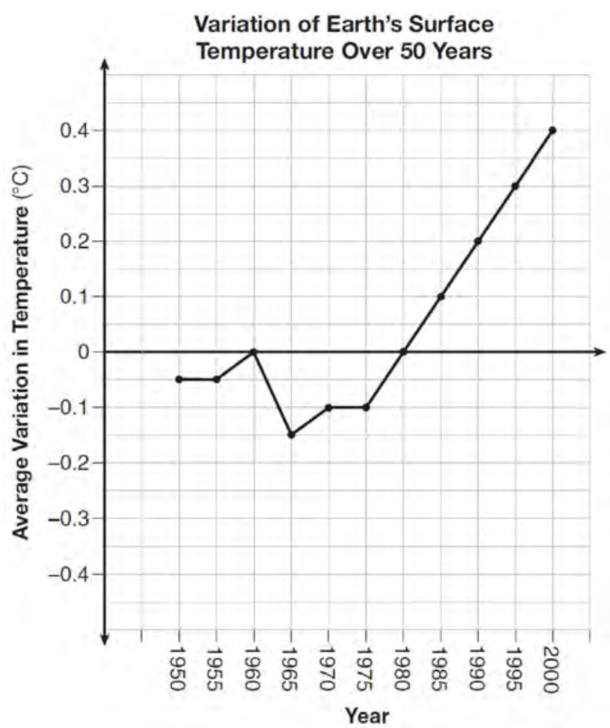
a. 0 to 1 hour
b. 1 hour to 1.5 hours
c. 2.5 hours to 5 hours
d. 5 hours to 8 hours

6. The table below shows the cost of mailing a postcard in different years. During which time interval did the cost increase at the greatest average rate?

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

a. 1898-1971
b. 1971-1985
c. 1985-2006
d. 2006-2012

7. The graph below shows the variation in the average temperature of Earth's surface from 1950-2000, according to one source.

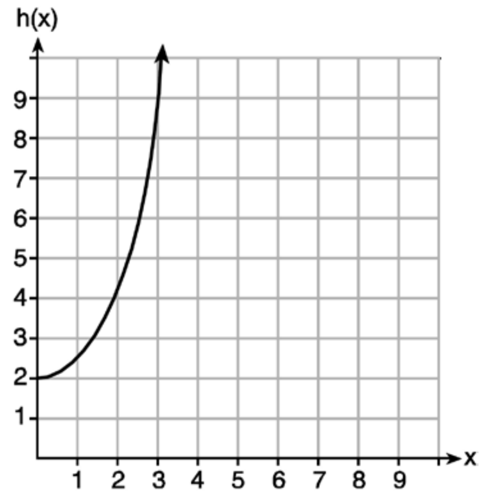


During which years did the temperature variation change the most per unit time? Explain how you determined your answer.

8. Given the functions g(x), f(x), and h(x) shown below:

$$g(x) = x^2 - 2x$$

x	f(x)
0	1
1	2
2	5
3	7



The correct list of functions ordered from greatest to least by average rate of change over the interval $0 \le x \le 3$ is

a. f(x), g(x), h(x)
b. h(x), g(x), f(x)
c. g(x), f(x), h(x)
d. h(x), f(x), g(x)

F.IF.6: Calculate and Interpret Rate of Change
Answer Section

1. ANS: A

Strategy: Equate speed with rate of change. $speed = \frac{\Delta y}{\Delta x} = \frac{rise}{run} = slope = rate\ of\ change$

Make a visual estimate of the steepest line segment on the graph, then use the slope formula to calculate the exact rates of change.

STEP 1. The line segment from (1, 40) to (2, 110) appears to be the steepest line segment in the graph. The line segment from (6, 230) to (8, 350) also seems very steep.

STEP 2. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$

The line segment from (1, 40) to (2, 110) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{110 - 40}{2 - 1} = \frac{70\ miles}{1\ hour}$.

The line segment from (6, 230) to (8, 350) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{350 - 230}{8 - 6} = \frac{120}{2} = \frac{60\ miles}{1\ hour}$.

PTS: 2 REF: 061418a1 NAT: F.IF.6 TOP: Rate of Change

2. ANS: A

Strategy: Use the formula for speed: speed$=\frac{\text{distance}}{\text{time}}$ and information from the problem to calculate average speed.

STEP 1. Calculate $d(t)$ for $t = 5$ and $t = 10$.

$d(t) = 0.8t^2$ and	$d(t) = 0.8t^2$
$d(5) = 0.8(5)^2$	$d(10) = 0.8(10)^2$
$d(5) = 0.8(25)$	$d(10) = 0.8(100)$
$d(5) = 20$	$d(5) = 80$

The rock had fallen 20 meters after 5 seconds and 80 meters after 10 seconds.
The total distance traveled was 60 meters in 5 seconds.

STEP 2: Use the speed formula to find average speed.

Substituting distance and time in the speed formula, speed$=\frac{\text{distance}}{\text{time}} = \frac{60\text{ meters}}{5\text{ seconds}} = \frac{12\text{ meters}}{1\text{ second}}$.

The rock's average speed between 5 and 10 seconds after being dropped was 12 meters per second.

DIMS? Does it make sense? Yes. The speed formula makes sense and the answer is expressed in meters per second as required by the problem.

PTS: 2 REF: 011521a1 NAT: F.IF.6 TOP: Rate of Change

3. ANS: C
Strategy: Use the slope formula and data from the table to calculate the exact rate of change over four enlargements.

STEP 1. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$ to compute the rate of change between (0, 15) and (4, 36.6).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{36.6 - 15}{4 - 0} = \frac{21.6}{4} = 5.4.$$

DIMS? Does it make sense? Yes. If you start with 15 and add 5.4 + 5.4 +5.4 + 5.4, you end up with 36.6. There were four enlargements and the average increase of each enlargement was 5.4 square inches.

PTS: 2 REF: 061511AI NAT: F.IF.6 TOP: Rate of Change

4. ANS: D
Strategy: Rate of change is the same as slope. Use the slope formula to find the rate of change between (20, 4.7) and (80, 2.3).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.3 - 4.7}{80 - 20} = \frac{-2.4}{60} = -0.04.$$

DIMS? Does it make sense? Yes. The average pupil diameter gets smaller very very slowly. Choices a and c are way too big and choices a and b indicate that the average pupil size is getting bigger rather than smaller.

PTS: 2 REF: 081414a1 NAT: F.IF.6 TOP: Rate of Change

5. ANS: A
Strategy: Equate rate of change with slope. Make a visual estimate of the steepest line segment on the graph, then use the slope formula to calculate the exact rates of change over given intervals.

STEP 1. The line segment from (0, 200) to (1, 700) appears to be the steepest line segment in the graph. The line segment from (1, 700) to (1.5, 900) also seems very steep. The rate of change gets slower as the temperature of the kiln gets hotter.

STEP 2. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$

The line segment from (0, 200) to (1, 700) has

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{700 - 200}{1 - 0} = \frac{500 \text{ degrees}}{1 \text{ hour}}.$$

The line segment from (1, 700) to (1.5, 900) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{900 - 700}{1.5 - 1} = \frac{200}{0.5} = \frac{400 \text{ degrees}}{1 \text{ hour}}$

The rate of change was greatest in the first hour.

DIMS? Does it make sense? Yes. The graph shows that rate of change slows down as time increases, so the first hour would have the greatest rate of change.

PTS: 2 REF: 081515ai NAT: F.IF.6 TOP: Rate of Change

6. ANS: D

Strategy: Find the average rate of change using the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

(a) $\dfrac{6-1}{1971-1898} = \dfrac{5}{73} \approx .07$

(b) $\dfrac{14-6}{1985-1971} = \dfrac{8}{14} \approx .57$

(c) $\dfrac{24-14}{2006-1985} = \dfrac{10}{21} \approx .48$

(d) $\dfrac{35-24}{2012-2006} = \dfrac{11}{6} \approx 1.83$

PTS: 2 REF: 011613ai NAT: F.IF.6 TOP: Rate of Change

7. ANS:
During 1960-1965, because the graph has the steepest slope during these years.

PTS: 2 REF: 011628ai NAT: F.IF.6 TOP: Rate of Change

8. ANS: D

Over the interval $0 \le x \le 3$, the average rate of change for $h(x) = \dfrac{9-2}{3-0} = \dfrac{7}{3}$, $f(x) = \dfrac{7-1}{3-0} = \dfrac{6}{3} = 2$, and $g(x) = \dfrac{3-0}{3-0} = \dfrac{3}{3} = 1$.

PTS: 2 REF: spr1301a1 NAT: F.IF.6 TOP: Rate of Change

N.Q.l: **Use Units to Solve Problems.**

N.Q.1: Use Units to Solve Problems

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

BIG IDEAS:

Units can cancel.

A **scale** is a ratio of the $\frac{\text{measurement of a model}}{\text{measurement of the real thing}}$.

Example. A toy car is 1 foot long. The real car it represents is 20 feet long. The scale of the model is:

$$\frac{\text{measurement of toy car}}{\text{measurement of real car}} = \frac{1\text{ feet}}{20\text{ feet}} = \frac{1}{20}$$

The word **scale** also refers to what you mark on the axes of a graph. The marks you make on a graph are called **scale** intervals, and the distance between each mark must be equal (represent the same number of units).

Conversions are sometimes necessary when working with units. A **conversion** occurs when you change the units of a scale.

Example: Suppose you are working with units that are expressed in feet, but want your answer to be in units expressed as inches.

Feet may be **converted** to inches by using the ratio of $\frac{12\text{ inches}}{1\text{ foot}}$.

Twelve feet may be **converted** to inches by using proportions (equivalent ratios), as follows:

$$\left.\frac{\text{inches}}{\text{feet}}\right| \frac{12}{1} = \frac{x}{12}$$

Using cross multiplication, we can solve for x.

$$12 \times 12 = 1 \times x$$

$$144 = x$$

The are 144 inches in 12 feet.

Per means "for each" when used with units.

miles **per** hour means miles for *each* hour and can be expressed as the ratio $\frac{\text{miles}}{1\text{ hour}}$

miles **per** gallon means miles for *each* gallon and can be expressed as the ratio $\frac{\text{miles}}{1\text{ gallon}}$

Cancellation of Units: Cancellation can be used with units.

Examples:

$$\frac{1\ yard}{3\ \cancel{feet}} \times \frac{27\ \cancel{feet}}{1} = \frac{1\ yard \times 27}{3} = \frac{27\ yards}{3}$$

To find the number of seconds in a year, use cancellation of units.

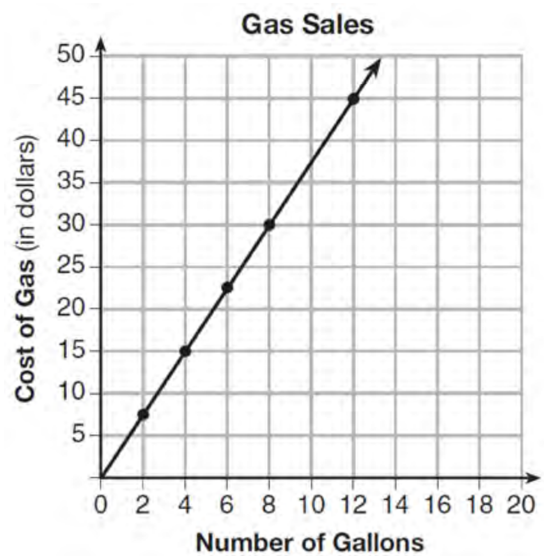
$$\frac{60 \text{ seconds}}{1 \cancel{\text{minute}}} \times \frac{60 \cancel{\text{minutes}}}{1 \cancel{\text{hour}}} \times \frac{24 \cancel{\text{hours}}}{1 \cancel{\text{day}}} \times \frac{365 \cancel{\text{days}}}{1 \text{ year}}$$

$$= \frac{60 \text{ seconds} \times 60 \times 24 \times 365}{1 \times 1 \times 1 \times 1 \text{ year}}$$

$$= \frac{30{,}536{,}000 \text{ seconds}}{1 \text{ year}}$$

REGENTS PROBLEMS

1. The graph below was created by an employee at a gas station.



Which statement can be justified by using the graph?

a. If 10 gallons of gas was purchased, \$35 was paid.
b. For every gallon of gas purchased, \$3.75 was paid.
c. For every 2 gallons of gas purchased, \$5.00 was paid.
d. If zero gallons of gas were purchased, zero miles were driven.

2. Peyton is a sprinter who can run the 40-yard dash in 4.5 seconds. He converts his speed into miles per hour, as shown below.

$$\frac{40\text{ yd}}{4.5\text{ sec}} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}}$$

Which ratio is *incorrectly* written to convert his speed?

a. $\frac{3\text{ ft}}{1\text{ yd}}$

b. $\frac{5280\text{ ft}}{1\text{ mi}}$

c. $\frac{60\text{ sec}}{1\text{ min}}$

d. $\frac{60\text{ min}}{1\text{ hr}}$

N.Q.l: Use Units to Solve Problems.
Answer Section

1. ANS: B
Strategy #1: Use the slope of the line to determine the cost per gallon of gas. Select any two points that are on intersections of vertical and horizontal gridlines, then substitute them into the slope formula to determine the rate of change, which is the cost per gallon of gas.

$$\textit{Select } (8,30) \textit{ and } (4,15)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30-15}{8-4} = \frac{15}{4} = \$3.75$$

or

$$\textit{Select } (12,45) \textit{ and } (8,30)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45-30}{12-8} = \frac{15}{4} = \$3.75$$

For every gallon of gas purchased. \$3.75 was paid.

Strategy #2. Eliminate wrong answers.
Choice (a) is wrong because the chart shows that 10 gallongs of gas costs \$37.50, not \$35.00.
Choice (b) is correct.
Choice (c) is wrong because the chart shows that 2 gallons of gas cost \$7.50, not \$5.00.
Choice (d) is wrong because the chart says nothing about the number of miles driven.

PTS: 2 REF: 011602ai NAT: A.CED.2 TOP: Graphing Linear Functions

2. ANS: B
Strategy: Work through each step of the problem and ask the DIMS question. Does It Make Sense.

STEP 1. $\dfrac{40\text{ yards}}{4.5\text{ seconds}} \times \dfrac{3\text{ feet}}{1\text{ yard}} = \dfrac{120\text{ feet}}{4.5\text{ seconds}}$ This makes sense. The yard units cancel and Peyton's speed becomes measured in feet per second instead of yards per second. We take the ratio of $\dfrac{120\text{ feet}}{4.5\text{ seconds}}$ to the next step in our analysis.

STEP 2. $\dfrac{120\text{ feet}}{4.5\text{ seconds}} \times \dfrac{5280\text{ feet}}{1\text{ mile}} = \dfrac{120 \times 5280\text{ feet}^2}{4.5\text{ second miles}}$. This does not make sense. The speed of a runner would not be measured in feet^2 per second miles. The problem is that the numerator and denominator are switched. It should be $\dfrac{1\text{ mile}}{5280\text{ feet}}$. When the numerator and denominator are changed, the problem becomes $\dfrac{120\text{ feet}}{4.5\text{ seconds}} \times \dfrac{1\text{ mile}}{5280\text{ feet}} = \dfrac{120\text{ miles}}{23{,}760\text{ seconds}}$. The feet units cancel and our measurement of Peyton's speed has distance over time, which makes sense. Answer choice b is selected to show that this ratio is *incorrectly* written.

STEP 3. Though we have solved the problem, we can continue our step by step analysis by taking the ratio of $\dfrac{120\text{ miles}}{23{,}760\text{ seconds}}$ to the next step in our analysis. The problem now becomes $\dfrac{120\text{ miles}}{23{,}760\text{ seconds}} ' \dfrac{60\text{ seconds}}{1\text{ minute}} = \dfrac{120 \times 60\text{ miles}}{23{,}760 \times 1\text{minutes}} = \dfrac{72{,}000\text{ miles}}{23{,}760\text{ minutes}}$. This makes sense. The seconds units cancel and we again have distance over miles. We take the ratio $\dfrac{72{,}000\text{ miles}}{23{,}760\text{ minutes}}$ to the next step.

STEP 4. $\dfrac{72{,}000\text{ miles}}{23{,}760\text{ minutes}} \times \dfrac{60\text{ minutes}}{1\text{ hour}} = \dfrac{72{,}000 \times 60\text{ miles}}{23{,}760 \times 1\text{ hours}} = \dfrac{432{,}000\text{ miles}}{23{,}760\text{ hours}} = 18\dfrac{2}{11}$ miles per hour. This makes sense. Peyton is a fast sprinter.

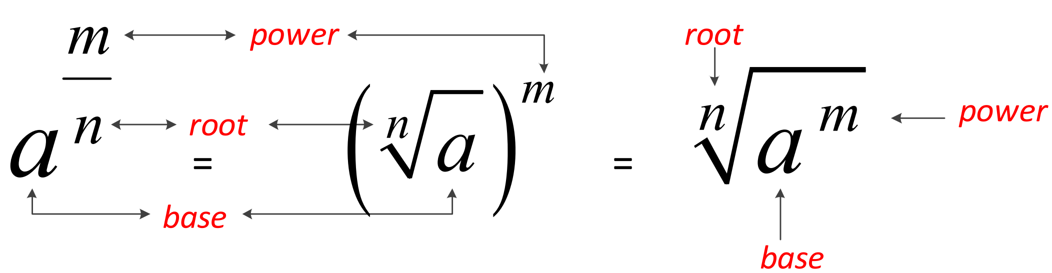
PTS: 2 REF: 011502a1 NAT: N.Q.1 TOP: Conversions

A.SSE.3c: Use Properties of Exponents to Transform Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.?3?

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* $(1.15)^t$ *can be rewritten as* $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*



Rules for Rational Exponents:

Rule: For any nonzero number a, $a^0 = 1$, and $a^{-n} = \dfrac{1}{a^n}$

Rule: For any nonzero number a and any rational numbers m and n, $a^m \cdot a^n = a^{m+n}$

Rule: For any nonzero number a and any rational numbers m and n, $(a^m)^n = a^{mn}$

Rule: For any nonzero numbers a and b and any rational number n $(ab)^n = a^n b^n$

Rule: For any nonzero number a and any rational numbers m and n, $\dfrac{a^m}{a^n} = a^{m-n}$

A number is in **<u>scientific notation</u>** if it is written in the form $a \times 10^n$, where n is an integer and $1 \le |a| < 10$

REGENTS PROBLEMS

1. Miriam and Jessica are growing bacteria in a laboratory. Miriam uses the growth function $f(t) = n^{2t}$ while Jessica uses the function $g(t) = n^{4t}$, where n represents the initial number of bacteria and t is the time, in hours. If Miriam starts with 16 bacteria, how many bacteria should Jessica start with to achieve the same growth over time?

a. 32	c. 8
b. 16	d. 4

2. Jacob and Jessica are studying the spread of dandelions. Jacob discovers that the growth over t weeks can be defined by the function $f(t) = (8) \cdot 2^t$. Jessica finds that the growth function over t weeks is $g(t) = 2^{t+3}$.

   Calculate the number of dandelions that Jacob and Jessica will each have after 5 weeks.

   Based on the growth from both functions, explain the relationship between $f(t)$ and $g(t)$.

A.SSE.3c: Use Properties of Exponents to Transform Expressions
Answer Section

1. ANS: D

Understanding the Problem.

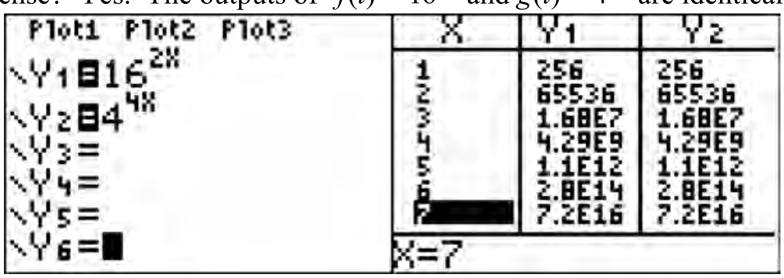
Miriam's exponential growth function is modeled by $f(t) = n^{2t}$. The problem tells us that n equals 16, so Miriam's exponential growth function can be rewritten as $f(t) = 16^{2t}$

Jessica's exponential growth function is modeled by $g(t) = n^{4t}$. The quantity n is unknown for Jessica's exponential growth function and the problem wants us to find the value of n that will make $f(t) = g(t)$.

Strategy: Substitute equivalent expresseions for $f(t)$ and $g(t)$, then solve for n.

$$\begin{array}{ccccc}
f(t) = g(t) & \text{or} & f(t) = g(t) & \text{or} & f(t) = g(t) \\
16^{2t} = n^{4t} & & 16^{2t} = n^{4t} & & 16^{2t} = n^{4t} \\
16^{2t} = \left(n^2\right)^{2t} & & 16^2 = n^4 & & 16^2 = n^4 \\
16 = n^2 & & 256 = n^4 & & \sqrt{16^2} = \sqrt{n^4} \\
4 = n & & 256^{\frac{1}{4}} = \left(n^4\right)^{\frac{1}{4}} & & 16 = n^2 \\
 & & 4 = n & & 4 = n
\end{array}$$

DIMS? Does It Make Sense? Yes. The outputs of $f(t) = 16^{2t}$ and $g(t) = 4^{4t}$ are identical.



PTS: 2 REF: 011519a1 NAT: A.SSE.3c TOP: Solving Exponential Equations

2. ANS:
Jacob and Jessica will both have 256 dandelions after 5 weeks.

$f(t) = 8 \cdot 2^t$	$g(t) = 2^{t+3}$
$f(5) = (8) \cdot 2^5$	$g(5) = 2^{5+3}$
$f(5) = 8 \cdot 32$	$g(5) = 2^8$
$f(5) = 256$	$g(5) = 256$

Both functions express the same mathematical relationships.

$f(t) = g(t)$

$8 \cdot 2^t = 2^{t+3}$

$8 \cdot 2^t = 2^t \cdot 2^3$

$8 \cdot 2^t = 2^t \cdot 8$

PTS: 2 REF: 011632ai NAT: A.SSE.3 TOP: Exponential Equations

F.IF.8b: Use Properties of Exponents to Interpret Expressions

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02)^t$, $y=(0.97)^t$, $y=(1.01)^{12t}$, $y=(1.2)^{\frac{t}{10}}$ and classify them as representing exponential growth or decay.

An **exponential function** is a function that contains a variable for an exponent.

Example: $y=2^x$

Exponential functions are useful for modeling real world events such as:

- population growth
- bacteria growth
- radioactive decay
- compound interest on investments
- concentrations of medicine in the body
- concentrations of pollutants in the environment

Exponential growth is modeled by the formula: $A=P(1+r)^t$

Exponential decay is modeled by the formula: $A=P(1-r)^t$

In both formulas:

A represents the amount after growth or decay.
P represents the initial (starting) amount.
r represents the rate of growth or decay for each time cycle.
t represents the number of time cycles.

Sample Problem:
On January 1, 1999, the price of gasoline was \$1.39 per gallon. If the price of gasoline increased by 0.5% per month, what was the cost of one gallon of gasoline, to the *nearest cent*, on January 1 one year later?

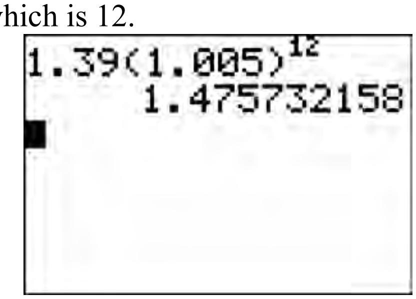
Solution:
This is exponential growth. The price is increasing.

A represents the amount after twelve months.
P represents the initial (starting) amount, which is \$1.39.
r represents the rate of growth or decay for each month, which is 0.5% per month..
t represents the number of time cycles, which is 12.

Write the exponential growth equation:

$A=\$1.39(1+0.5\%)^{12}$

$A=1.39(1.005)^{12}$



The cost of gasoline one year later would be \$1.48.

REGENTS PROBLEMS

1. The value in dollars, $v(x)$, of a certain car after x years is represented by the equation $v(x) = 25{,}000(0.86)^x$. To the *nearest dollar*, how much more is the car worth after 2 years than after 3 years?
   a. 2589
   b. 6510
   c. 15,901
   d. 18,490

2. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, y, of one account that was left inactive for a period of x years. What is the y-intercept of this equation and what does it represent?
   a. 0.98, the percent of money in the account initially
   b. 0.98, the percent of money in the account after x years
   c. 5000, the amount of money in the account initially
   d. 5000, the amount of money in the account after x years

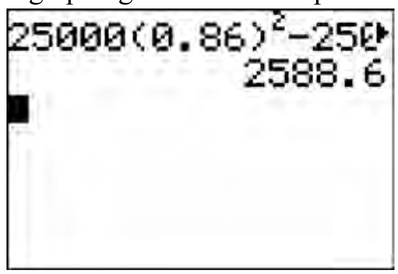
3. For a recently released movie, the function $y = 119.67(0.61)^x$ models the revenue earned, y, in millions of dollars each week, x, for several weeks after its release. Based on the equation, how much more money, in millions of dollars, was earned in revenue for week 3 than for week 5?
   a. 37.27
   b. 27.16
   c. 17.06
   d. 10.11

4. Dylan invested \$600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

5. The number of carbon atoms in a fossil is given by the function $y = 5100(0.95)^x$, where x represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

F.IF.8b: Use Properties of Exponents to Interpret Expressions
Answer Section

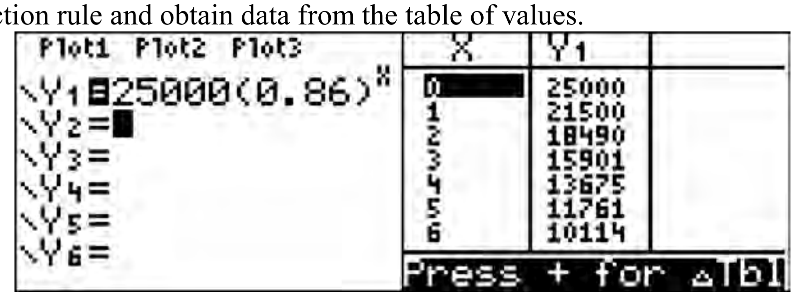
1. ANS: A
   Strategy #1
   Input $25,000(0.86)^2 - 25,000(0.86)^3$ into a graphing calculator and press enter.



$$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$$

Strategy #2: Input the function rule in a graphing calculator and obtain the value of the car after 2 years and 3 years from the table of values. Then, compute the difference.

STEP 1: Input the function rule and obtain data from the table of values.

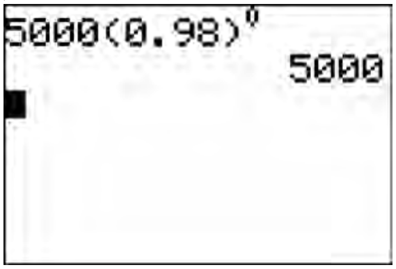


STEP 2: Compare the value of the car after 2 years and after 3 years.
The car is worth \$18,490 after 2 years.
The car is worth \$15,901 after 3 years.
The difference is $18490 - 15901 = 2589$
$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$

PTS: 2 REF: 011508a1 NAT: F.IF.8b TOP: Evaluating Exponential Expressions

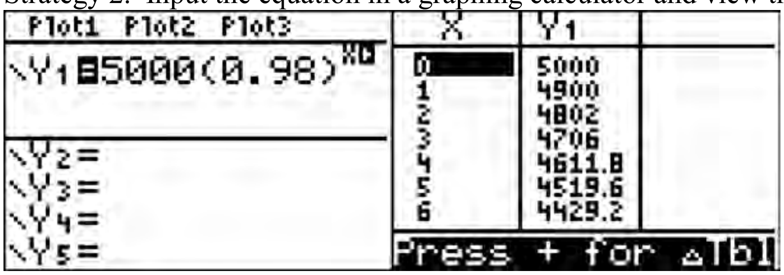
2. ANS: C

Strategy 1: The y-intercept of a function occurs when the value of x is 0. The strategy is to evaluate the function $y = 5000(0.98)^x$ for $x = 0$



This represents the amount of money in the account before exponential decay begins.

Strategy 2. Input the equation in a graphing calculator and view the table of values.

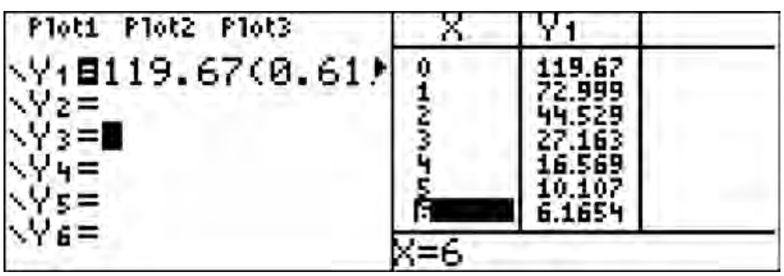


The table of values clearly shows the initial value of the account and its exponential decay.

PTS: 2 REF: 011515a1 NAT: F.IF.8b TOP: Modeling Exponential Equations

3. ANS: C

Strategy #1. Input the function rule in a graphing calculator, then use the table of values to identify the revenues earned in weeks 3 and 5, then compute the difference.



The table of values shows that the movie earned 27.163 million dollars in week 3.
The table of values shows that the movie earned 10.107 million dollars in week 5.
The difference is $(27.163 - 10.107) = 17.056$

Strategy #2. Use a graphing calculator to evaluate the expression $119.67(0.61)^5 - 119.67(0.61)^3$, which equals 17.056..

PTS: 2 REF: 011603ai NAT: F.IF.2 TOP: Evaluating Functions

4. ANS:
After 2 years, the balance in the account is $619.35.

Strategy: Write an exponential growth equation to model the problem. Then solve the equation for two years.

STEP 1: Exponential growth is modeled by the formula $A(t) = P(1 + r)^t$, where:
A represents the amount after t cycles of growth,
P represents the starting amount, which is $600.
r represents the rate of growth, which is 1.6% or .016 as a decimal, and
t represents the number of cycles of growth, which are measured in years with annual compounding.

The equation is: $A(t) = 600(1 + .016)^t$

STEP 2: Solve for two years growth.

$$A(t) = 600(1 + .016)^t$$
$$A(2) = 600(1 + .016)^2$$
$$A(2) = 600(1.016)^2$$
$$A(2) = 600(1.032256)$$
$$A(2) = 619.35$$

DIMS: Does It Make Sense? Yes. Each year, the interest on each $100 is $1.60, so the first year, there will be $6 \times \$1.60 = \$9.60$ interest. The second year interest will be another $9.60 for the original $600 plus 1.6% on the $9.60. The total interest after two years will be $\$9.60 + \$9.60 + .016(\$9.60) \approx \$19.35$. Add this interest to the original $600 and the amount in the account will be $619.35.

PTS: 2 REF: 061529AI STA: F.IF.B TOP: Modeling Exponential Equations
NOT: NYSED classifies this problem as A.CED.1

5. ANS:
The percent of change each year is 5%.

Strategy: Use information from the problem together with the standard formula for exponential decay, which is $A = P(1-r)^t$, where A represents the amount remaining, P represents the initial amount, r represents the rate of decay, and t represents the number of cycles of decay.

$$A = P(1-r)^t$$

$$y = 5100(0.95)^x$$

The structures of the equations show that $(1-r) = 0.95$.

Solving for r shows that $r = 0.05$, or 5%.

$$(1-r) = 0.95$$
$$-r = 0.95 - 1$$
$$-r = -0.05$$
$$r = 0.05$$

PTS: 2 REF: 081530ai NAT: F.IF.8b TOP: Modeling Exponential Functions
NOT: NYSED classifies this problem as F.LE.5.

EQUATIONS AND INEQUALITIES

A.SSE.1: Terms, Factors, & Coefficients of Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

Vocabulary

Equation An **equation** consists of two *expressions* connected by an equal sign. The equal sign indicates that both *expressions* have the same (equal) value. The two expressions in an **equation** are typically called the *left expression* and the *right expression*.

Expression An **expression** is a mathematical statement or phrase consisting of one or more *terms*. *Terms* are the building blocks of **expressions**, similar to the way that letters are the building blocks of words.

Term A **term** is a *number*, a *variable*, or the *product* of numbers and variables.

- **Terms** in an expression are always separated by a plus sign or minus sign.
- **Terms** in an expression are always either positive or negative.
- Numbers and variables connected by the operations of division and multiplication are parts of the same **term**.
- **Terms**, together with their signs, can be moved around within the same expression without changing the value of the expression. If you move a **term** from the left expression to the right expression, or from the right expression to the left expression (across the equal sign), the plus or minuse sign associated with the term must be changed.

Variable A variable is a quantity whose value can change or vary. In algebra, a letter is typically used to represent a variable. The value of the letter can change. The letter x is commonly used to represent a variable, but other letters can also be used. The letters s, o, and sometimes l are avoided by some students because they are easily confused in equations with numbers.

Variable Expression A mathematical phrase that contains at least one variable.

Example: The equation $2x+3 = 5$ contains a left expression and a right expression. The two expressions are connected by an equal sign. The expression on the left is a variable expression containing two terms, which are $+2x$ and $+3$. The expression on the right contains only one term, which is $+5$.

Coefficient: A **coefficient** is the numerical factor of a term in a polynomial. It is typically thought of as the number in front of a variable.
Example: 14 is the **coefficient** in the term $14x^3y$.

Factor: A **factor** is:

1) a whole number that is a **divisor** of another number, or
2) an algebraic expression that is a **divisor** of another algebraic expression.

Examples:

- 1, 2, 3, 4, 6, and 12 all divide the number 12,
  so 1, 2, 3, 4, 6, and 12 are all factors of 12.
- $(x-3)$ and $(x+2)$ will divide the trinomial expression x^2-x-6,
  so $(x-3)$ and $(x+2)$ are both factors of the x^2-x-6.

BIG IDEAS

Exponential growth can be modeled by the function $A = P(1+r)^t$, where:
A represents the current amount,
P represents the starting amount,
$(1+r)$ represents the rate of growth per cycle, and
t represents the number of gowth cycles
Students should be able to interpet any part of of an exponential growth function in terms of the real world context that the function represents.

REGENTS PROBLEMS

1. The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. The yearly rate of appreciation of the comic book is

   a. 17%
   b. 1.7%
   c. 1.017%
   d. 0.017%

2. The owner of a small computer repair business has one employee, who is paid an hourly rate of \$22. The owner estimates his weekly profit using the function $P(x) = 8600 - 22x$. In this function, x represents the number of

   a. computers repaired per week
   b. hours worked per week
   c. customers served per week
   d. days worked per week

3. To watch a varsity basketball game, spectators must buy a ticket at the door. The cost of an adult ticket is \$3.00 and the cost of a student ticket is \$1.50. If the number of adult tickets sold is represented by a and student tickets sold by s, which expression represents the amount of money collected at the door from the ticket sales?

a. $4.50as$
b. $4.50(a+s)$
c. $(3.00a)(1.50s)$
d. $3.00a + 1.50s$

4. The equation $A = 1300(1.02)^7$ is being used to calculate the amount of money in a savings account. What does 1.02 represent in this equation?

a. 0.02% decay
b. 0.02% growth
c. 2% decay
d. 2% growth

A.SSE.1: Terms, Factors, & Coefficients of Expressions
Answer Section

1. ANS: B

   Strategy: Identify each of the parts of the function $V(t) = 1350(1.017)^t$, then answer the question.

   $V(t)$ represents the current value of the comic book in dollars.
   1350 represents the original value of the comic book when it was purchased.
   (1.017) represents the growth factor, which consists of (1+r), where r is the rate of growth per year. The value of r is 0.017, which is found by subtracting 1 from (1.017).
   t represents the number of years since its purchase.

   The problem wants to know the value of r, which is 0.017. However, all of the answer choices are expressed as percents rather than decimals. A decimal may be converted to a percent as follows:

   $$\frac{.017}{1} = \frac{x\%}{100\%}$$
   $$.017 \times 100 = x\%$$
   $$1.7\% = x\%$$
   $$\frac{.017}{1} = \frac{1.7\%}{100\%}$$

   The yearly appreciation rate of the comic book is 1.7% and the correct answer is b.

   DIMS? Does It Make Sense? The appreciation rate seems to make sense, but it is difficult to understand why someone would originally pay $1,350 for a comic book.

   PTS: 2 REF: 061517AI NAT: A.SSE.1b TOP: Modeling Exponential Equations

2. ANS: B

   The problem states that the employee is paid an hourly rate of $22.
   In the equation $P(x) = 8600 - 22x$, the hourly rate of $22 appears next to the letter x, which is a *variable* representing the number of hours that the employee works.

   DIMS (Does it Make Sense?)
   Yes. The equation $P(x) = 8600 - 22x$ says that the owner's profit (P) is a function of how much the employee gets paid. As the value of x increases, the employee gets paid more and the owner's profits get smaller.

   PTS: 2 REF: 011501a1 NAT: A.SSE.1a TOP: Modeling Linear Equations

3. ANS: D

   Strategy: Translate the words into mathematical expressions.

   a × 3.00 + s × 1.50
   The cost of an adult ticket is $3.00 and the cost of a student ticket is $1.50.

   $$a(3.00) + s(1.50)$$
   $$3.00a + 1.50s$$

   PTS: 2 REF: 081503ai NAT: A.SSE.1 TOP: Modeling Linear Equations

4. ANS: D

Strategy: Use the formula for exponential growth or decay, which is $A = P(1 \pm r)^t$, where
A represents the amount after t growth or decay cycles.
P represents the starting amount.
r represents the rate of growth expressed as a decimal, and
t represents the number of growth or decay cycles.

In the equation $A = 1300(1.02)^7$, the number 1.02 corresponds to $(1 \pm r)$, so write

$$\begin{aligned} 1.02 &= 1 \pm r \\ 1.02 - 1 &= r \\ .02 &= r \\ 2\% &= r \end{aligned}$$

1.02 means that the growth rate is 2%.

PTS: 2 REF: 011608ai NAT: A.SSE.1 TOP: Modeling Exponential Functions

A.CED.1: Create Equations and Inequalities in One Variable

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

BIG IDEAS

Translating words into mathematical symbols is an important skill in mathematics. The process involves first identifying key words and operations and second converting them to mathematical symbols.

Sample Regents Problem

Tanisha and Rachel had lunch at the mall. Tanisha ordered three slices of pizza and two colas. Rachel ordered two slices of pizza and three colas. Tanisha's bill was \$6.00, and Rachel's bill was \$5.25. What was the price of one slice of pizza? What was the price of one cola?

Step 1: Underline key terms and operations
Tanisha ordered three slices of pizza and two colas.
Rachel ordered two slices of pizza and three colas.
Tanisha's bill was \$6.00, and
Rachel's bill was \$5.25

Step 2.:
Convert to mathematic symbolism

Tanisha ordered $\frac{3P}{\text{three slices of pizza}}$ $\frac{+}{\text{and}}$ $\frac{2C}{\text{two colas.}}$

Rachel ordered $\frac{2P}{\text{two slices of pizza}}$ $\frac{+}{\text{and}}$ $\frac{3C}{\text{three colas}}$.

Tanisha's bill was $\frac{6}{\$6.00}$, and

Rachel's bill was $\frac{5.25}{\$5.25}$

Step 3.:
Write the final expressions/equations
Tanisha: $3P+2C=6$
Rachel: $2P+3C=5.25$

Different Views of a Function

Students should understand the relationships between **four views of a function** and be able to move from one view to any other view with relative ease, The four views of a function are:

1) the description of the function in words
2) the function rule (equation) form of the function
3) the graph of the function, and
4) the table of values of the function.

REGENTS PROBLEMS

1. Sam and Jeremy have ages that are consecutive odd integers. The product of their ages is 783. Which equation could be used to find Jeremy's age, j, if he is the younger man?
   a. $j^2 + 2 = 783$
   b. $j^2 - 2 = 783$
   c. $j^2 + 2j = 783$
   d. $j^2 - 2j = 783$

2. The length of the shortest side of a right triangle is 8 inches. The lengths of the other two sides are represented by consecutive odd integers. Which equation could be used to find the lengths of the other sides of the triangle?
   a. $8^2 + (x + 1) = x^2$
   b. $x^2 + 8^2 = (x + 1)^2$
   c. $8^2 + (x + 2) = x^2$
   d. $x^2 + 8^2 = (x + 2)^2$

3. John has four more nickels than dimes in his pocket, for a total of \$1.25. Which equation could be used to determine the number of dimes, x, in his pocket?
   a. $0.10(x + 4) + 0.05(x) = \$1.25$
   b. $0.05(x + 4) + 0.10(x) = \$1.25$
   c. $0.10(4x) + 0.05(x) = \$1.25$
   d. $0.05(4x) + 0.10(x) = \$1.25$

4. A cell phone company charges \$60.00 a month for up to 1 gigabyte of data. The cost of additional data is \$0.05 per megabyte. If d represents the number of additional megabytes used and c represents the total charges at the end of the month, which linear equation can be used to determine a user's monthly bill?
   a. $c = 60 - 0.05d$
   b. $c = 60.05d$
   c. $c = 60d - 0.05$
   d. $c = 60 + 0.05d$

5. Connor wants to attend the town carnival. The price of admission to the carnival is \$4.50, and each ride costs an additional 79 cents. If he can spend at most \$16.00 at the carnival, which inequality can be used to solve for r, the number of rides Connor can go on, and what is the maximum number of rides he can go on?

   a. $0.79 + 4.50r \le 16.00$; 3 rides
   b. $0.79 + 4.50r \le 16.00$; 4 rides
   c. $4.50 + 0.79r \le 16.00$; 14 rides
   d. $4.50 + 0.79r \le 16.00$; 15 rides

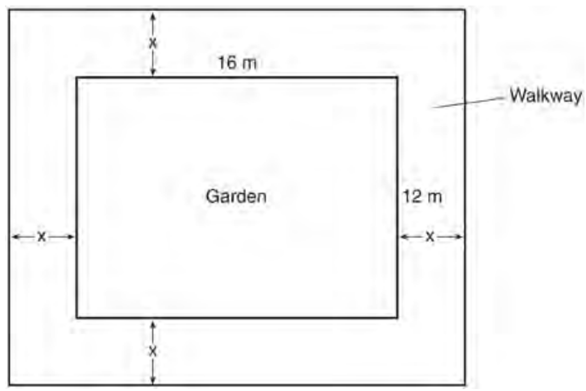
6. A rectangular picture measures 6 inches by 8 inches. Simon wants to build a wooden frame for the picture so that the framed picture takes up a maximum area of 100 square inches on his wall. The pieces of wood that he uses to build the frame all have the same width. Write an equation or inequality that could be used to determine the maximum width of the pieces of wood for the frame Simon could create. Explain how your equation or inequality models the situation. Solve the equation or inequality to determine the maximum width of the pieces of wood used for the frame to the *nearest tenth of an inch*.

7. Jackson is starting an exercise program. The first day he will spend 30 minutes on a treadmill. He will increase his time on the treadmill by 2 minutes each day. Write an equation for $T(d)$, the time, in minutes, on the treadmill on day d. Find $T(6)$, the minutes he will spend on the treadmill on day 6.

8. Donna wants to make trail mix made up of almonds, walnuts and raisins. She wants to mix one part almonds, two parts walnuts, and three parts raisins. Almonds cost \$12 per pound, walnuts cost \$9 per pound, and raisins cost \$5 per pound. Donna has \$15 to spend on the trail mix. Determine how many pounds of trail mix she can make. [Only an algebraic solution can receive full credit.]

9. A school is building a rectangular soccer field that has an area of 6000 square yards. The soccer field must be 40 yards longer than its width. Determine algebraically the dimensions of the soccer field, in yards.

10. A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x, the width of the walkway. Describe how your equation models the situation. Determine and state the width of the walkway, in meters.

11. New Clarendon Park is undergoing renovations to its gardens. One garden that was originally a square is being adjusted so that one side is doubled in length, while the other side is decreased by three meters. The new rectangular garden will have an area that is 25% more than the original square garden. Write an equation that could be used to determine the length of a side of the original square garden. Explain how your equation models the situation. Determine the area, in square meters, of the new rectangular garden.

12. A landscaper is creating a rectangular flower bed such that the width is half of the length. The area of the flower bed is 34 square feet. Write and solve an equation to determine the width of the flower bed, to the *nearest tenth of a foot*.

13. A typical cell phone plan has a fixed base fee that includes a certain amount of data and an overage charge for data use beyond the plan. A cell phone plan charges a base fee of \$62 and an overage charge of \$30 per gigabyte of data that exceed 2 gigabytes. If C represents the cost and g represents the total number of gigabytes of data, which equation could represent this plan when more than 2 gigabytes are used?

a. $C = 30 + 62(2 - g)$	c. $C = 62 + 30(2 - g)$
b. $C = 30 + 62(g - 2)$	d. $C = 62 + 30(g - 2)$

14. A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by x, and the area of the garden is 108 square meters. Determine, algebraically, the dimensions of the garden in meters.

15. Joe has a rectangular patio that measures 10 feet by 12 feet. He wants to increase the area by 50% and plans to increase each dimension by equal lengths, x. Which equation could be used to determine x?

a. $(10 + x)(12 + x) = 120$	c. $(15 + x)(18 + x) = 180$
b. $(10 + x)(12 + x) = 180$	d. $(15)(18) = 120 + x^2$

A.CED.1: Create Equations and Inequalities
Answer Section

1. ANS: C
   Strategy: Deconstruct the problem to find the information needed to write the equation.

   Let j represent Jeremy's age. The last sentence says j represents Jeremy's age.

   Let $\left(j+2\right)$ represent Sam's age. The problem tells us that Sam and Jeremy have ages that are **consecutive odd integers**. The consecutive odd integers that could be ages are $\{1,3,5,7,9,\ldots\ldots\}$ and each odd integer is 2 more that the odd integer before it. Thus, if Jeremy is 2 years younger than Sam, as the problem says, then Sam's age can be represented as $\left(j+2\right)$.

   The second sentence says, "The product of their ages is 783." Product is the result of multiplication, so we can write $j\left(j+2\right)=783$. Since this is not an answer choice, we must manipulate the equation:

$$j\left(j+2\right)=783$$

$$j^2+2j=783$$

   Our equation is now identical to answer choice *c*, which is the correct answer.

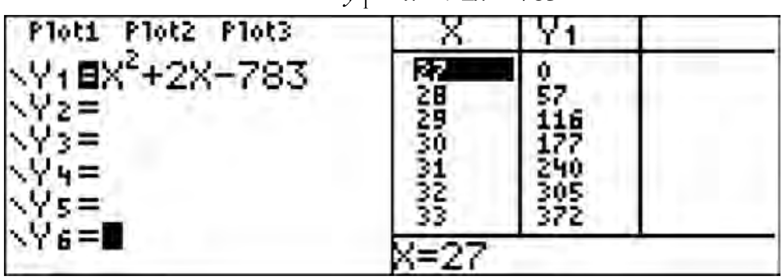
   DIMS? Does It Make Sense? Yes. Jeremy is 27 and Sam is 29. The product of their ages is $27\times 29=783$. In order to input this into a graphing calculator, the equation must be transformed as follows:

$$j^2+2j=783$$

$$j^2+2j-783=0$$

$$0=j^2+2j-783$$

$$y_1=x^2+2x-783$$



   PTS: 2 REF: 081409a1 NAT: A.CED.1 TOP: Modeling Quadratics

2. ANS: D
Strategy: Use the Pythagorean Theorem, the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

The shortest side must be one of the legs, since the longest side is always the hypotenuse.
Substitute 8 for a in the equation.

$$a^2 + b^2 = c^2$$

$$8^2 + b^2 = c^2$$

The lengths of sides b and c are consecutive odd integers. Let x represent the smaller odd integer and let $(x+2)$ represent the larger consecutive odd integer. Side c must be represented by $(x+2)$ because side c represents the hypotenuse, which is always the longest side of a right triangle. Therefore, side b is represented by x and side c is represented by $(x+2)$. Substitute these values into the equation.

$$8^2 + b^2 = c^2$$

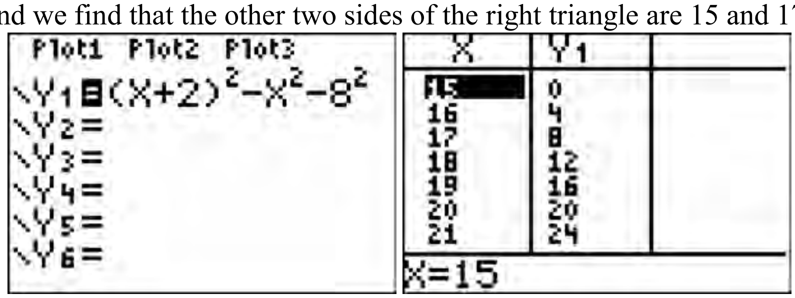
$$8^2 + x^2 = (x+2)^2$$

By using the commutative property to rearrange the two terms in the right expression, we obtain the same equation as answer choice d.

$$8^2 + x^2 = (x+2)^2$$

$$x^2 + 8^2 = (x+2)^2$$

DIMS? Does It Make Sense? Yes. Transorm the equation for input into a graphing calculator as follows:
$0 = (x+2)^2 - x^2 - 8^2$ and we find that the other two sides of the right triangle are 15 and 17.



By the Pythagorean Theorem, $8^2 + 15^2 = 17^2$

$$64 + 225 = 289$$

$$289 = 289$$

Everything checks!

PTS: 2 REF: spr1304a1 NAT: A.CED.1 TOP: Geometric Applications of Quadratics

3. ANS: B
Strategy: This is a coin problem, and the value of each coin is important.

Let x represent the number of dimes, as required by the problem.
Let .10x represent the value of the dimes. (A dime is worth $0.10)

The problem says that John has 4 more nickels than dimes.
Let $(x+4)$ represent the number of nickels that John has.
Let $.05(x+4)$ represent the value of the nickles. (A nickel is worth $0.05)

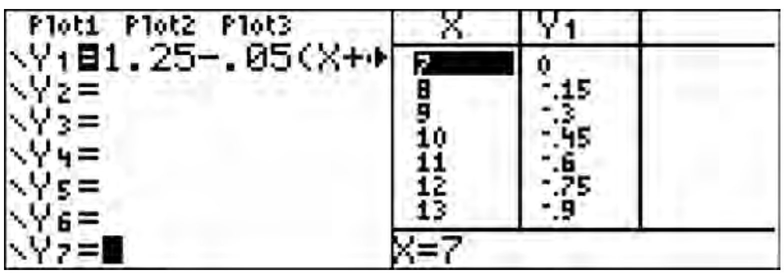
The total amount of money that John has is $1.25.
The total amount of money that John has can also be represented by $.10x+.05(x+4)$
These two expressions are both equal, so write:

$$.10x+.05(x+4)=\$1.25$$

This is not an answer choice, but using the commutative property, we can rearrange the order of the terms in the left expression $.05(x+4)+.10x=\$1.25$, which is the same as answer choice b.

DIMS? Does It Make Sense? Yes. Transform the equation for input into a graphing calculator as follows:
$.05(x+4)+.10x=\$1.25$

$0=\$1.25-.05(x+4)-.10x$



John has 7 dimes and 11 nickles. The dimes are worth 70 cents and the nickels are word 55 cents. In total, John has $1.25.

PTS: 2 REF: 061416a1 NAT: A.CED.1 TOP: Modeling Linear Equations

4. ANS: D
Strategy: Translate the words into algebraic terms and expressions. Then eliminate wrong answers.

The problem tells us to:
Let c represent the total charges at the end of the month.
Let 60 represent the cost of 1 gigabyte of data.
Let d represent the cost of each megabyte of data after the first gigabyte.

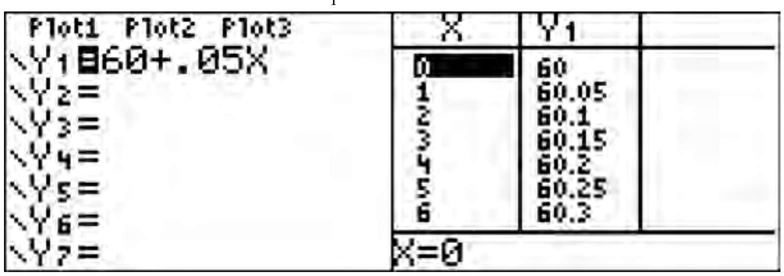
The total charges equal 60 plus .05d.
Write $c = 60 + .05d$. This is answer choice d.

DIMS? Does It Make Sense? Yes. $c = 60 + .05d$ could be used to represent the user's monthly bill. First, transpose the formula for input into the graphing calculator:

$$c = 60 + .05d$$

$$0 = 60 + .05x$$

$$Y_1 = 60 + .05x$$



The table of values shows that the monthly charges increase 5 cents for every additional megabyte of data.

PTS: 2 REF: 061422a1 NAT: A.CED.1 TOP: Modeling Linear Equations

5. ANS: C
Strategy: Write and solve an inequality that relates total costs to how much money Connor has.

STEP 1. Write the inequality:
The price of admission comes first and is \$4.50. Write +4.50
Each ride (r) costs an additional 0.79. Write $+0.79r$
Total costs can be expressed as: $4.50 + 0.79r$
$4.50 + 0.79r$ must be less than or equal to the \$16 Connor has.
Write: $4.50 + 0.79r \le 16.00$

STEP 2: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$4.50 + 0.79r$	$\le$	16.00
Subtract 4.50 from both expressions	$0.79r$	$\le$	11.50
Divide both expressions by 0.79	r	$\le$	$\frac{11.50}{.79}$
Simplify	r	$\le$	14.55696203
Interpret	r	$\le$	14 rides

The correct answer choice is c: $4.50 + 0.79r \le 16.00$; 14 rides

DIMS? Does It Make Sense? Yes. Admissions costs \$4.50 and 14 rides cost $14 \times .79 = \$11.06$. After 14 rides, Connor will only have 45 cents left, which is not enough to go on another ride.

$$\$16 - (\$4.50 + \$11.05)$$

$$\$16 - (\$15.55)$$

$$\$0.45$$

PTS: 2 REF: 011513a1 NAT: A.CED.1 TOP: Modeling Linear Inequalities

6. ANS:
The maximum width of the frame should be 1.5 inches.

Strategy: Write an inequality, then solve it.

STEP 1: Write the inequality.
The picture is 6 inches by 8 inches. The area of the picture is (6×8) square inches.
The width of the frame is an unknown variable represented by x.

Two widths of the frame (2x) must be added to the length and width of the picture. Therefore, the area of the picture with frame is $(6+2x)(8+2x)$ square inches
The area of the picture with frame, $(6+2x)(8+2x)$ square inches, must be less than or equal (≤) to 100.
Write $(6+2x)(8+2x) \leq 100$

STEP 2: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$(6+2x)(8+2x)$	$\leq$	100
Use Distributive Property to Clear Parentheses	$48+12x+16x+4x^2$	$\leq$	100
Commutative Property	$4x^2+12x+16x+48$	$\leq$	100
Combine Like Terms	$4x^2+28x+48$	$\leq$	100
Subtract 100 from both expressions	$4x^2+28x-52$	$\leq$	0

Use the Quadratic Formula: a=4, b=28, c=-52

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-28 \pm \sqrt{28^2-4(4)(-52)}}{2(4)}$$

$$x = \frac{-28 \pm \sqrt{1616}}{8}$$

$$x = \frac{-28 \pm \sqrt{1616}}{8}$$

$$x = \frac{-28 \pm 40.1995}{8}$$

$$x = \frac{-28 + 40.1995}{8}$$

$$x = \frac{12.1995}{8}$$

$x = 1.5$ inches

DIMS? Does It Make Sense? Yes. If the frame is 1.5 inches wide, then the total picture with frame will be

$$(6+2\times 1.5)(8+2\times 1.5)$$

$$(9)(11)$$

$$99 \text{ square inches}$$

PTS: 6 REF: 081537ai NAT: A.CED.1 TOP: Geometric Applications of Quadratics

7. ANS:

$T(d)=2d+28$

Jackson will spend 40 minutes on the treadmill on day 6.

Strategy: Start with a table of values, then write an equation that models both the table view and the narrative view of the function. Then, use the equation to determine the number of minutes Jackson will spend on the treadmill on day 6.

STEP 1: Model the narrative view with a table view.

d	1	2	3	4	5	6	7	8	9
$T(d)$	30	32	34	36	38	40	42	44	46

STEP 2: Write an equation.

$$T(d)=30+2(d-1)$$

$$T(d)=30+2d-2$$

$$T(d)=28+2d$$

STEP 3: Use the equation to find the number of minutes Jackson will spend on the treadmill on day 6.

$$T(d)=28+2d$$

$$T(6)=28+2(6)$$

$$T(6)=40$$

DIMS? Does It Make Sense? Yes. Both the equation and the table of values predict that Jackson will spend 40 minutes on the treadmill on day 6.

PTS: 2 REF: 081532ai NAT: A.CED.1 TOP: Modeling Linear Functions

8. ANS:
Donna can make 2 pounds of trail mix.

Strategy 1: Determine the costs of six pounds of mix, then scale the amount down to $15 of mix.

STEP 1. The mix will have six parts. If each part is 1 pound, the costs of the mix can be determined as follows:
$12 for one part almonds @ $12 per pound,
$18 for two parts walnuts @ $9 per pound, and
$15 for three parts raisins @ $5 per pound.
$45 for six pounds of mix.

STEP 2: Scale the amount down to $15 of mix

$$\left.\frac{Cost}{Pounds}\right| \quad \frac{\$45}{6} = \frac{\$15}{x}$$

$$45x = 6(15)$$

$$45x = 90$$

$$x = 2$$

Donna can make 2 pounds of trail mix.

DIMS? Does It Make Sense? Yes. If 2 pounds of the mix cost $15, 3 times as much should cost $45.

Strategy 2. Write an expression that scales the costs of the mix to $15.

Let x represent the scale factor.

$$\text{Write}\begin{bmatrix}\text{(1lb. almonds @ \$12 per lb.)} \times \text{scale factor} + \\ \text{(2lbs. walnuts @ \$9 per lb.)} \times \text{scale factor} + \\ \text{(3lbs. raisins @ \$5 per lb.)} \times \text{scale factor}\end{bmatrix} = \$15$$

$$12x + (2 \times 9)x + (3 \times 5)x = 15$$

$$12x + 18x + 15x = 15$$

$$45x = 15$$

$$x = \frac{15}{45}$$

$$x = \frac{1}{3}$$

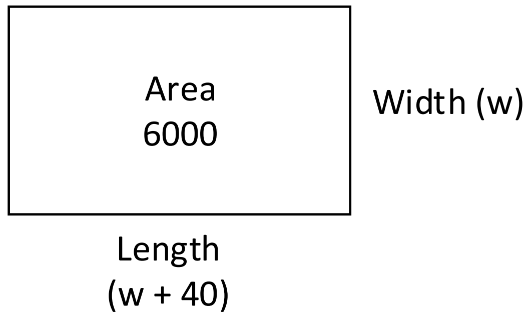
The scale factor is $\frac{1}{3}$. If an entire batch of trail mix contains 6 pounds of ingredients, Donna needs to scale the recipe down and make only $\frac{1}{3}$ of that amount. In other words, Donna needs to make $\frac{1}{3} \times 6 = 2$ pounds of trail mix if she only has $15 to spend.

PTS: 2 REF: spr1305a1 NAT: A.CED.1 TOP: Modeling Linear Equations

9. ANS:
The soccer field is 60 yards wide and 100 yards long.

Strategy: Draw and label a picture, then use the picture to write and solve an equation based on the area formula: Area = length × width

STEP 1: Draw and label a picture.



STEP 2: Write and solve an equation based on the area formula: Area = width × length

$$\begin{aligned}
6000 &= w(w+40) \\
6000 &= w^2 + 40w \\
0 &= w^2 + 40w - 6000 \\
0 &= (w+100)(w-60) \\
w &= -100 \text{ reject - distance should be positive} \\
w &= 60 \\
w + 40 &= 100
\end{aligned}$$

DIMS? Does It Make Sense? Yes. If the width of the soccer field is 60 yards and the length of the soccer field is 100 yards, then the area of the soccer field will be 6,000 square yards, as required by the problem.

PTS: 4 REF: 081436a1 NAT: A.CED.1 TOP: Geometric Applications of Quadratics

10. ANS:
a) $396 = (16 + 2x)(12 + 2x)$.
b) The length, $16 + 2x$, and the width, $12 + 2x$, are multiplied and set equal to the area.
c) The width of the walkway is 3 meters.

Strategy: Use the picture, the area formula ($Area = length \times width$), and information from the problem to write an equation, then solve the equation.

STEP 1. Use the area formula, the picture, and information from the problem to write an equation.

$$Area = length \times width$$

$$396 = (16 + 2x)(12 + 2x)$$

STEP 2. Solve the equation.

$$396 = (16 + 2x)(12 + 2x)$$

$$396 = (16 \times 12) + (16 \times 2x) + (2x \times 12) + (2x \times 2x)$$

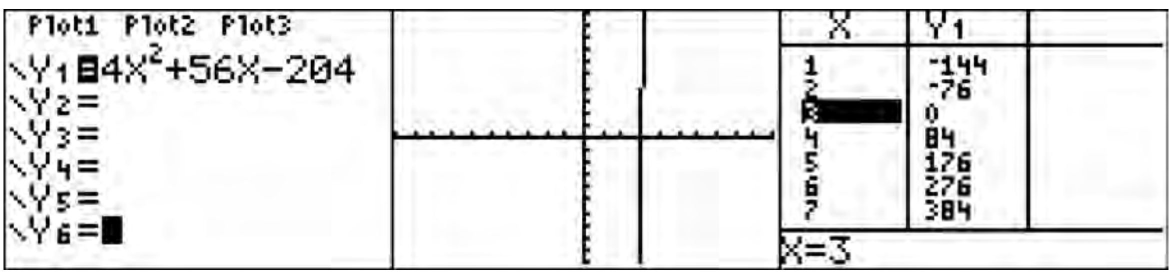
$$396 = 192 + 32x + 24x + 4x^2$$

$$396 = 192 + 56x + 4x^2$$

$$396 = 4x^2 + 56x + 192$$

$$0 = 4x^2 + 56x + 192 - 396$$

$$0 = 4x^2 + 56x - 204$$



The width of the walkway is 3 meters.

DIMS? Does It Make Sense? Yes. The garden plus walkway is $16 + 2(3) = 22$ meters long and $12 + 2(3) = 18$ meters wide. $Area = 22 \times 18 = 396$, which fits the information in the problem.

PTS: 4 REF: 061434a1 NAT: A.CED.1 TOP: Geometric Applications of Quadratics

11. ANS:

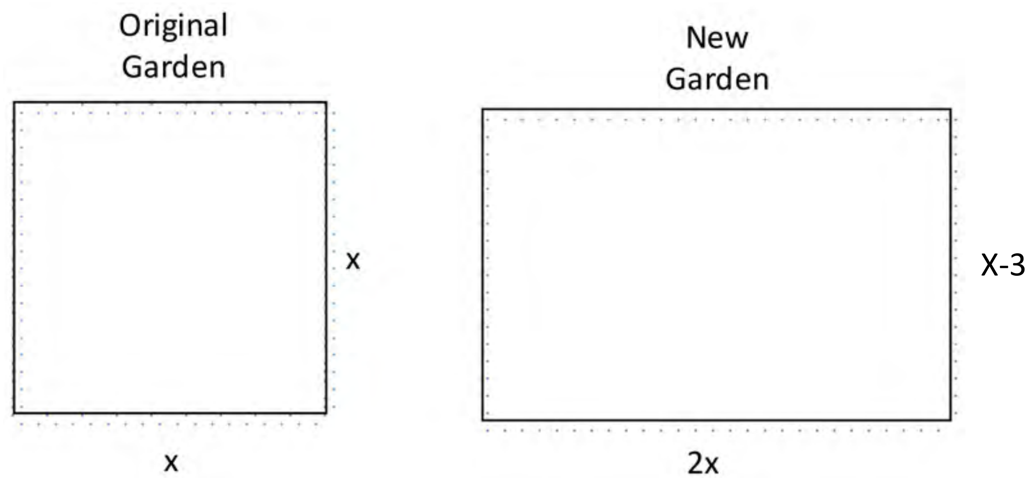
a) $1.25x^2 = (2x)(x-3)$

b) Because the original garden is a square, x^2 represents the original area, $x-3$ represents the side decreased by 3 meters, $2x$ represents the doubled side, and $1.25x^2$ represents the new garden with an area 25% larger.

c) The length of a side of the original square garden was 8 meters.
The area of the new rectangular garden is 80 square meters.

Strategy: Draw two pictures: one picture of the garden as it was in the past and one picture of the garden as it will be in the future. Then, write and solve an equation to determine the length of a side of the original garden.

STEP 1. Draw 2 pictures.



Area of original garden is x^2. Area of new garden is $1.25x^2$.

STEP 2: Use the area formula, A = length × width, to write an equation for the area of the new garden.

$$A = \text{length} \times \text{width}$$

$$1.25x^2 = (2x)(x-3)$$

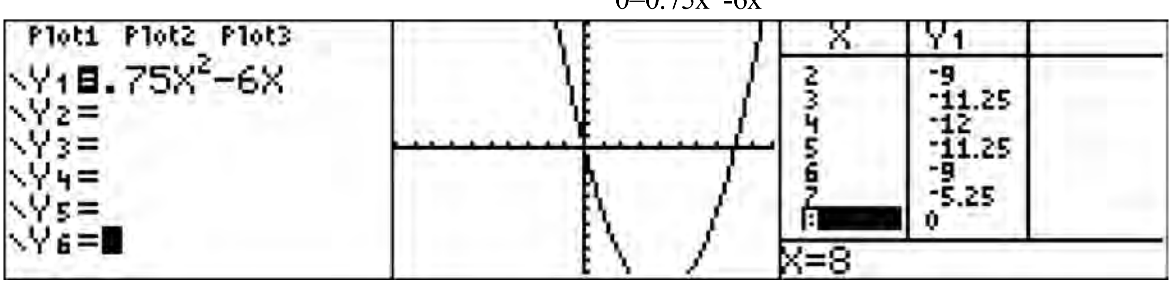
STEP 3: Transform the equation for input into a graphing calculator and solve.

$$1.25x^2 = (2x)(x-3)$$

$$1.25x^2 = 2x^2 - 6x$$

$$0 = 2x^2 - 1.25x^2 - 6x$$

$$0 = 0.75x^2 - 6x$$



The length on a side of the original square garden was 8 meters.

The area of the new garden is $1.25(8)^2 = 1.25(64) = 80$ square meters.

DIMS? Does It Make Sense? Yes. The dimensions of the original square garden are 8 meters on each side and the area was 64 square meters. The dimensions of the new rectangular garden are 16 meters length and 5 meters width. The new garden will have area of 80 meters. The area of the new garden is 1.25 times the area of the original garden.

PTS: 6 REF: 011537a1 NAT: A.CED.1 TOP: Geometric Applications of Quadratics

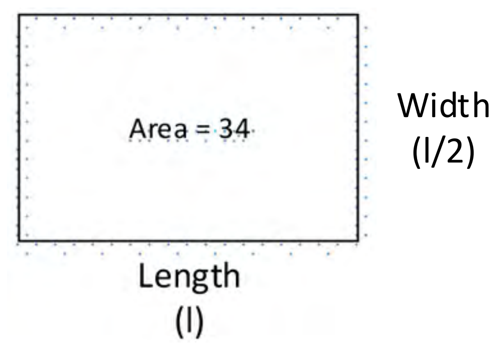
12. ANS:

a) Equation$34 = l\left(\frac{1}{2}l\right)$

b) The width of the flower bed is approximately 4.1 feet.

Strategy: Draw a picture, then write and solve an equation based on the area formula, Area = length × width.

STEP 1. Draw a picture.



STEP 2: Write and solve an equation based on the area formula.

$$\text{Area} = \text{length} \times \text{width}.$$

$$34 = l\left(\frac{l}{2}\right)$$

$$34 = \frac{l^2}{2}$$

$$68 = l^2$$

$$\sqrt{68} = \sqrt{l^2}$$

$$8.2 \approx l$$

$$4.1 \approx w$$

PTS: 2 REF: 061532AI NAT: A.CED.1 TOP: Geometric Applications of Quadratics

13. ANS: D
Strategy: Translate the words into algebraic terms and expressions. Then eliminate wrong answers.

The problem tells us to:
Let C represent the total cost.
Let g represent the number of gigabytes used.

The first sentence, "*A typical cell phone plan has a* ***fixed base fee*** *that includes a certain amount of data and an* ***overage charge*** *for data use beyond the plan*." tells us that total cost equals a base fee plus an overage charge. From this, we know that the basic equation will look something like

$$C = fixed\ base\ fee + overage\ ch\arg e$$

The second sentence tells us that "A cell phone plan charges a base fee of \$62 ...." so we can substitute this specific information into our general equation and we have

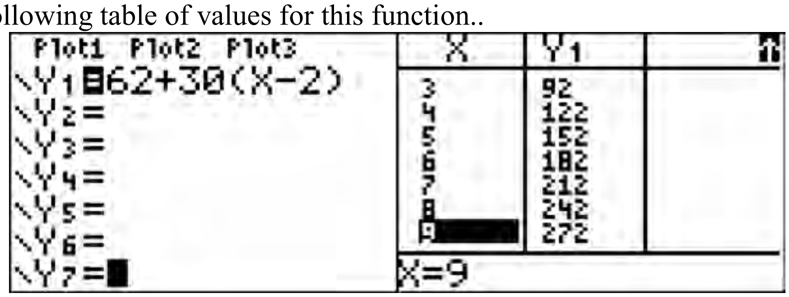
$$C = \$62 + overage\ ch\arg e$$

We can eliminate answer choices *a* and *b*. The correct answer is either *c* or d.

The second sentence also tells us that the overage charge is "...\$30 per gigabyte of data that exceed 2 gigabytes." We can use this information to choose between answer choices *c* and *d*.

Answer choice *c* is $C = 62 + 30(2 - g)$. This doesn't make sense, because the value of the term $30\left(2 - g\right)$ becomes negative if the number of gigabytes used is greater than 2, and the total cost becomes negative if the number of gigabytes used is 5 or more. Answer choice c can be eliminated. Answer choice d is the only choice left, and is the correct answer.

DIMS? Does It Make Sense? Yes. $C = 62 + 30(g - 2)$ could represent the plan *when more than 2 gigabytes are used*, as shown in the following table of values for this function..

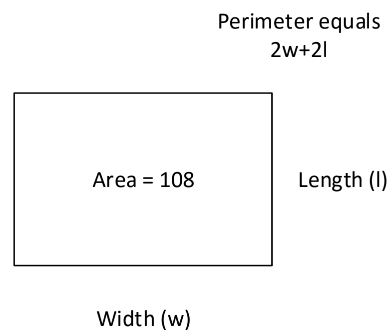


PTS: 2 REF: 081508ai NAT: A.CED.1 TOP: Modeling Linear Functions

14. ANS:
The garden is a rectangle that measures 18 meters by 6 meters.

Strategy: . Solve as a system of two equations, because the question requires solving for two variables: length and width.

STEP 1. Draw a picture that illustrates the information in the problem.



STEP 2. Using the picture, write two equations using length and area formulas for rectangles. Let l represent the unknown *length* of the garden and let w represent the unknown *width* of the garden.

The first equation, Eq_1, is based on the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

The second equation, Eq_2, is based on the area formula for rectangles, which is $A = lw$

$$Eq_1 \qquad 48 = 2l + 2w$$

$$Eq_2 \qquad lw = 108$$

STEP 2. Isolate the length variable Eq_2

$$lw = 108$$

$$Eq_{2_a} \qquad l = \frac{108}{w}$$

STEP 3. Solve Eq_1 and Eq_{2_a} as a system using substitution, as follows:

$$Eq_1 \qquad 48 = 2l + 2w$$

$$Eq_{2_a} \qquad l = \frac{108}{w}$$

$$48 = 2\left(\frac{108}{w}\right) + 2w$$

$$48w = 2(108) + 2w^2$$

$$2w^2 - 48w + 216 = 0$$

$$2w^2 - 48w = -216$$

$$w^2 - 24w = -108$$

$$w^2 - 24w + (-12)^2 = -108 + (-12)^2$$

$$(w-12)^2 = -108 + 144$$

$$(w-12)^2 = 36$$

$$w = \pm 6$$

The garden is 6 meters wide. The length of the garden can be found using $Eq_{2_a} \qquad l = \frac{108}{w}$.

$$Eq_{2_a} \qquad l = \frac{108}{w}$$

$$l = \frac{108}{6}$$

$$l = 18$$

PTS: 4 REF: 011636ai NAT: A.CED.1 TOP: Geometric Applications of Quadratics

15. ANS: B
Strategy: STEP 1. First, determine the area of the current rectangular patio and increase its size by 50%, which will be the size of the new patio. STEP 2. Then, increase each dimension of the current rectangular patio by x, as follows:
STEP 1.

$$\text{Area} = \text{length} \times \text{width}$$

$$\text{Current Patio}$$

$$A = 10 \times 12$$

$$A = 120$$

$$\text{New Patio}$$

$$A = 120 \times 150\%$$

$$A = 120 \times 1.5$$

$$A = 180$$

The new patio will have an area of 180 square feet. Eliminate choice (a).
STEP 2.

$$(10 + x)(12 + x) = 180$$

Choose answer b.

PTS: 2 REF: 011611ai NAT: A.CED.1 TOP: Geometric Applications of Quadratics

A.CED.2: Create Equations and Inequalities in Two Variables

Create equations that describe numbers or relationships.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

BIG IDEAS

In addition to creating equations that show relationships between variables, students must also be able to represent the relationships between variables using tables and graphs. This can require skills and fluency in reproducing or explain results obtained from a graphing calculator.

Four Views of a Function

Students should understand the relationships between **four views of a function** and be able to move from one view to any other view with relative ease, The four views of a function are:

1) the description of the function in words
2) the function rule (equation) form of the function
3) the graph of the function, and
4) the table of values of the function.

The **TI83+ graphing calculator** is an excellent tool for navigating between **three views of a function** (it does not provid a narrative description of the function in words).

- Students should know how to transform any linear equation into slope intercept form and input the equation into the $\boxed{\text{Y=}}$ feature of the graphing calculator. Once the equation is input into the graphing calculator, students should know how to access
  - the graph of the function using the $\boxed{\text{GRAPH}}$ feature of the calculator, and
  - the table of values of the function using the $\boxed{\text{TABLE}}$ feature of the calculator.
- Students should also know how to
  - Use the $\boxed{\text{WINDOW}}$ features of the calculator to manipulate the characteristics of the graph, and
  - Use the $\boxed{\text{TBLSET}}$ features of the calculator to manipulate the characteristics of the table of values.

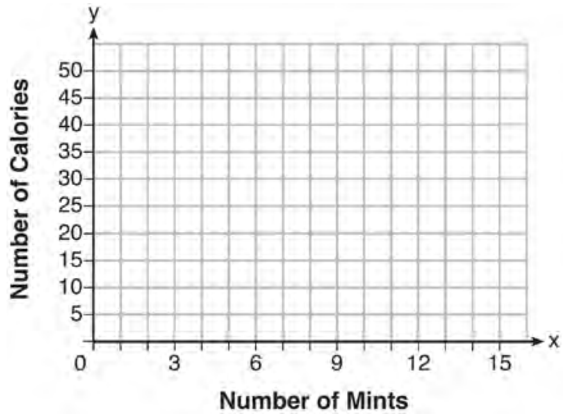
Regents Problems

1. In 2013, the United States Postal Service charged \$0.46 to mail a letter weighing up to 1 oz. and \$0.20 per ounce for each additional ounce. Which function would determine the cost, in dollars, $c(z)$, of mailing a letter weighing z ounces where z is an integer greater than 1?
   a. $c(z) = 0.46z + 0.20$
   b. $c(z) = 0.20z + 0.46$
   c. $c(z) = 0.46(z - 1) + 0.20$
   d. $c(z) = 0.20(z - 1) + 0.46$

2. Rowan has \$50 in a savings jar and is putting in \$5 every week. Jonah has \$10 in his own jar and is putting in \$15 every week. Each of them plots his progress on a graph with time on the horizontal axis and amount in the jar on the vertical axis. Which statement about their graphs is true?
   a. Rowan's graph has a steeper slope than Jonah's.
   b. Rowan's graph always lies above Jonah's.
   c. Jonah's graph has a steeper slope than Rowan's.
   d. Jonah's graph always lies above Rowan's.

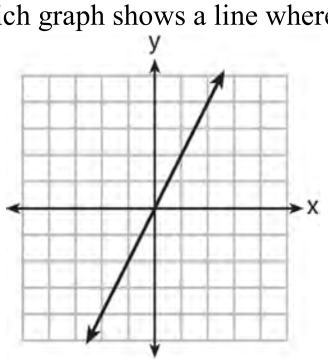
3. An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday. Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday. Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer. Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

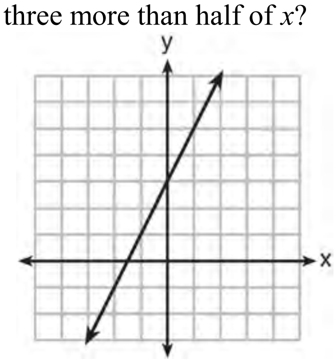
4. Max purchased a box of green tea mints. The nutrition label on the box stated that a serving of three mints contains a total of 10 Calories. On the axes below, graph the function, C, where $C(x)$ represents the number of Calories in x mints.

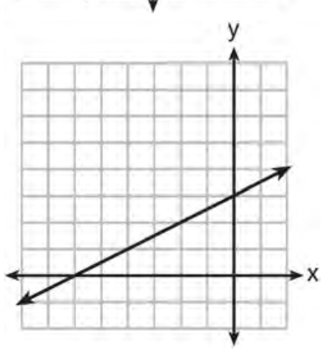


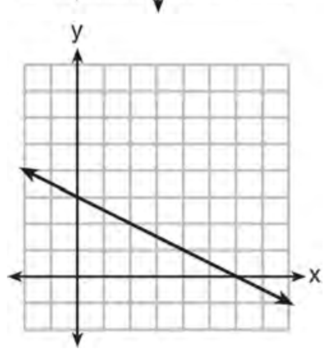
Write an equation that represents $C(x)$. A full box of mints contains 180 Calories. Use the equation to determine the total number of mints in the box.

5. Which graph shows a line where each value of y is three more than half of x?

a. 

c. 

b. 

d. 

A.CED.2: Create and/or Graph Equations
Answer Section

1. ANS: D
Strategy: Eliminate wrong answers.

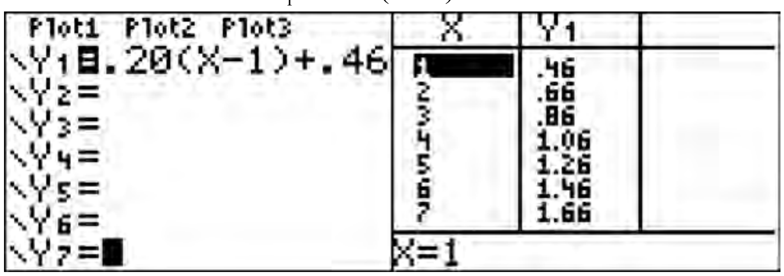
The problem states that there is a flat charge of \$0.46 to mail a letter. This flat charge applies regardless of what the letter weighs. Eliminate any answer that multiplies this flat charge by the weight of the letter. Eliminate answer choices *a* and *c*.

The difference between answer choices b and d is in the terms $0.20z$ and $0.20(z-1)$, where z represents the weight of the letter in ounces. Choice b charges 20 cents for every ounce. Choice d charges 20 cents for every ounce in excess of the first ounce. Choice d is the correct answer.

DIMS? Does It Make Sense? Yes. Transform answer choice c for input into the graphing calculator.

$$c(z) = 0.20(z-1) + 0.46$$

$$Y_1 = 0.20(x-1) + 0.46$$



The table shows \$0.46 to mail a letter weighing up to 1 oz. and \$0.20 per ounce for each additional ounce.

PTS: 2 REF: 011523a1 NAT: A.CED.2 TOP: Modeling Linear Equations

2. ANS: C
Strategy: Create equations that model Rowan's and Jonah's savings plans, then compare the slopes.

STEP 1. Create Equations
$y = ax + b$, where a represents the slope of the line

$R(w) = 50 + 5w$

$R(w) = 5w + 50$ The slope of Rowan's graph is $\frac{5 \text{ rise}}{1 \text{ run}}$, or simply 5.

$J(w) = 10 + 15w$

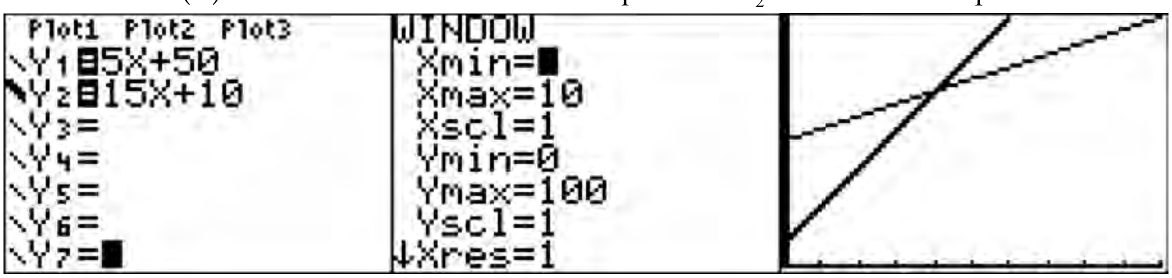
$J(w) = 15w + 10$ The slope of Jonah's graph is $\frac{15 \text{ rise}}{1 \text{ run}}$, or simply 15.

STEP 2. Compare the slopes.
Jonah's slope is greater than Rowan's slope because $15 > 5$. Therefore, Jonah's graph will have a steeper slope.

DIMS? Does It Make Sense? Yes. Input both equations in a graphing calculator, as follows:

$R(w) = 5w + 50$ Transform Rowan's equation to $\text{Y}_1 = 5x + 50$ for input.

$J(w) = 15w + 10$ Transform Jonah's equation to $\text{Y}_2 = 15w + 10$ for input.



Jonah's graph, which is bold, is steeper than Rowan's graph.

PTS: 2 REF: 081502ai NAT: A.CED.2 TOP: Graphing Linear Systems

3. ANS:
a) $2.35c + 5.50d = 89.50$
b) Pat's numbers are not possible, bercause the equation does not balance using Pat's numbers.
c) There were 10 cats in the shelter on Wednesday

Strategy: Use information from the first two sentences to write the equation, then use the equation to see if Pat is correct, then modify the equation for the last part of the question.

STEP 1: Write the equation
Let c represent the number of cats in the shelter.
Let d represent the number of dogs in the shelter.

$$2.35c + 5.50d = 89.50$$

STEP 2: Use the equation to see if Pat is correct.

$$\begin{aligned} 2.35c + 5.50d &= 89.50 \\ 2.35(8) + 5.50(14) &\neq 89.50 \\ 18.80 + 77.00 &\neq 89.50 \\ 95.80 &\neq 89.50 \end{aligned}$$

STEP 3: Modify the equation to reflect the total number of animals in the shelter.
Let c represent the number of cats in the shelter.
Let *(22-c)* represent the number of dogs in the shelter.

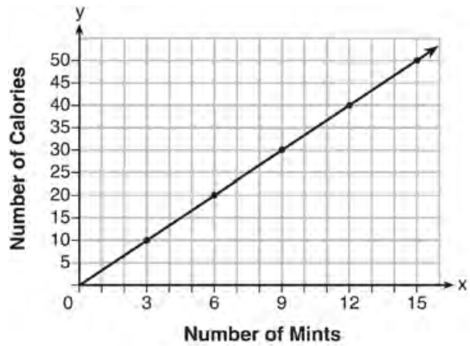
$$\begin{aligned} 2.35c + 5.50(22 - c) &= 89.50 \\ 2.35c + 121 - 5.50c &= 89.50 \\ -3.15c &= -31.50 \\ c &= 10 \end{aligned}$$

DIMS? Does It Make Sense? Yes. If there were 10 cats in the shelter and 12 dogs, the total costs of caring for the animals would be $89.50.

$$\begin{aligned} 2.35c + 5.50d &= 89.50 \\ 2.35(10) + 5.50(12) &= 89.50 \\ 23.50 + 66 &= 89.50 \\ 89.50 &= 89.50 \end{aligned}$$

PTS: 4 REF: 061436a1 NAT: A.CED.2 TOP: Modeling Linear Equations

4. ANS:

a) 

b) $C(x) = \frac{10}{3}x$

c) A full box contains 54 mints.

Strategy: Write the equation, then graph the equation, then use the equation and 180 calories to determine the number of mints in a full box.

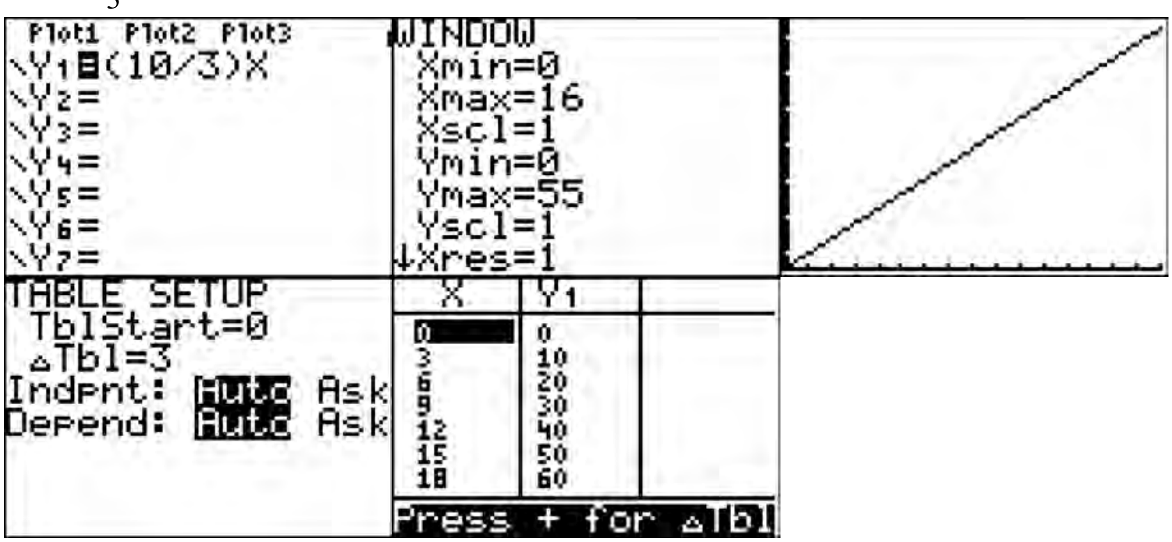
STEP 1. Write the equation.

If 3 mints contain ten calories, then one mint contains $\frac{10}{3}$ calories, and x number of mints contains $\frac{10}{3}x$ calories. Therefore: $C(x) = \frac{10}{3}x$.

STEP 2: Transform the equation and input the equation into a graphing calculator.

$$C(x) = \frac{10}{3}x$$

$$Y_1 = \frac{10}{3}x$$



STEP 3. Transfer the graph from the calculators table of values to the paper graph and complete the graph.

STEP 4. Substitute 180 calories for $C(x)$ in the equation and solve for x (the number of mints)

$$C(x) = \frac{10}{3}x$$

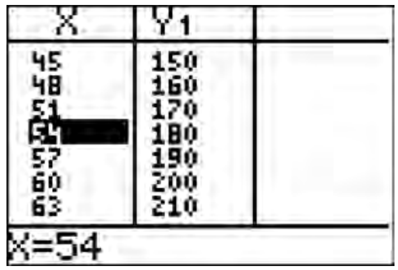
$$180 = \frac{10}{3}x$$

$$540 = 10x$$

$$54 = x$$

There are 54 mints in a full box.

DIMS: Does It Make Sense? Yes. The table view of the function also shows that 180 calories is paired with 54 mints.



PTS: 4 REF: fall1308a1 NAT: A.CED.2 TOP: Graphing Linear Functions

5. ANS: B

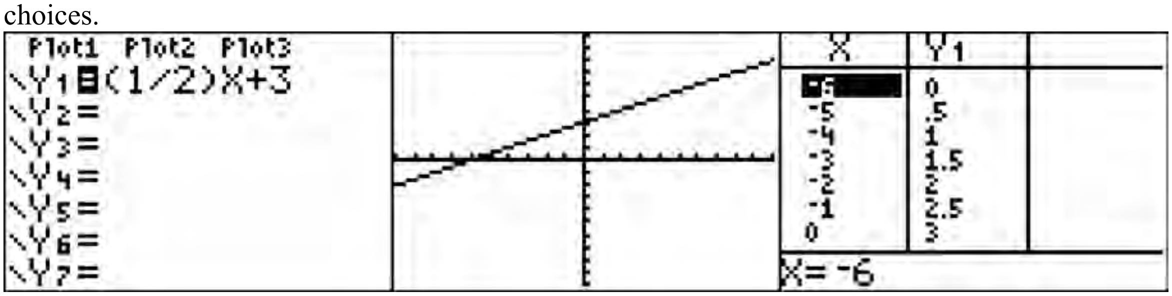
Strategy: Convert the narrative view to a function rule, then graph it.

STEP 1. Write the function rule.

$$\begin{array}{cccc} y & = & +3 & +\frac{1}{2}x \\ \hline \left(\text{each value of y}\right) & \text{is} & (\text{three more}) & \text{than (half of x)} \end{array}$$

$$y = \frac{1}{2}x + 3$$

STEP 2. Input the function rule in a graphing calculator and compare the graph view of the function to the answer choices.



Answer choice *b* is correct.

DIMS? Does It Make Sense? Yes. The x and y intercepts are reflected in both the graph and the table of values.

PTS: 2 REF: 081413a1 NAT: A.CED.2 TOP: Graphing Linear Functions
KEY: bimodalgraph

A.REI.10: Interpret Graphs as Sets of Solutions

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

BIG IDEAS

What a graph represents:

The graph of an equation represents the set of all points that satisfy the equation (make the equation balance).

Each and every point on the graph of an equation represents a coordinate pair that can be substituted into the equation to make the equation true.

If a point is on the graph of the equation, the point is a solution to the equation.

Graphing Linear Equations:

To graph a linear equation, you need to know either of the following:

- The coordinates of two points on the line, or
- The coordinates of one point on the line and the slope of the line.

If you know two points on the line, simply plot both of them them and draw a line passing through the two points.

If you know one point on the line and the slope of the line, plot the point and use the slope to find a second point. Then, draw a line passing through the two points.

REGENTS PROBLEMS

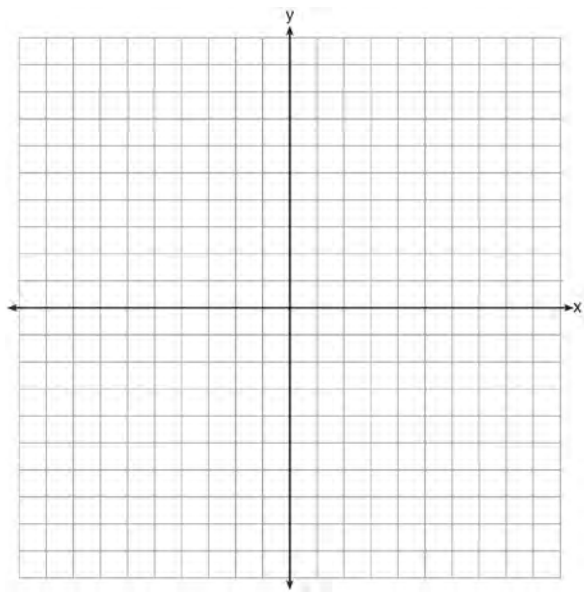
1. The graph of a linear equation contains the points $(3,11)$ and $(-2,1)$. Which point also lies on the graph?

a. $(2,1)$	c. $(2,6)$
b. $(2,4)$	d. $(2,9)$

2. Which point is *not* on the graph represented by $y = x^2 + 3x - 6$?

a. $(-6,12)$
b. $(-4,-2)$
c. $(2,4)$
d. $(3,-6)$

3. On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



Is the point (3,2) a solution to the equation? Explain your answer based on the graph drawn.

A.REI.10: Interpret Graphs as Sets of Solutions
Answer Section

1. ANS: D
Strategy: Find the slope of the line between the two points, then use $y - mx + b$ to find the y-intercept, then write the equation of the line and determine which answer choice is also on the line.

STEP 1. Find the slope of the line that passes through the points $(3, 11)$ and $(-2, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 11}{-2 - 3} = \frac{-10}{-5} = 2$$

Write $y = 2x + b$

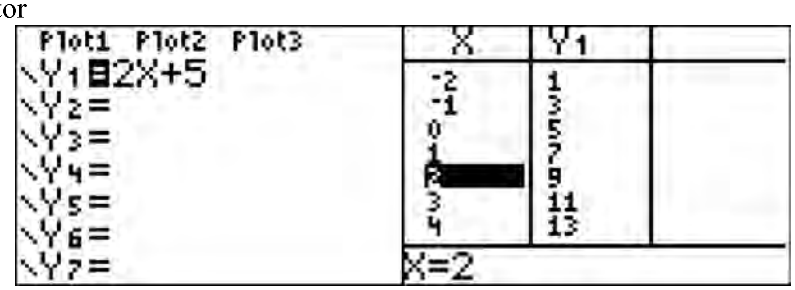
STEP 2. Use either given point and the equation $y = 2x + b$ to solve for b, the y-intercept. The following calculation uses the point (3,11).

$$y = 2x + b$$
$$11 = 2(3) + b$$
$$11 = 6 + b$$
$$5 = b$$

Write $y = 2x + 5$

STEP 3 Determine which answer choice balances the equation $y = 2x + 5$.

Use a graphing calculator



or simply solve the equation $y = 2x + 5$ for y when $x = 2$.

$$y = 2x + 5$$
$$y = 2(2) + 5$$
$$y = 4 + 5$$
$$y = 9$$

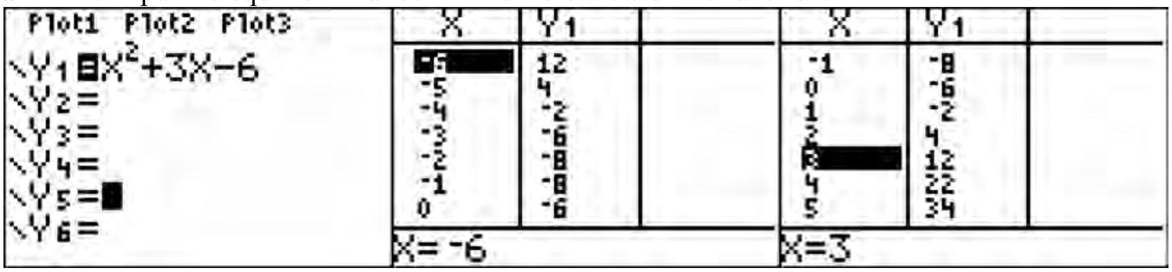
The point (2, 9) is also on the line.

PTS: 2 REF: 011511a1 NAT: A.REI.10 TOP: Graphing Linear Functions

2. ANS: D

Straegy: Input the equation in a graphing calculator, then use the table of values to eliminate wrong answers.

STEP 1. Input the equation and look at the table view of the function.



STEP 2. Eliminate answers that are *on* the graph.

The point (-6, 12) is on the graph, so eliminate answer choice a.

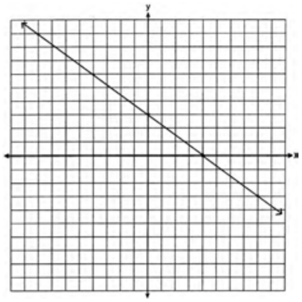
The point (-4, -2) is on the graph, so eliminate answer choice b.

The point (2, 4) is on the graph, so eliminate answer choice c.

The point (3, -6) is *not* on the graph, so answer choice d is the correct answer.

PTS: 2 REF: 081405a1 NAT: A.REI.10 TOP: Graphing Quadratic Functions

3. ANS:



No, because (3,2) is not on the graph.

Strategy #1. Use the y-intercept and the slope to plot the graph of the line, then determine if the point (3,2) is on the graph.

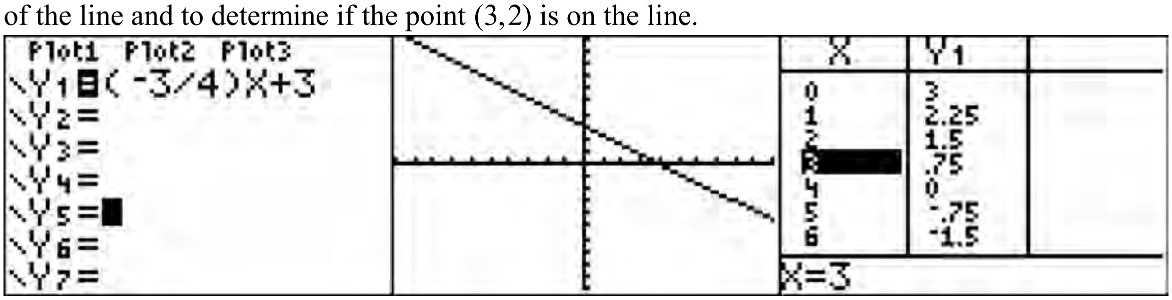
STEP 1. Plot the y-intercept.
Plot (0,3). The given equation is in the slope intercept form of a line, $y = mx + b$, where b is the y-intercpet. The value of b is 3, so the graph of the equation crosses the y axis at (0,3).

STEP 2. Use the slope of the line to find and plot a second point on the line. The given equation is in the slope intercept form of a line, $y = mx + b$, where m is the slopet. The value of m is $\frac{-3}{4}$, so the graph of the equation has a negative slope that goes down three units and across four units. Starting at the y-intercept, (0,3), if you go down 3 and over 4, the graph of the line will pass through the point (4,0).

STEP 3. Use a straightedge to draw a line that passes through the points (0,3) and (4,0).

STEP 4. Inspect the graph to determine if the point (3,2) is on the line. It is not.

Strategy #2. Input the equation of the line into a graphing calculator, then use the table of values to plot the graph of the line and to determine if the point (3,2) is on the line.



Be sure to explain your answer in terms of the graph and not in terms of the table of values or the function rule.

PTS: 2 REF: 061429a1 NAT: A.REI.10 TOP: Graphing Linear Functions

F.IF.4: Identify and Interpret Key Features of Graphs

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

Vocabulary

***x*-intercept** The point at which the graph of a relation intercepts the x-axis. The ordered pair for this point has a value of $y = 0$.

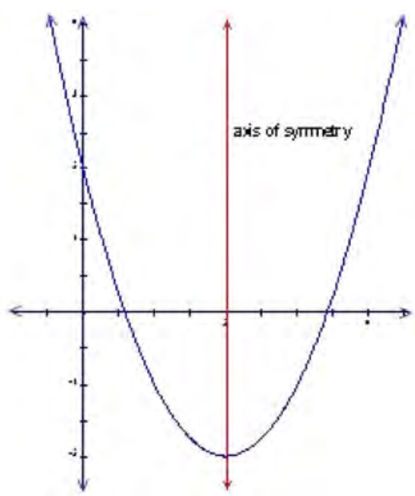
Example: The equation y = 8 + 2x has an x-intercept of -4.

***y*-intercept** The point at which a graph of a relation intercepts the y-axis. The ordered pair for this point has a value of $x = 0$.

Example: The equation y = 8 + 2x has a y-intercept of 8.

axis of symmetry ***(G)*** A line that divides a plane figure into two congruent reflected halves; Any line through a figure such that a point on one side of the line is the same distance to the axis as its corresponding point on the other side.

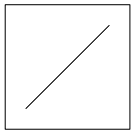
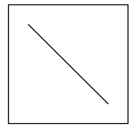
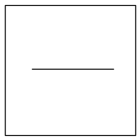
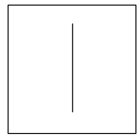
Example:



This is a graph of the parabola
$y = x^2 - 4x + 2$ together with its axis of symmetry $x = 2$.

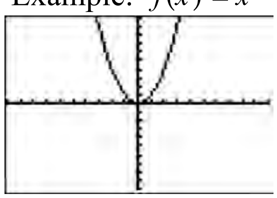
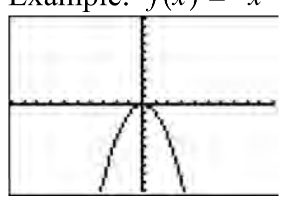
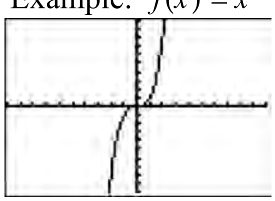
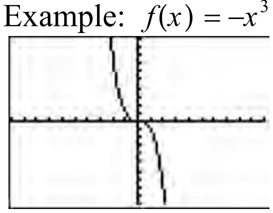
period (of a function) *(A2T) The horizontal distance after which the graph of a function starts repeating itself.* The smallest value of k in a function f for which there exists some constant k such that $f(t) = f(t+k)$ for every number t in the domain of f.

Slope

 **Positive Slope** Goes up from left to right.	 **Negative Slope**. Goes down from left to right.
 **Zero Slope**. A horizontal line has a slope of zero.	 **Undefined Slope**. A vertical line has an undefined slope.

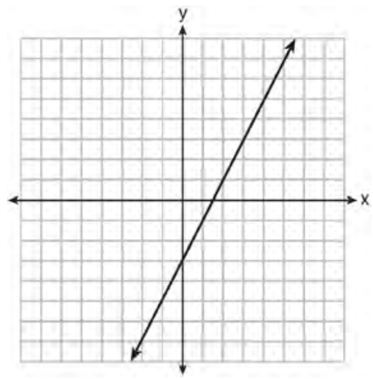
End Behaviors

The **end behaviors** of a graph refers to the directions (behaviors) of the graph of $f(x)$ as x approaches infinity in either direction. To determine the end behavior of the graph of any polynomial function, you need to know the degree of the polynomial and whether the leading coefficient is positive or negative. The table below shows the four possible sets of end behaviors of a polynomial function.

	Leading Coefficient is **Positive**	Leading Coefficient is **Negative**
Degree of function is **Even**	Example: $f(x) = x^2$  **End Behaviors** Left tail increases Right tail increases	Example: $f(x) = -x^2$  **End Behaviors** Left tail decreases Right tail decreases
Degree of function is **Odd**	Example: $f(x) = x^3$  **End Behaviors** Left tail decreases Right tail increases	Example: $f(x) = -x^3$  **End Behaviors** Left tail increases Right tail decreases

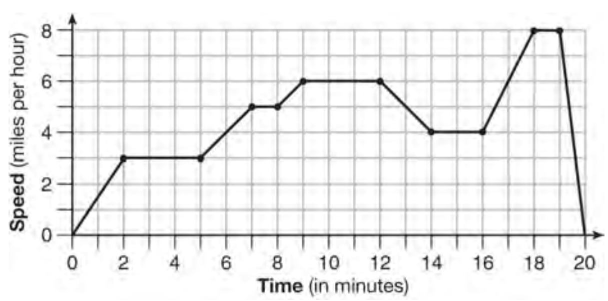
REGENTS PROBLEMS

1. Which function has the same y-intercept as the graph below?



a. $y = \dfrac{12 - 6x}{4}$ c. $6y + x = 18$
b. $27 + 3y = 6x$ d. $y + 3 = 6x$

2. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.



Which statement best describes what the jogger was doing during the 9 – 12 minute interval of her jog?
a. She was standing still. c. She was decreasing her speed
b. She was increasing her speed. d. She was jogging at a constant rate.

3. The value of the x-intercept for the graph of $4x - 5y = 40$ is

a. 10

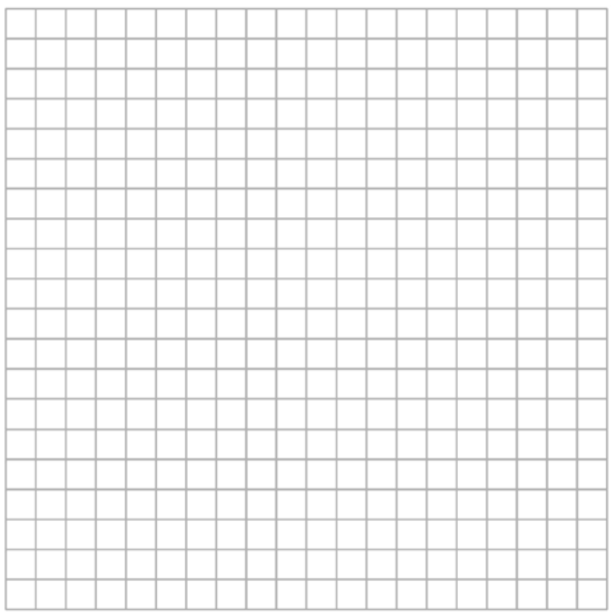
b. $\frac{4}{5}$

c. $-\frac{4}{5}$

d. -8

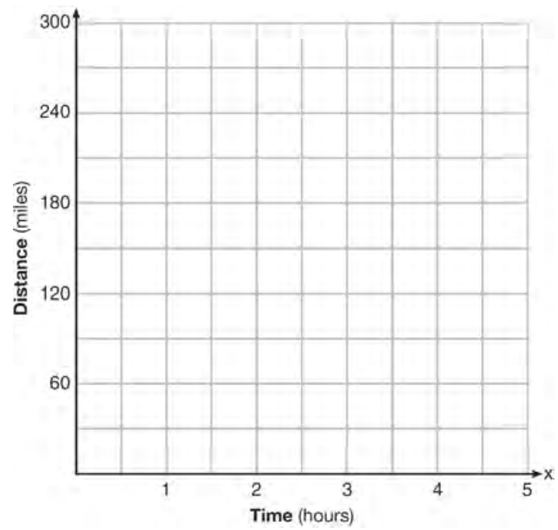
4. During a snowstorm, a meteorologist tracks the amount of accumulating snow. For the first three hours of the storm, the snow fell at a constant rate of one inch per hour. The storm then stopped for two hours and then started again at a constant rate of one-half inch per hour for the next four hours.

a) On the grid below, draw and label a graph that models the accumulation of snow over time using the data the meteorologist collected.



b) If the snowstorm started at 6 p.m., how much snow had accumulated by midnight?

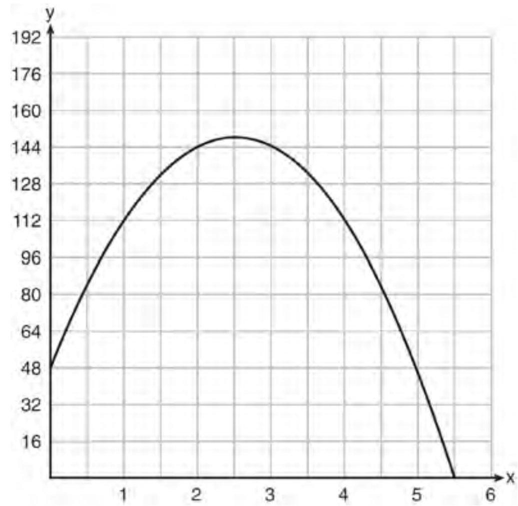
5. A driver leaves home for a business trip and drives at a constant speed of 60 miles per hour for 2 hours. Her car gets a flat tire, and she spends 30 minutes changing the tire. She resumes driving and drives at 30 miles per hour for the remaining one hour until she reaches her destination. On the set of axes below, draw a graph that models the driver's distance from home.



6. Let $h(t) = -16t^2 + 64t + 80$ represent the height of an object above the ground after t seconds. Determine the number of seconds it takes to achieve its maximum height. Justify your answer.

State the time interval, in seconds, during which the height of the object *decreases*. Explain your reasoning.

7. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds.



For which interval is the ball's height always *decreasing*?

a. $0 \le x \le 2.5$

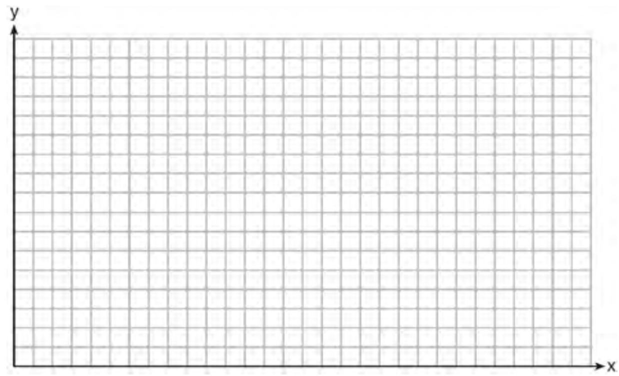
b. $0 < x < 5.5$

c. $2.5 < x < 5.5$

d. $x \ge 2$

8. A toy rocket is launched from the ground straight upward. The height of the rocket above the ground, in feet, is given by the equation $h(t) = -16t^2 + 64t$, where t is the time in seconds. Determine the domain for this function in the given context. Explain your reasoning.

9. A football player attempts to kick a football over a goal post. The path of the football can be modeled by the function $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$, where x is the horizontal distance from the kick, and $h(x)$ is the height of the football above the ground, when both are measured in feet. On the set of axes below, graph the function $y = h(x)$ over the interval $0 \le x \le 150$.



Determine the vertex of $y = h(x)$. Interpret the meaning of this vertex in the context of the problem. The goal post is 10 feet high and 45 yards away from the kick. Will the ball be high enough to pass over the goal post? Justify your answer.

F.IF.4: Identify and Interpret Key Features of Graphs
Answer Section

1. ANS: D
Strategy: Identify the y-intercept in the graph, then test each answer choice to see if it has the same y-intercept.

STEP 1. Identify the y-intercept in the graph.
The y-intercept is can be defined as the y-value of the coordinate where the graph intercepts (passes through) the y-axis. The graph shows that the function passes through the y-axis at the point $\left(0,-3\right)$, so the value of the y-intercept is -3.

STEP 2. Test the other equations to see if the point $\left(0,-3\right)$works.

a	$y = \dfrac{12-6x}{4}$ Does not work $-3 = \dfrac{12-6(0)}{4}$ $-3 = \dfrac{12}{4}$ $-3 \neq 3$	c	$6y + x = 18$ Does not work $6(-3) + (0) = 18$ $-18 \neq 18$
b	$27 + 3y = 6x$ Does not work $27 + 3(-3) = 6(0)$ $27 - 9 = 0$ $18 \neq 0$	d	$y + 3 = 6x$ $\left(0,-3\right)$works! $(-3) + 3 = 6(0)$ $0 = 0$

PTS: 2 REF: 011509a1 NAT: F.IF.4 TOP: Graphing Linear Functions

2. ANS: D
Strategy: Pay close attention to the labels on the x-axis and the y-axis, then eliminate wrong answers. NOTE: A horizontal line (no slope) means that speed is not changing.

Answer a can be eliminated because she would have a speed of 0 if she were standing still. She was only standing still at the start and end of her jog.
Answer b can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer c can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer d is the correct choice because a horizontal line (no slope) means that speed is not changing.

PTS: 2 REF: 061502AI NAT: F.IF.4 TOP: Relating Graphs to Events

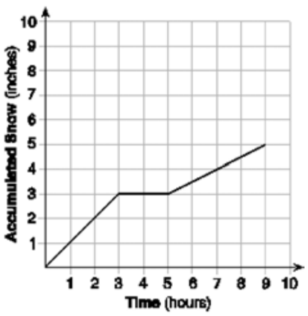
3. ANS: A
Strategy: Find the value of x when y equals 0. NOTE: The x-intercept can be defined as the x-value of the coordinate where the graph intercepts (passes through) the x-axis.

$$4x - 5y = 40$$
$$4x + 5(0) = 40$$
$$4x = 40$$
$$x = 10$$

The value of the x-intercept is 10.

PTS: 2 REF: 081408a1 NAT: F.IF.4 TOP: Graphing Linear Functions

4. ANS:



At midnight, 6 hours after the storm began, $3\frac{1}{2}$ inches of snow have fallen.

Strategy - Part a). Label the x and y axes and the corresponding intervals, then use the rates of change from the problem to complete the graph.

STEP 1. Plot the rate of change for the first three hours of the storm. The rate of change during this time is 1 inch per 1 hour.

STEP 2. Plot no change in accumulation for the two hours when the storm is stopped.

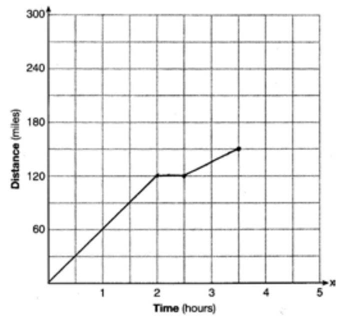
STEP 3. Plot the rate of change for the next four hours. During this interval, the rate of change is $\frac{1}{2}$ inch per 1 hour.

Strategy: Part b). Determine which point on the graph corresponds to midnight.

Midnight it six hours after 6 p.m., so the coordinate $\left(6, 3\frac{1}{2}\right)$ can be used to determine the amount of accumulation at midnight. The amount of snow accumulation at midnight is $3\frac{1}{2}$ inches.

PTS: 4 REF: spr1307a1 NAT: F.IF.4 TOP: Relating Graphs to Events

5. ANS:



Strategy - Use the speed of the car as the rate of change to complete the graph.

STEP 1. Plot 2 hours at 60 miles per hour slope, based on the language "... a constant speed of 60 miles per hour for 2 hours."

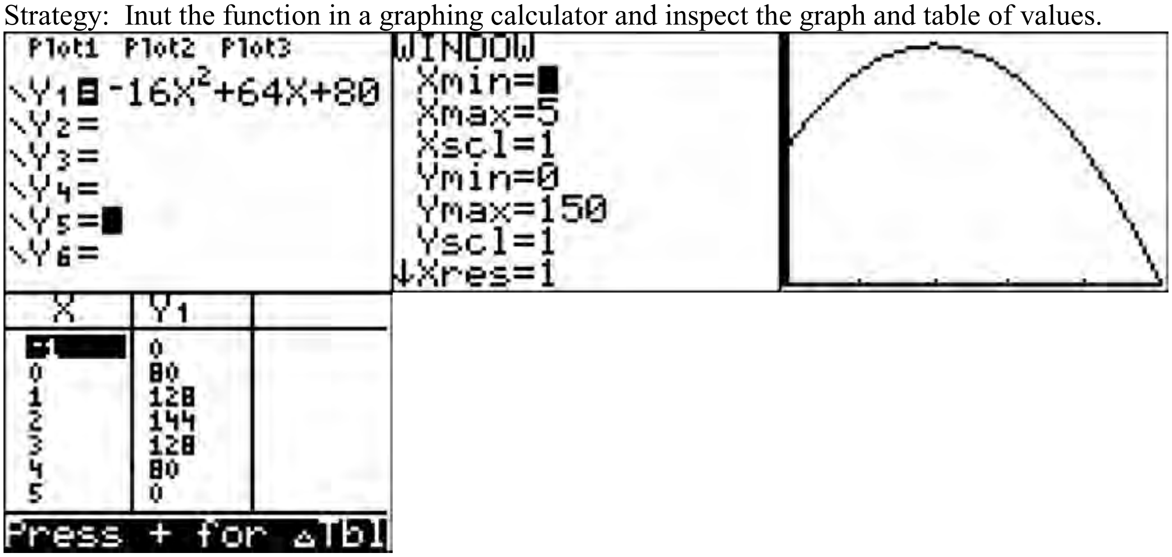
STEP 2. Plot $\frac{1}{2}$ hour at 0 slope based on the language "...she spends 30 minutes changing the tire."

STEP 3. Plot 1 hour at 30 miles per hour slope based on the language "...drives at 30 miles per hour for the remaining one hour..."

PTS: 2 REF: 081528ai NAT: F.IF.4 TOP: Relating Graphs to Events

6. ANS:
The object reaches its maximum height at 2 seconds.
The height of the object decreases between 2 seconds and 5 seconds.

Strategy: Inut the function in a graphing calculator and inspect the graph and table of values.



PTS: 4 REF: 011633ai NAT: F.IF.4 TOP: Graphing Quadratic Functions

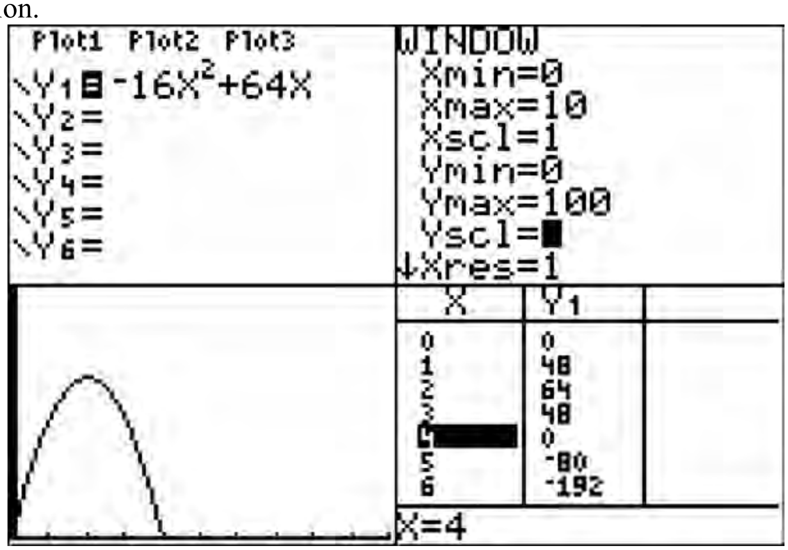
7. ANS: C
Strategy: Identify the domain of x that corresponds to a negative slope (decreasing height) in the function, then eliminate wrong answers.

STEP 1. The axis of symmetry for the parabola is $x = 2.5$ and the graph has a negative slope after $x = 2.5$ all the way to $x = 5.5$, meaning that the height of the ball is decreasing over this interval.

STEP 2. Eliminate wrong answers.
Answer choice a can be eliminated because the the slope of the graph increases over the interval $0 \le x \le 2.5$.
Answer choice b can be eliminated because the the slope of the graph both increases and decreases over the interval $0 \le x \le 2.5$.
Answer choice c is the correct choice, because it shows the domain of x where the graph has a negative slope.
Answer choice d can be eliminated because the the slope of the graph increases from $x \ge 2$ until $x = 2.5$.

PTS: 2 REF: 061409a1 NAT: F.IF.4 TOP: Graphing Quadratic Functions

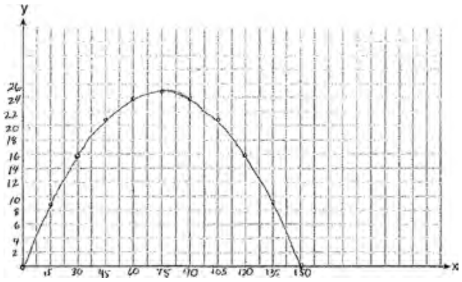
8. ANS:
The rocket launches at $t = 0$ and lands at $t = 4$, so the domain of the function is $0 \le x \le 4$.
Strategy: Input the function into a graphing calculator and determine the flight of the rocket using the graph and table views of the function.



The toy rocket is in the air between 0 and 4 seconds, so the domain of the function is $0 \le x \le 4$.

PTS: 2 REF: 081531ai NAT: F.IF.4 TOP: Graphing Quadratic Functions

9. ANS:

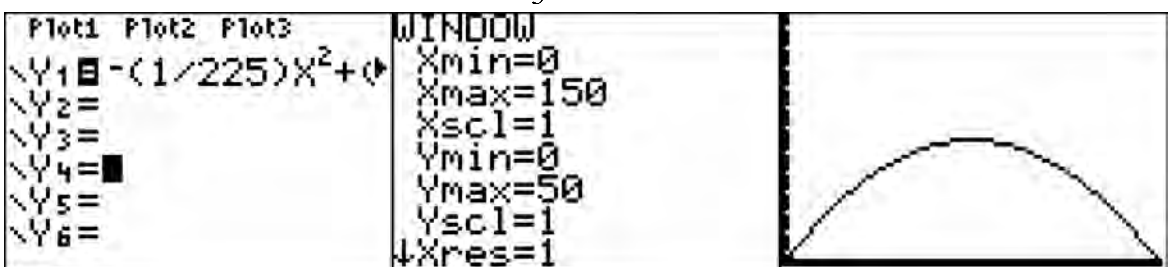
a) 

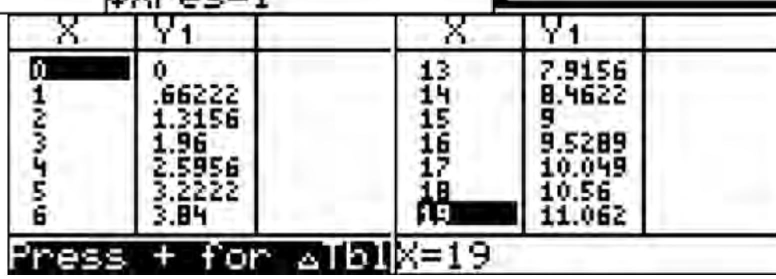
b) The vertex is at $(75,25)$. This means that the ball will reach it highest (25 feet) when the horizontal distance is 75 feet.

c) No, the ball will not clear the goal post because it will be less than 10 feet high.

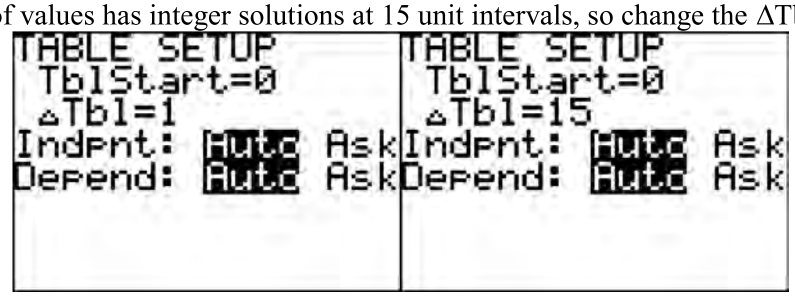
Strategy: Input the equation into a graphing calculator and use the table and graph views to complete the graph on paper, then find the vertex and determine if the ball will pass over the goal post.

STEP 1. Input $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$ into a graphing calculator. Set the window to reflect the interval $0 \le x \le 150$ and estimate the height to be approximately $\frac{1}{3}$ the domain of x.





Observe that the table of values has integer solutions at 15 unit intervals, so change the ΔTbl to 15.



The change in ΔTbl results in a table of values that is easier to graph on paper.

X	Y1	
0	0	
15	9	
30	16	
45	21	
60	24	
75	25	
90	24	

Press + for ΔTbl

X	Y1	
0	0	
15	9	
30	16	
45	21	
60	24	
75	25	
90	24	

Press + for ΔTbl

Use the graph view and the table of values to complete the graph on paper.

STEP 2. Use the table of values to find the vertex. The vertex is located at $(75,25)$.

X	Y1	
30	16	
45	21	
60	24	
75	25	
90	24	
105	21	
120	16	

X=75

STEP 3. Convert 45 yards to 135 feet and determine if the the ball will be 10 feet or higher when $x = 135$.

X	Y1	
90	24	
105	21	
120	16	
135	9	
150	0	
165	-11	
180	-24	

X=135

or $y = -\frac{1}{225}(135)^2 + \frac{2}{3}(135) = -81 + 90 = 9$

The ball will be 9 feet above the ground and will not go over the 10 feet high goal post.

PTS: 6 REF: 061537AI NAT: F.IF.4 TOP: Graphing Quadratic Functions

A.CED.3: Interpret Solutions

Create equations that describe numbers or relationships.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Big Idea:

Solutions to inequalities can be graphed on a number line which, by definition, represents the set of all real numbers. There are times, however, when the solution set of real numbers must be replaced with a more restricted set of numbers. This replacement process is also known as interpreting solutions,

For example, if an inequality is created to represent the number of students absent from class of 34 students on any given day, it might be written and graphed as follows:

$$0 \le \#\ of\ absent\ students \le 34$$

If you study this graph and think about all the numbers it represents, it doesn't make any sense. Nobody would ever turn in an attendance roster saying that $2\frac{1}{2}$ students or π students were absent, yet these and infinitely more rational and irrational numbers are included in the above inequality solution. In reality, only whole students are counted as absent or present when attendance is taken, and it would be more meaningful if we replace our inequality with a restricted set containing only whole numbers, as shown below:

{0, 1, 2, 3 … 32, 33, 34}

When a solution set is limited to a specified range or certain types of numbers, those numbers are known as the **replacement set**. When a replacement set is specified for a problem, a **3-step procedure** is followed:

1. Solve the inequality for all real numbers (the regular solution set).
2. See if any elements of the specified replacement set are part of the solution set.
3. Replace the solution set with all numbers from the replacement set that make the inequality true.

Replacement Set Notation Using Three Dots

{,,, -3, -2, -1, 0, 1, 2, 3 …}	The three dots at the beginning and end of this set indicate that the pattern continues in both directions.
{ 0, 1, 2, 3 …}	The three dots at the end of this set indicate that the pattern continues in a positive direction. This is the set of whole numbers.
{,,, -3, -2, -1)	The three dots at the beginning of this set indicate that the pattern continues in a negative direction. This is the set of negative integers.
{0, 1, 2, 3 …26, 27, 28}	The three dots in the middle of this set indicate that the pattern continues between 3 and 26.

Big Idea:

A linear inequality describes a region of the coordinate plane that has a **boundary line**. Every point in the region is a **solution of the inequality**.

Two or more linear inequalities together form a **system of linear inequalities**. Note that there are two or more boundary lines in a system of linear inequalities.
A **solution of a system of linear inequalities** makes each inequality in the system true. The graph of a system shows all of its solutions.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

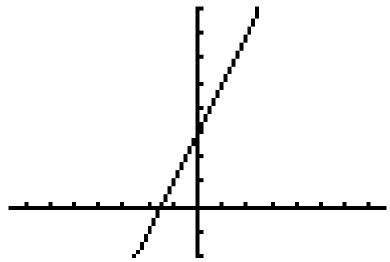
- When the inequality sign **contains** an equality bar beneath it, use a solid line for the boundary.
- When the inequality sign **does not contain** an equality bar beneath it, use a dashed or dotted line for the boundary

Step Two. Restore the inequality sign and test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. However, if the boundary line passes through the point (0,0), another point not on the boundary line must be selected for testing.

- If the test point makes the inequality true, shade the side of the boundary line that includes the test point.
- If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.

<u>Example</u> Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$. This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.



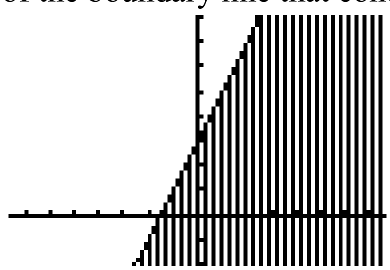
Next, **<u>test a point</u>** to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ True, so the solution of the inequality is the region that contains the point $(0,0)$.

Therefore, we shade the side of the boundary line that contains the point (0,0).



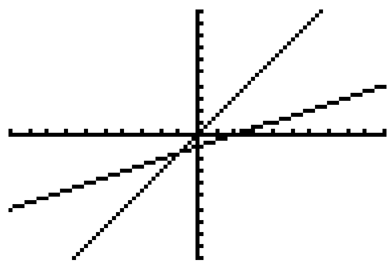
Note: The TI-83+ graphing calculator does not have the ability to distinguish between solid and dashed lines on a graph of an inequality. The less than and greater than symbols are input using the far-left column of symbols that can be accessed through the $\boxed{\text{Y=}}$ feature.

<u>Graphing a System of Linear Inequalities</u>. Systems of linear inequalities are graphed in the same manner as systems of equations are graphed. The solution of the system of inequalities is the region of the coordinate plane that is shaded by both inequalities.

<u>Example</u>: Graph the system: $\begin{array}{l} 4y \geq 6x \\ -3x+6y \leq -6 \end{array}$

First, convert both inequalities to slope-intercept form and graph.

$4y \geq 6x$ $\dfrac{4y}{4} \geq \dfrac{6x}{4}$ $y \geq \dfrac{3}{2}x$ $\text{m}=\dfrac{3}{2}, \quad \text{b}=0$	$-3x+6y \leq -6$ $6y \leq -6+3x$ $6y \leq 3x-6$ $D_2 \quad \dfrac{6y}{6} \leq \dfrac{3x}{6} - \dfrac{6}{6}$ $y \leq \dfrac{1}{2}x-1$ $\text{m}=\dfrac{1}{2}, \quad \text{b}=-1$



Next, test a point in each inequality and shade appropriately.

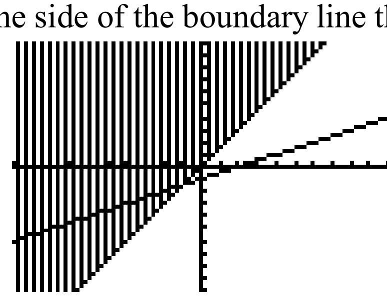
- Since point (0,0) is on the boundary line of $y \geq \frac{3}{2}x$, select another point, such as (0,1).

  $y \geq \dfrac{3}{2}x$

  Test (0,1)

  $1 \geq \dfrac{3}{2}(0)$

  $1 \geq 0$ This is true, so the point (0,1) is in the solution set of this inequality. Therefore, we shade the side of the boundary line that includes point (0,1).



- Since (0,0) is not on the boundary line of $y \leq \frac{1}{2}x-1$, we can use (0,0) as our test point, as follows:

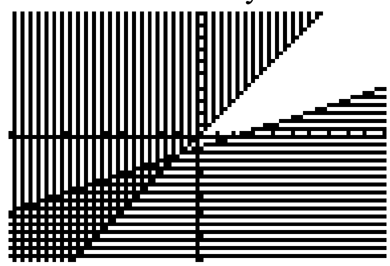
$y \le \frac{1}{2}x - 1$

Test (0,0)

$0 \le \frac{1}{2}(0) - 1$

$0 \le -1$ This is not true, so the point (0,0) is not in the solution set of this inequality.

We therefore must shade the side of the boundary line that does not incude the point (0,0).



Note that the system of inequalities divides the coordinate plane into four sections. The solution set for the system of inequalities is the area where the two shaded regions overlap.

Remember The Big Rule for Solving Inequalities:

All the rules for solving equations apply to inequalties – plus one:

When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.

REGENTS PROBLEMS

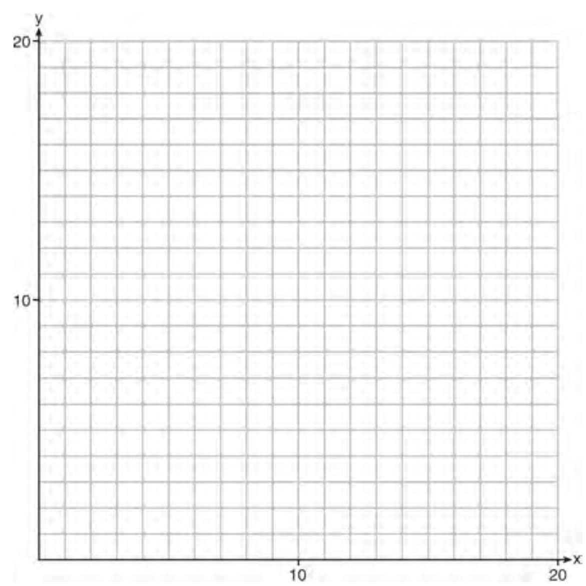
1. The cost of a pack of chewing gum in a vending machine is \$0.75. The cost of a bottle of juice in the same machine is \$1.25. Julia has \$22.00 to spend on chewing gum and bottles of juice for her team and she must buy seven packs of chewing gum. If *b* represents the number of bottles of juice, which inequality represents the maximum number of bottles she can buy?

a. $0.75b + 1.25(7) \ge 22$
b. $0.75b + 1.25(7) \le 22$
c. $0.75(7) + 1.25b \ge 22$
d. $0.75(7) + 1.25b \le 22$

2. Natasha is planning a school celebration and wants to have live music and food for everyone who attends. She has found a band that will charge her \$750 and a caterer who will provide snacks and drinks for \$2.25 per person. If her goal is to keep the average cost per person between \$2.75 and \$3.25, how many people, p, must attend?

   a. $225 < p < 325$
   b. $325 < p < 750$
   c. $500 < p < 1000$
   d. $750 < p < 1500$

3. Edith babysits for x hours a week after school at a job that pays \$4 an hour. She has accepted a job that pays \$8 an hour as a library assistant working y hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least \$80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation. Graph these inequalities on the set of axes below.

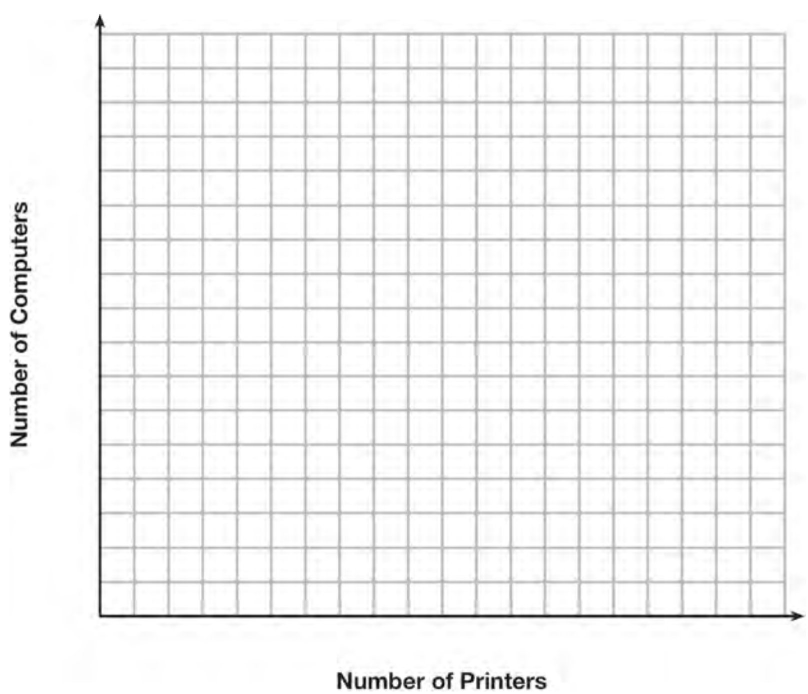


Determine and state one combination of hours that will allow Edith to earn *at least* \$80 per week while working *no more than* 15 hours.

4. David has two jobs. He earns \$8 per hour babysitting his neighbor's children and he earns \$11 per hour working at the coffee shop. Write an inequality to represent the number of hours, x, babysitting and the number of hours, y, working at the coffee shop that David will need to work to earn a minimum of \$200. David worked 15 hours at the coffee shop. Use the inequality to find the number of full hours he must babysit to reach his goal of \$200.

5. A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are \$6 before the day of the show and \$9 on the day of the show. To meet the expenses of the show, the club must sell at least \$5,000 worth of tickets.
a) Write a system of inequalities that represent this situation.
b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.

6. An on-line electronics store must sell at least \$2500 worth of printers and computers per day. Each printer costs \$50 and each computer costs \$500. The store can ship a maximum of 15 items per day. On the set of axes below, graph a system of inequalities that models these constraints.



Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.

A.CED.3: Interpret Solutions
Answer Section

1. ANS: D
Strategy: Examine the answer choices and eliminate wrong answers.

STEP 1. Eliminate answer choices *a* and *c* because both of them have greater than or equal signs. Julia must spend less than she has, not more.

STEP 2. Choose between answer choices *b* and *d*. Answer choice *d* is correct because the term 0.75(7) means that Julia must buy 7 packs of chewing gum @ \$0.75 per pack. Answer choice b is incorrect because the term 1.25(7) means that Julia will buy 7 bottles of juice.

DIMS? Does It Make Sense? Yes. Answer choice d shows in the first term that Julia will buy 7 packs of gum and the total of the entire expression must be equal to or less than \$22.00.

PTS: 2 REF: 081505ai NAT: A.CED.3 TOP: Modeling Linear Inequalities

2. ANS: D
Strategy:

STEP1. Use the definition of average cost.

$$\text{Average Cost} = \frac{\text{total costs}}{\text{number of persons sharing the cost}}$$

Total costs for the band and the caterer are: $\$750 + \$2.25p$

If the average cost is \$3.25, the formula is $\$3.25 = \dfrac{\$750 + \$2.25p}{p}$

Solve for p

$$\$3.25p = \$750 + \$2.25p$$

$$p = 750$$

If the average cost is \$2.75, the formula is $\$2.75 = \dfrac{\$750 + \$2.25p}{p}$

Solve for p

$$\$2.75p = \$750 + \$2.25p$$

$$.50p = 750$$

$$p = 1500$$

DIMS? Does It Make Sense? Yes. If 750 people attend, the average cost is \$2.25 per person. If 1500 people attend, the average cost is \$3.25 per person. For any number of people between 750 and 1500, the average cost per person will be between \$2.25 and \$3.25.

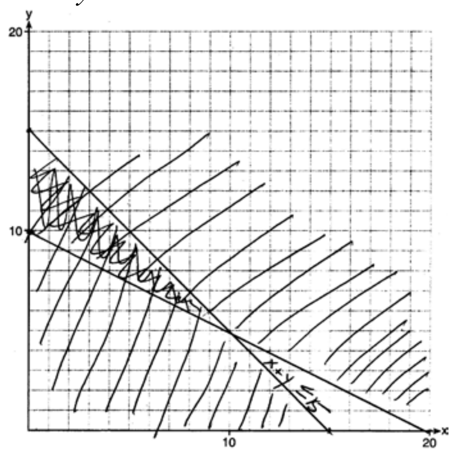
PTS: 2 REF: 061524AI NAT: A.CED.3 TOP: Modeling Linear Inequalities

3. ANS:

a) $x+y \le 15$

$4x+8y \ge 80$

b)



c) Zero hours at school and 15 hours at the library.

Strategy: Write two inequalities, then input them into a graphing calculator and transfer the graph view to the paper, then answer the questions.

STEP 1. Write two inequalities.

Let x represent the number of hours Edith babysits.

Let y represent the number of hours Edith works at the library.

Write: Eq. 1 $x+y \le 15$

Eq.2 $4x+8y \ge 80$

STEP 2. Transform both inequalities for input into a graphing calculator

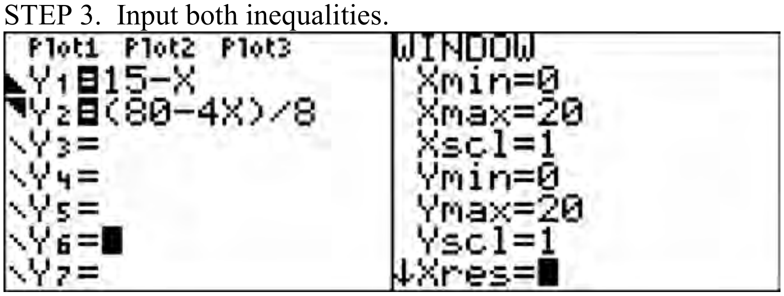
Eq. 1 $x+y \le 15$

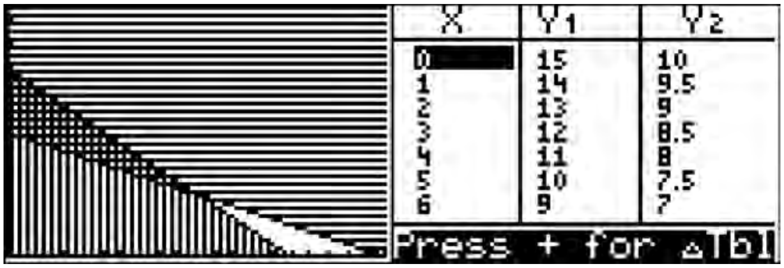
$y \le 15-x$

Eq.2 $4x+8y \ge 80$

$y \ge \dfrac{80-4x}{8}$

STEP 3. Input both inequalities.





Use data from the table of values to construct the graph on paper.

STEP 3. Test one combination of hours in the solution set (the dark shaded area).
Test (0, 15).

Eq. 1 $x+y \le 15$

$0+15 \le 15$

$15 \le 15$

Eq.2 $4x+8y \ge 80$

$4(0)+8(15) \ge 80$

$120 \ge 80$

DIMS? Does It Make Sense? Yes.

PTS: 6 REF: 081437a1 NAT: A.CED.3 TOP: Modeling Systems of Linear Inequalities

4. ANS:
David must babysit five full hours to reach his goal of $200.

Strategy: Write an inequality to represent David's income from both jobs, then use it to solve the problem, then interpret the solution.

STEP 1. Write the inequality.
Let x represent the number of hours that David babysits.
Let y represent the number of hours that David works at the coffee shop.
Write: $8x + 11y \geq 200$

STEP 2. Substitute 15 for y and solve for x.

$$\begin{aligned} 8x + 11y &\geq 200 \\ 8x + 11(15) &\geq 200 \\ 8x + 165 &\geq 200 \\ 8x &\geq 200 - 165 \\ 8x &\geq 35 \\ x &\geq \frac{35}{8} \\ x &\geq 4.375 \end{aligned}$$

STEP 3. Interpret the solution.
The problem asks for the number of full hours, so the solution, $x \geq 4.375$, must be rounded up to 5 full hours.

DIMS? Does It Make Sense? Yes. If David works 15 hours at the coffee shop and 5 hours at the library, he will earn more than 200

$$\begin{aligned} 8(5) + 11(15) &\geq 200 \\ 40 + 165 &\geq 200 \\ 205 &\geq 200 \end{aligned}$$

What does not make sense is why David earns $8 per hour babysitting and Edith, in the previous problem, only earns $4 per hour.

PTS: 4 REF: fall1309a1 NAT: A.CED.3 TOP: Modeling Linear Inequalities

5. ANS:

a) $Eq.1 \quad p + d \le 800$

$Eq.2 \quad \$6p + \$9d \ge 5000$

b) Yes, it is possible. They will need to sell 263 or more tickets on the day of the show. They have 360 tickets left.

Strategy: Write a system of equations, then use it to answer part b.

STEP 1.

Let p represent the number of tickets sold before the day of the show.

Let d represent the number of tickets sold on the day of the show.

Write: $Eq.1 \quad p + d \le 800$

$Eq.2 \quad \$6p + \$9d \ge \$5000$

STEP 2. Substitute 440 for p in both equations and solve.

$Eq.1$

$$p + d \le 800$$
$$440 + d \le 800$$
$$d \le 800 - 440$$
$$d \le 360$$

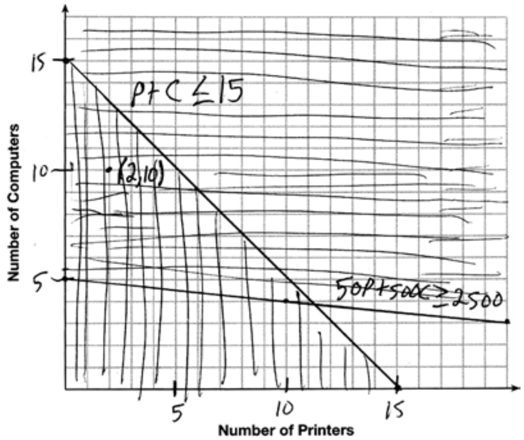
$Eq.2$

$$\$6p + \$9d \ge \$5000$$
$$\$6(440) + \$9d \ge \$5000$$
$$\$2640 + \$9d \ge \$5000$$
$$\$9d \ge \$5000 - \$2640$$
$$\$9d \ge \$2360$$
$$d \ge \frac{\$2360}{\$9}$$
$$d \ge 262.2$$

DIMS? Does It Make Sense? Yes. They could cover their costs by selling 263 tickets and make almost \$9000 over costs if they sell 360 tickets on the day of the show.

PTS: 2 REF: spr1306a1 NAT: A.CED.3 TOP: Modeling Systems of Linear Inequalities

6. ANS:



a)

b) A combination of 2 printers and 10 computers meets all the constraints because $(2,10)$ is in the solution set of the graph.

Strategy: Write a system of inequalities, transform and input both inequalities into a graphing calculator, draw the graph on the paper using the table of values view in the calculator, then use the graph to answer the question.

STEP 1. Write the system of inequalities.

Let p represent the number of printers shipped each day.

Let c represent the number of computers shipped each day.

Write: $Eq.1 \quad p + c \le 15$

$$Eq.2 \quad \$50p + \$500c \ge \$2500$$

STEP 2. Transform both equations and input them into the graphing calculator.

$$Eq.1 \quad p + c \le 15$$

$$c \le 15 - p$$

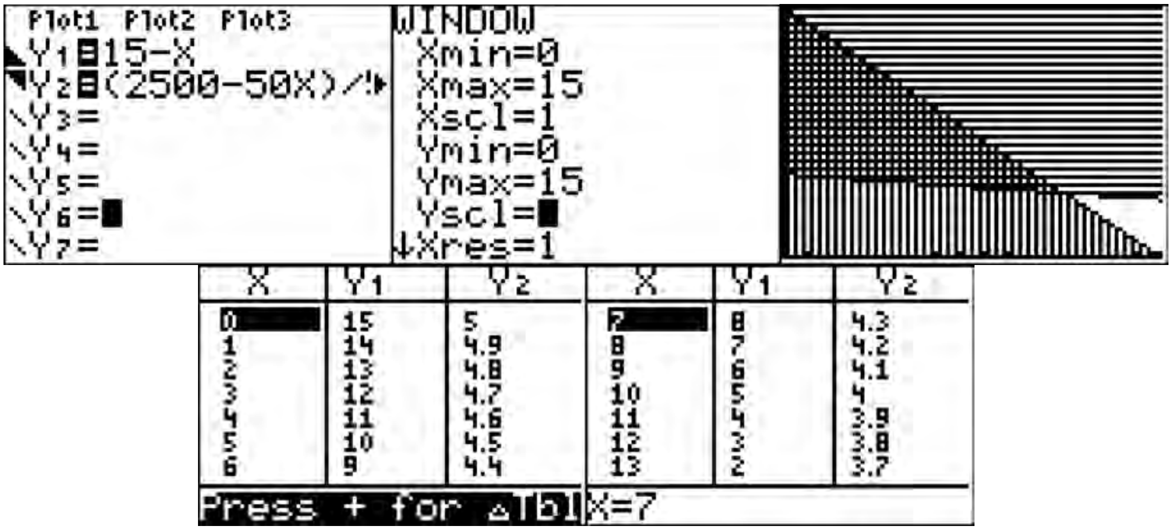
$$y \le 15 - x$$

$$Eq.2 \quad \$50p + \$500c \ge \$2500$$

$$\$500c \ge \$2500 - \$50p$$

$$c \ge \frac{\$2500 - \$50p}{\$500}$$

$$y \ge \frac{\$2500 - \$50x}{\$500}$$



STEP 3. Use information from the graphing calculator to construct the graph (see above).

STEP 4. Select (2, 10), or any other point in the heavily shaded area, as a combination of printers and computers that would allow the electronics store to meet all of the constraints.

DIMS? Does It Make Sense? Yes. The point (2, 10) satisfies both inequalities, as shown below:.

$$Eq.1 \quad \begin{aligned} p + c &\le 15 \\ 2 + 10 &\le 15 \\ 12 &\le 15 \end{aligned} \qquad\qquad Eq.2 \quad \begin{aligned} \$50p + \$500c &\ge \$2500 \\ \$50(2) + \$500(10) &\ge \$2500 \\ \$100 + \$5000 &\ge \$2500 \\ \$5100 &\ge \$2500 \end{aligned}$$

PTS: 4 REF: 061535AI NAT: A.CED.3 TOP: Modeling Systems of Linear Inequalities

A.CED.4: Transform Formulas

Create equations that describe numbers or relationships.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Vocabulary

Formula: A **formula** is an equation that shows the relationship between two or more variables.

Transform: To **transform** something is to change its form or appearance. In mathematical equations, a **transformaion** changes form and appearance, but does not change the relationships between variables. To **transform** a formula or equation usually means to isolate a specific variable.

Field properties and operations can be used to transform **formulas** to isolate different variables in the same ways that equations are manipulated to isolate a variable.

Example: The **formula** $P = 2l + 2w$ can be used to find the perimeter of a rectangle. In English, $P = 2l + 2w$ translates as "The *perimeter equals two times the length plus two times the width*." In the **formula** $P = 2l + 2w$, the P variable is already isolated. You can isolate the l variable or the w variables, as follows. (*Note that the steps and operations are the same as with regular equations.*)

To isolate the l variable: Start with the formula: $P = 2l + 2w$ Move the term 2w to the left expression. $P - 2w = 2l$ Divide both sides of the equation by 2. $\frac{P - 2w}{2} = l$ You now have a formula for l in terms of P and w.	To isolate the w variable: Start with the formula: $P = 2l + 2w$ Move the term $2l$ to the left expression. $P - 2l = 2w$ Divide both sides of the equation by 2. $\frac{P - 2l}{2} = w$ You now have a formula for l in terms of P and w.

Regents Problems

1. The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of r, in terms of h and V, is

a. $r = \sqrt{\frac{V}{\pi h}}$

b. $r = \sqrt{V\pi h}$

c. $r = 2V\pi h$

d. $r = \frac{V}{2\pi}$

2. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The radius, r, of the cone may be expressed as

a. $\sqrt{\frac{3V}{\pi h}}$

b. $\sqrt{\frac{V}{3\pi h}}$

c. $3\sqrt{\frac{V}{\pi h}}$

d. $\frac{1}{3}\sqrt{\frac{V}{\pi h}}$

3. The distance a free falling object has traveled can be modeled by the equation $d = \frac{1}{2}at^2$, where a is acceleration due to gravity and t is the amount of time the object has fallen. What is t in terms of a and d?

a. $t = \sqrt{\frac{da}{2}}$

b. $t = \sqrt{\frac{2d}{a}}$

c. $t = \left(\frac{da}{d}\right)^2$

d. $t = \left(\frac{2d}{a}\right)^2$

4. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of A, h, and b_2. The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

5. The volume of a large can of tuna fish can be calculated using the formula $V = \pi r^2 h$. Write an equation to find the radius, r, in terms of V and h. Determine the diameter, to the nearest inch, of a large can of tuna fish that has a volume of 66 cubic inches and a height of 3.3 inches.

6. Michael borrows money from his uncle, who is charging him simple interest using the formula $I = \mathrm{P}rt$. To figure out what the interest rate, r, is, Michael rearranges the formula to find r. His new formula is r equals

a. $\frac{I-P}{t}$

b. $\frac{P-I}{t}$

c. $\frac{I}{Pt}$

d. $\frac{Pt}{I}$

A.CED.4: Transform Formulas
Answer Section

1. ANS: A
Strategy: Use the four column method to isolate r.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

PTS: 2 REF: 011516a1 NAT: A.CED.4 TOP: Transforming Formulas

2. ANS: A
Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\frac{1}{3}\pi r^2 h$
Multiply both expressions by 3	$3V$	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{3V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{3V}{\pi h}$	=	r^2
Take square root of both sides.	$\sqrt{\frac{3V}{\pi h}}$	=	r

PTS: 2 REF: 061423a1 NAT: A.CED.4 TOP: Transforming Formulas

3. ANS: B
Strategy: Use the four column method. Isolate t.

Notes	Left Expression	Sign	Right Expression
Given	d	=	$\frac{1}{2}at^2$
Multiply both expressions by 2	$2d$	=	at^2
Divide both expressions by a	$\frac{2d}{a}$	=	$\frac{at^2}{a}$
Simplify	$\frac{2d}{a}$	=	t^2
Take square root of both expressions	$\sqrt{\frac{2d}{a}}$	=	t

PTS: 2 REF: 061519AI NAT: A.CED.4 TOP: Transforming Formulas

4. ANS:

a) $b_1 = \frac{2A}{h} - b_2$

b) The other base is 8 feet.

Strategy: Use the four column method to isolate b_1 and create a new formula, then use it to find the length of the other base.

Notes	Left Expression	Sign	Right Expression
Given	A	=	$\frac{1}{2}h(b_1 + b_2)$
Multiply both expressions by 2	$2A$	=	$h(b_1 + b_2)$
Divide both expressions by h	$\frac{2A}{h}$	=	$\frac{h(b_1 + b_2)}{h}$
Simplify	$\frac{2A}{h}$	=	$b_1 + b_2$
Subtract b_2 from both expressions	$\frac{2A}{h} - b_2$	=	b_1

Substitute the values stated in the problem in the formula.

$$A = 60,\ h = 6,\ b_2 = 12$$

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2(60)}{6} - 12$$

$$b_1 = \frac{120}{6} - 12$$

$$b_1 = 20 - 12$$

$$b_1 = 8 \text{ feet}$$

PTS: 4 REF: 081434a1 NAT: A.CED.4 TOP: Transforming Formulas

5. ANS:

a) $r = \sqrt{\dfrac{V}{\pi h}}$

b) 5 inches

Strategy: Use the four column method to isolate *r* and create a new formula, then use the new formula to answer the problem.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\dfrac{V}{\pi h}$	=	$\dfrac{\pi r^2 h}{\pi h}$
Simplify	$\dfrac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\dfrac{V}{\pi h}}$	=	r

Substitute the values from the problem into the new equation.

$$V = 66,\ \ h = 3.3$$

$$r = \sqrt{\frac{V}{\pi h}}$$

$$r = \sqrt{\frac{66}{\pi(3.3)}}$$

$$r = \sqrt{\frac{20}{\pi}}$$

$$r \approx \sqrt{6.4}$$

$$r \approx 2.52$$

If the radius is approximately 2.5 inches, the diameter is approximately 5 inches.

PTS: 4 REF: 081535ai NAT: A.CED.4 TOP: Transforming Formulas

6. ANS: C

Strategy: Isolate r, as follows:

$$I = \Pr t$$

$$I = Pt(r)$$

$$\frac{I}{Pt} = r$$

PTS: 2 REF: 011606ai NAT: A.CED.4 TOP: Transforming Formulas

F.LE.5: Interpret Parts of an Expression or Equation

Interpret expressions for functions in terms of the situation they model.
5. Interpret the parameters in a linear or exponential function in terms of a context.

Big Idea

Each number, variable, or product of a number and variable in an expression can be represented in narrative form.

Vocabulary

parameter *(A)(G)(A2T)* A quantity or constant whose value varies with the circumstances of its application.
Example: In $y = ax^2$, a is a parameter

REGENTS PROBLEMS

1. A company that manufactures radios first pays a start-up cost, and then spends a certain amount of money to manufacture each radio. If the cost of manufacturing r radios is given by the function $c(r) = 5.25r + 125$, then the value 5.25 best represents
   a. the start-up cost
   b. the profit earned from the sale of one radio
   c. the amount spent to manufacture each radio
   d. the average number of radios manufactured

2. A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function?
   a. y is the total cost, x is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.
   b. y is the total cost, x is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.
   c. x is the total cost, y is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.
   d. x is the total cost, y is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.

3. The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?
   a. The commercial costs \$0 to produce and \$110 per airing up to \$900.
   b. The commercial costs \$110 to produce and \$900 each time it is aired.
   c. The commercial costs \$900 to produce and \$110 each time it is aired.
   d. The commercial costs \$1010 to produce and can air an unlimited number of times.

4. The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

5. The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where $C(m)$ is the total cost for m months of membership. State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

F.LE.5: Interpret Parts of an Expression or Equation
Answer Section

1. ANS: C
   Strategy: Interpret the the function $c(r) = 5.25r + 125$ in narrative (word) form.

$c(r)$	$=$	5.25	r	$+$	125
the cost of manufacturing r radios	=	$5.25	for each radio	plus	a start-up cost of $125

$5.25 for each radio represents the amount spent to manufacture each radio, which is answer choice c.

PTS: 2 REF: 061407a1 NAT: F.LE.5 TOP: Modeling Linear Equations

2. ANS: B
   Strategy: Interpret the the function $y = 40 + 90x$ in narrative (word) form.

y	$=$	40	$+$	90	x
total cost	=	a one time installation fee of $40	plus	a $90 service charge	times the number of months

PTS: 2 REF: 081402a1 NAT: F.LE.5 TOP: Modeling Linear Equations

3. ANS: C
   Strategy: Interpret the the function $C(n) = 110n + 900$ in narrative (word) form, then eliminate wrong answers.

$C(c)$	$=$	110	n	$+$	900
The costs of a commercial	=	$110	times the number of times the commerical airs	plus	a production cost of $900

Answer choice a is wrong because the the production costs are not $0.
Answer choice b is wrong because the production costs and costs per airing are reversed.
Answer choice c is correct.
Answer choice d in wrong because it makes no sense.

PTS: 2 REF: 061501AI NAT: F.LE.5 TOP: Modeling Linear Equations

4. ANS:
   0.5 represents the rate of decay and 300 represents the initial amount of the compound.

Strategy: Use information from the problem together with the standard formula for exponential decay, which is $A = P(1-r)^t$, where A represents the amount remaining, P represents the initial amount, r represents the rate of decay, and t represents the number of cycles of decay.

$$A = P(1-r)^t$$

$$p(t) = 300(0.5)^t$$

The structures of the equations show that $P = 300$ and $(1-r) = 0.5$.
Accordingly, 300 represents the initial amount of chemical substance in milligrams and 0.5 represents the rate of decay each year.

PTS: 2 REF: 061426a1 NAT: F.LE.5 TOP: Modeling Exponential Equations

5. ANS:

$$y = mx + b$$
$$y = (\text{slope})x + (y\text{-intercept})$$
$$C(x) = 50(m) + (79.50)$$

The slope is 50 and represents the amount paid each month for membewrship in the gym.
The y-intercept is 79.50 and represents the initial cost of membership.

PTS: 2 REF: 011629ai NAT: F.LE.5 TOP: Modeling Linear Functions

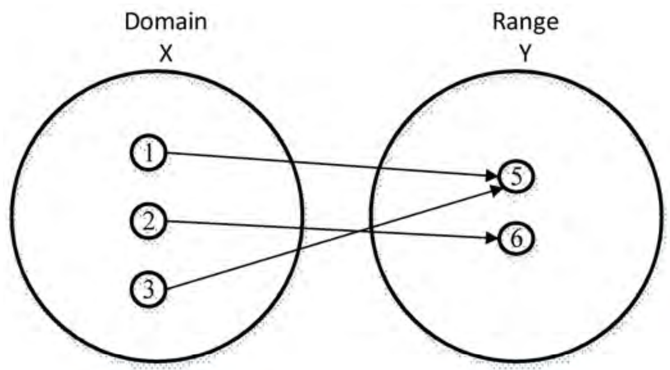
F.IF.1: Define Functions

Understand the concept of a function and use function notation.

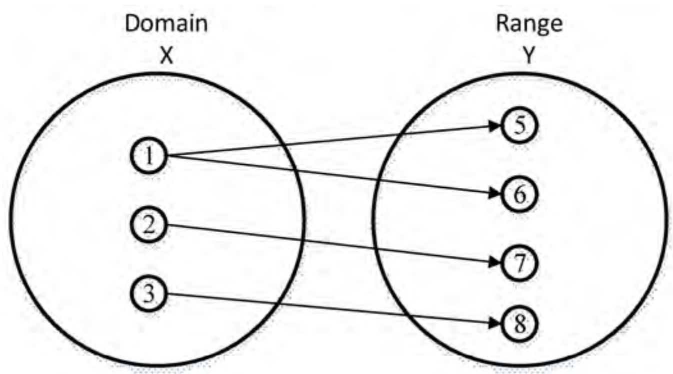
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Function: A rule that assigns to each number x in the **function's** *domain* (x-axis) a unique number $f(x)$ in the function's *range (y-axis)*. A function takes the input value of an independent variable and pairs it with *one and only one* output value of a dependent variable.

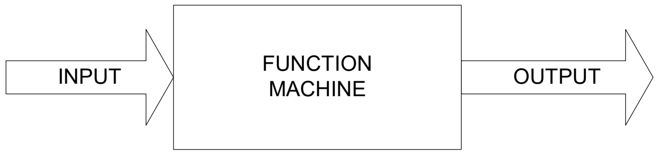
Function



Not a Function

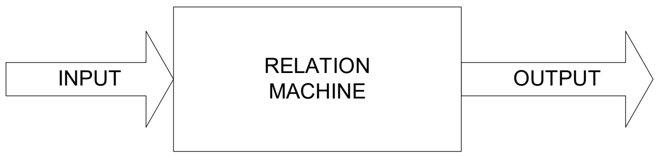


Expressed as ordered Pairs:
Function: (1,5) (2,6) (3,5)
Not a Function: (1,5) (2,7) (3,8) (1,6)



<u>Function</u>: A **<u>function</u>** is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A **<u>function</u>** is always a relation.

Example: y=2x



<u>Relation</u>: A relation may produce more than one output for a given input. A relation may or may not be a function.

Example: $y^2 = x$
$y = \sqrt{x}$

This is not a function, because when x=16, there is more than one y-value. $\sqrt{16} = \pm 4$.

A function can be represented four ways. These are:

- A verbal description
- a function rule (equation)
- a table of values
- a graph.

<u>Function Rules</u> show the relationship between dependent and independent variables in the form of an equation with two variables.

- The **<u>independent</u>** variable is the **<u>input</u>** of the function and is typically denoted by the x-variable.
- The **<u>dependent</u>** variable is the **<u>output</u>** of the function and is typically denoted by the y-variable.

<u>All linear equations</u> in the form $y = mx + b$ **<u>are functions except vertical lines</u>**.

<u>2nd degree and higher equations</u> may or may not be functions.

<u>Tables of Values</u> show the relationship between dependent and independent variables in the form of a table with columns and rows:

- The **<u>independent</u>** variable is the **<u>input</u>** of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- The **<u>dependent</u>** variable is the **<u>output</u>** of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

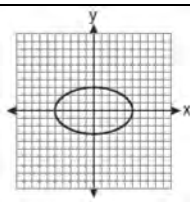
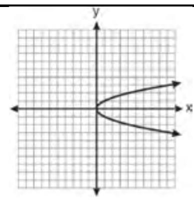
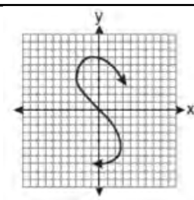
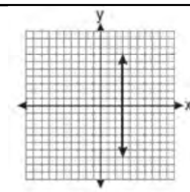
Function			Not A Function	
x	y		x	y
1	5		1	5
2	6		2	6
3	7		3	7
4	8		4	8
5	9		2	9

<u>Graphs</u> show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:

- The value of **<u>independent</u>** variable is **<u>input</u>** of the function and is typically shown on the **<u>x-axis</u>** (horizontal axis) of the coordinate plane.
- The value of the **<u>dependent</u>** variable is the **<u>output</u>** of the function and is typically shown on the **<u>y-axis</u>** (vertical axis) of the coordinate plane.

<u>Vertical Line Test</u>: If a vertical line passes through a graph of an equation more than once, the graph is *not* a graph of a **<u>function</u>**.

If you can draw a vertical line through any value of x in a relation, and the vertical line intersects the graph in two or more places, the relation is not a function.

 Circles and Ellipses …are **<u>not functions</u>**.	 Parabola-like graphs that open to the side …are **<u>not functions</u>**.	 S-Curves …are **<u>not functions</u>**	 Vertical lines …are **<u>not functions</u>**.

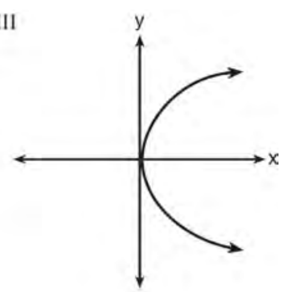
REGENTS PROBLEMS

1. Which representations are functions?

I

x	y
2	6
3	–12
4	7
5	5
2	–6

III



II $\{(1,1), (2,1), (3,2), (4,3), (5,5), (6,8), (7,13)\}$ IV $y = 2x + 1$

a. I and II
b. II and IV
c. III, only
d. IV, only

2. Which table represents a function?

a.

x	2	4	2	4
f(x)	3	5	7	9

b.

x	0	−1	0	1
f(x)	0	1	−1	0

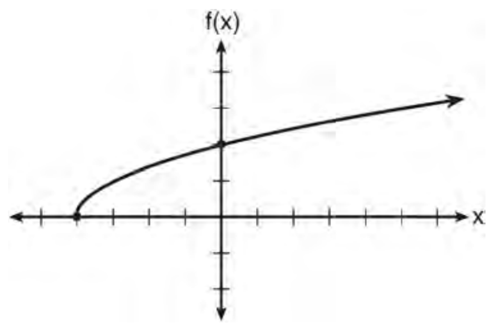
c.

x	3	5	7	9
f(x)	2	4	2	4

d.

x	0	1	−1	0
f(x)	0	−1	0	1

3. The graph of the function $f(x) = \sqrt{x+4}$ is shown below.



The domain of the function is

a. $\{x \mid x > 0\}$

b. $\{x \mid x \geq 0\}$

c. $\{x \mid x > -4\}$

d. $\{x \mid x \geq -4\}$

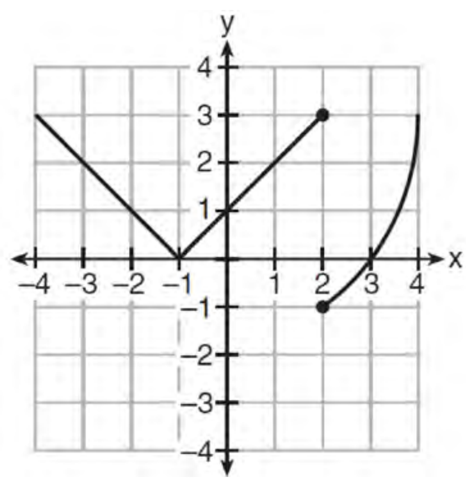
4. The function f has a domain of $\{1,3,5,7\}$ and a range of $\{2,4,6\}$. Could f be represented by $\{(1,2),(3,4),(5,6),(7,2)\}$? Justify your answer.

5. A function is shown in the table below.

x	f(x)
–4	2
–1	–4
0	–2
3	16

If included in the table, which ordered pair, $(-4,1)$ or $(1,-4)$, would result in a relation that is no longer a function? Explain your answer.

6. Marcel claims that the graph below represents a function.



State whether Marcel is correct. Justify your answer.

F.IF.1: Define Functions
Answer Section

1. ANS: B
Strategy: Determine if each of the for views are functions, then select from the answer choices. A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable.

I ***is not*** a function because when $x = 2$, y can equal both 6 and -6.
II **is** a function because there are no values of x that have more than one value of y.
III ***is not*** a function because it fails the vertical line test, which means there are values of x that have more than one value of y.
IV ***is*** a function because it is a straight line that is not vertical.

Answer choice b is the correct answer.

PTS: 2 REF: 081511ai NAT: F.IF.1 TOP: Defining Functions

2. ANS: C
Strategy: Eliminate wrong answers. A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable.

Answer choice a ***is not*** a function because there are two values of y when $x = 2$.
Answer choice b ***is not*** a function because there are two values of y when $x = 0$.
Answer choice c ***is*** a function because only one value of y is paired with each value of x.
Answer choice d ***is not*** a function because there are two values of y when $x = 0$.

.

PTS: 2 REF: 061504AI NAT: F.IF.1 TOP: Defining Functions

3. ANS: D
Strategy: Use the number line of the x-axis, the fact that the graph begins with a solid dot, indicating that -4 is included in the domain, and the fact that the graph includes an arrow indicating that the graph continues to positive infinity, to select answer choice d.

PTS: 2 REF: 061509AI NAT: F.IF.1 TOP: Domain and Range

4. ANS:
Yes, because every element of the domain is assigned one unique element in the range.

Strategy: Determine if any value of x has more that one associated value of y. A function has one and only one value of y for every value of x.

PTS: 2 REF: 061430a1 NAT: F.IF.1 TOP: Defining Functions

5. ANS:
$(-4, 1)$, because then every element of the domain is not assigned one unique element in the range.

PTS: 2 REF: 011527a1 NAT: F.IF.1 TOP: Defining Functions

6. ANS:
Marcel is not correct, because the relation does not pass the vertical line test. If you draw the vertical line $x = 2$, there will be more than one value of y. A function can have one and only one value of y for every value of x.

PTS: 2 REF: 011626ai NAT: F.IF.1 TOP: Defining Functions
KEY: graphs

F.IF.2: Use Function Notation

Understand the concept of a function and use function notation.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Function Notation:

In a function, the dependent y variable (on the y-axis of the graph) is paired with a a specific value of the independent x variable (on the x-axis of the graph). In function notation, $f(x)$ is used instead of the letter y. When graphing using function notation, the label of the y-axis is changed to show the f(x) axis. There are three primary advantages to using function notation:

1. The use of function notation indicates that the relationship is a function, and
2. The use of function notation simplifies evaluation of the value of f(x) for specific values of x.
3. The use of function notation allows greater flexibility and specificity in naming variables.

Examples:

The equation	...can be written using function notation as...			
$y=3x^2+4$	$f(x)=3x^2+4$	$x\xrightarrow{f}3x^2+4$	$f:x\mapsto 3x^2+4$	$f=\left\{(x,y)\middle\vert 3x^2+4\right\}$
$y=2x$	$f(x)=2x$	$x\xrightarrow{f}2x$	$f:x\mapsto 2x$	$f=\left\{(x,y)\middle\vert 2x\right\}$

Note that the y variable can be replaced with many forms in function notation. The letters f and x are often replaced with other letter, so you might see something like $g(h)=3x^2+4$. In this example, $g(h)$ still represents the value of y, the dependent variable.

To evaluate a function, substitute the indicated number of expression for the variable.

Example:

Given the function rule $f(x)=3x^2+4$, find the value of $f(5)$ as follows:

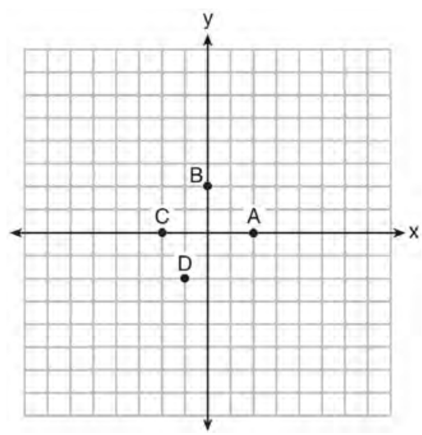
$$f(x)=3x^2+4$$
$$f(5)=3(5)^2+4$$
$$f(5)=3(25)+4$$
$$f(5)=75+4$$
$$f(5)=79$$

REGENTS PROBLEMS

1. If $f(x) = \frac{1}{3}x + 9$, which statement is always true?

a. $f(x) < 0$
b. $f(x) > 0$
c. If $x < 0$, then $f(x) < 0$.
d. If $x > 0$, then $f(x) > 0$.

2. The graph of $y = f(x)$ is shown below.



Which point could be used to find $f(2)$?

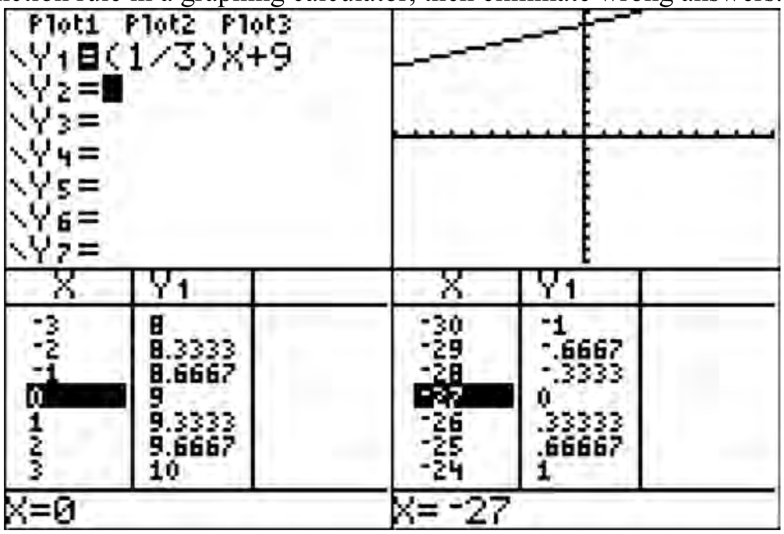
a. A
b. B
c. C
d. D

3. If $f(x) = \frac{\sqrt{2x+3}}{6x-5}$, then $f\left(\frac{1}{2}\right) =$

a. 1
b. -2
c. -1
d. $-\frac{13}{3}$

F.IF.2: Use Function Notation
Answer Section

1. ANS: D
   Strategy: Inspect the function rule in a graphing calculator, then eliminate wrong answers.



   Answer choice *a* can be eliminated because the table clearly shows $f(x)$ values greater than zero.
   Answer choice *b* can be eliminated because the table clearly shows $f(x)$ values less than zero.
   Answer choice *c* can be eliminated because if x is greater than -27, then $f(x) > 0$.
   Choose answer choice *d* because the graph and table clearly show that all values of $f(x)$ are positive when values of x are positive.

   PTS: 2 REF: 061417a1 NAT: F.IF.2 TOP: Domain and Range

2. ANS: A
   Strategy: Understand that the meaning of $f(2)$ is the value of y when $x = 2$, then eliminate wrong answers.

   Choose answer choice A because represents $f(2)$ with coordinates $(2,0)$. $f(2) = 0$.
   Answer choice b is wrong because if represents $f(0)$. $f(0) = 2$
   Answer choice c is wrong because if represents $f(-2)$. $f(-2) = 0$
   Answer choice d is wrong because if represents $f(-1)$. $f(-1) = -2$

   PTS: 2 REF: 061420a1 NAT: F.IF.2 TOP: Functional Notation

3. ANS: C

Strategy: Substitute $\frac{1}{2}$ for x, and solve.

$$f(x) = \frac{\sqrt{2x+3}}{6x-5}$$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{2\left(\frac{1}{2}\right)+3}}{6\left(\frac{1}{2}\right)-5}$$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{4}}{-2}$$

$$f\left(\frac{1}{2}\right) = \frac{2}{-2}$$

$$f\left(\frac{1}{2}\right) = -1$$

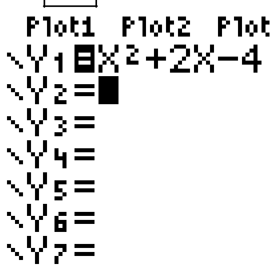
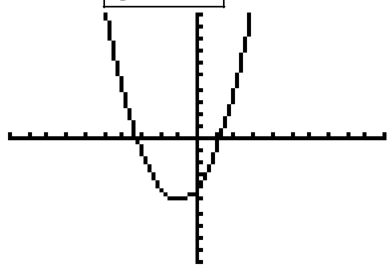
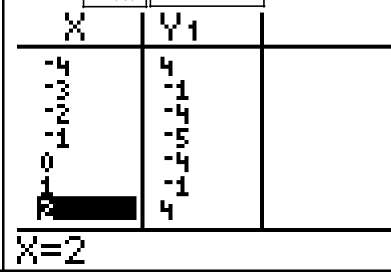
PTS: 2 REF: 081512ai NAT: F.IF.2 TOP: Functional Notation

F.IF.9: Four Views of a Function

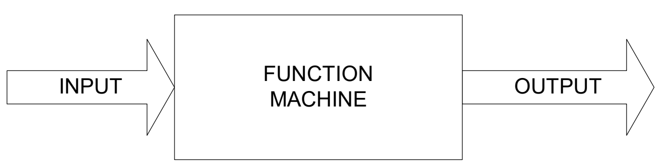
Analyze functions using different representations.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

BIG IDEAS:
A function can be represented mathematically through four inter-related views. These are:
- #1 a function rule (equation)
- #2 a table of values
- #3 a graph.
- #4 words

The TI-83+ graphing calculator allows you to input the function rule and access the graph and table of values, as shown below:

Y= Function Rule View	GRAPH View	2nd TABLE View
		

Definition of a Function: a function takes the input value of an independent variable and pairs it with one and only one output value of a dependent variable.



Function: A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A function is always a relation.
Example: $y=2x$

Function Rules show the relationship between dependent and independent variables in the form of an equation with two variables.
- § The **independent** variable is the **input** of the function and is typically denoted by the x-variable.
- § The **dependent** variable is the **output** of the function and is typically denoted by the y-variable.

When inputting function rules in a TI 83+ graphing calculator, the y-value (dependent variable) must be isolated as the left expression of the equation.

Tables of Values show the relationship between dependent and independent variables in the form of a table with columns and rows:
- § The **independent** variable is the **input** of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- § The **dependent** variable is the **output** of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

Graphs show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:

- § The value of **independent** variable is the **input** of the function and is typically shown on the **x-axis** (horizontal axis) of the coordinate plane.
- § The value of the **dependent** variable is the **output** of the function and is typically shown on the **y-axis** (vertical axis) of the coordinate plane.

REGENTS PROBLEMS

1. Given the following quadratic functions:

$$g(x) = -x^2 - x + 6$$

and

x	−3	−2	−1	0	1	2	3	4	5
n(x)	−7	0	5	8	9	8	5	0	−7

Which statement about these functions is true?

a. Over the interval $-1 \le x \le 1$, the average rate of change for $n(x)$ is less than that for $g(x)$.

b. The y-intercept of $g(x)$ is greater than the y-intercept for $n(x)$.

c. The function $g(x)$ has a greater maximum value than $n(x)$.

d. The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

2. Which quadratic function has the largest maximum?

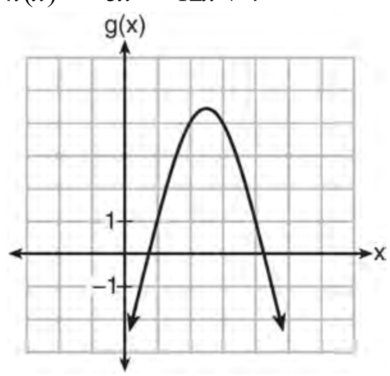
a. $h(x) = (3 - x)(2 + x)$

b.

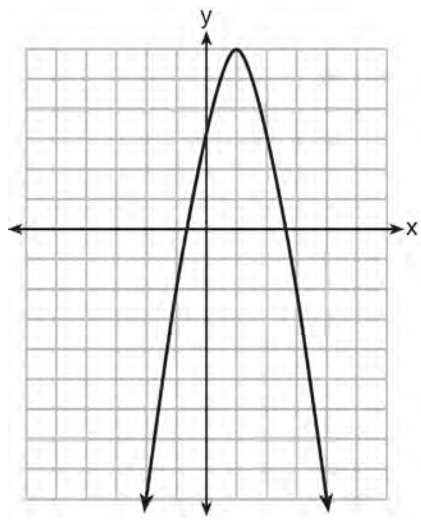
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

c. $k(x) = -5x^2 - 12x + 4$

d.

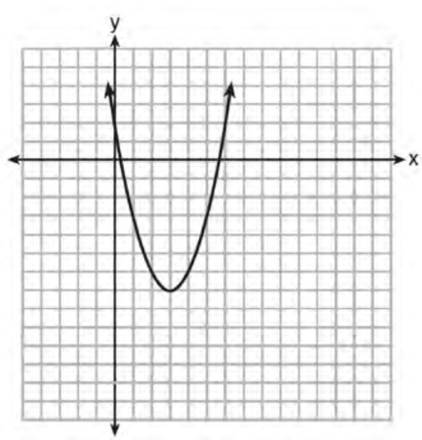


3. Let f be the function represented by the graph below.



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

4. The graph representing a function is shown below.



Which function has a minimum that is *less* than the one shown in the graph?

a. $y = x^2 - 6x + 7$
b. $y = |x + 3| - 6$
c. $y = x^2 - 2x - 10$
d. $y = |x - 8| + 2$

F.IF.9: Four Views of a Function
Answer Section

1. ANS: D

Strategy: Each answer choice must be evaluated using a different strategy.

a. Use the slope formula to find the rate of change for

$$m_{g(x)} = \frac{[g(1)]-[g(-1)]}{[1]-[-1]} = \frac{4-6}{2} = \frac{-2}{2} = -1$$

$$m_{n(x)} = \frac{[n(1)]-[n(-1)]}{[1]-[-1]} = \frac{9-5}{2} = \frac{4}{2} = 2$$

Statement a is false. The average rate of change for $n(x)$ is *more* than that for $g(x)$.

b. Compare the y-intercepts for both functions. The y-intercepts occur when $x = 0$.

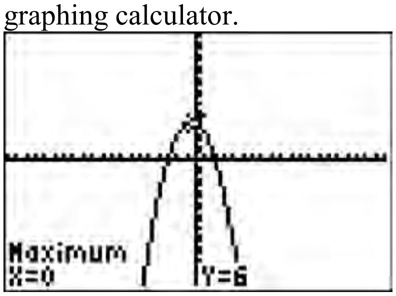
The y-intercept for g(x) = 6. $g(0) = -0^2 - 0 + 6 = 6$

The y-intercept for n(x) = 8 from the table.

Statement b is false. The y-intercept of $g(x)$ is *less* than the y-intercept for $n(x)$.

c. Compare the maxima of both functions.

The maxima of $g(x) = -x^2 - x + 6$ is 6. This can be found manually or with a graphing calculator.



The maxima of n(x) = 9, which can be seen in the table.

Statement c is false. The function $g(x)$ has a *smaller* maximum value than $n(x)$.

d. Compare the sum of the roots for both functions.

The sum of the roots for g(x) = $-3+2=-1$ from a graphing calculator.

X	Y1	
-3	0	
-2	4	
-1	6	
0	6	
1	4	
2	0	
3	-6	

X= -3

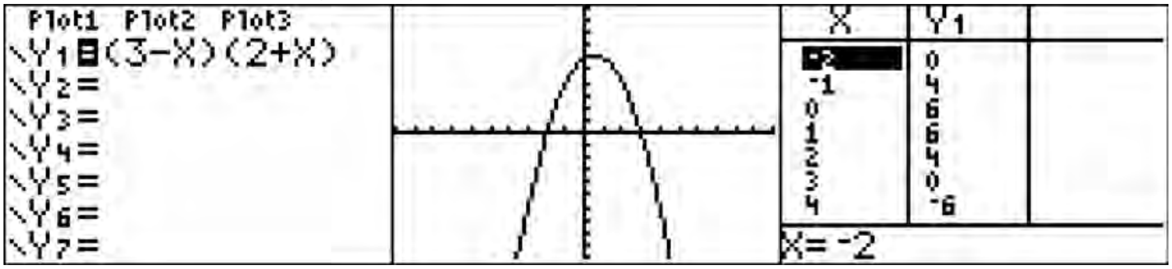
The sum of the roots for n(x) = $-2+4=2$ from the table.

Statement d is true. The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

PTS: 2 REF: 081521ai NAT: F.IF.9 TOP: Graphing Quadratic Functions

2. ANS: C
Strategy: Each answer choice needs to be evaluated for the largest maximum using a different strategy..

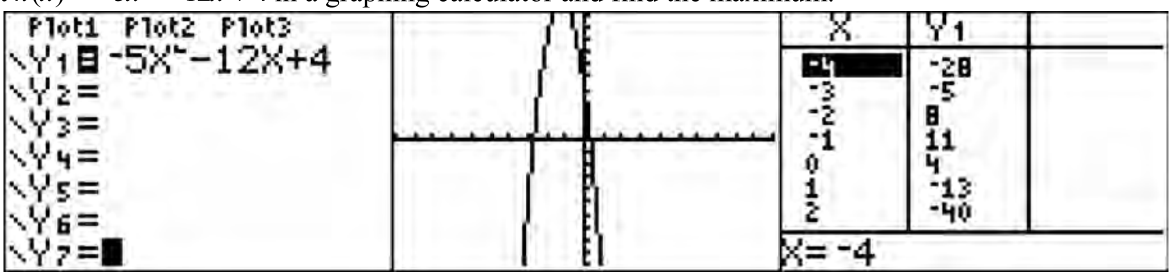
a) Input $h(x) = (3 - x)(2 + x)$ in a graphing calculator and find the maximum.



The maximum for answer choice *a* is a little more than 6.

b) The table shows that the maximum is a little more than 9.

c) Input $k(x) = -5x^2 - 12x + 4$ in a graphing calculator and find the maximum.



The table of values shows that the maximum is 11 or more.

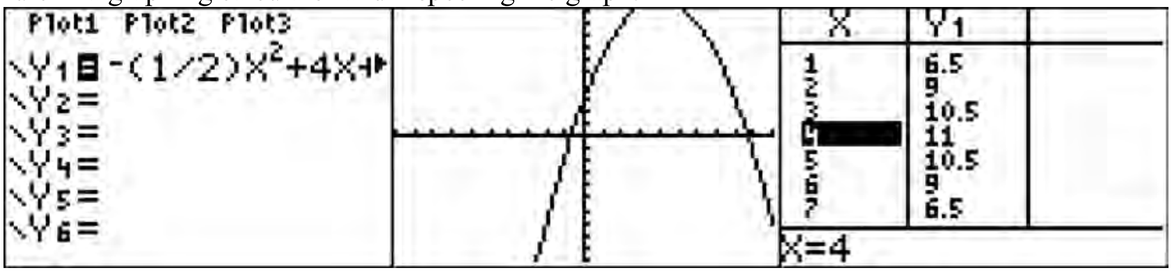
d) The graph shows that the maximum is a little more than 4.

Answer choice *c* is the best choice.

PTS: 2 REF: 061514AI NAT: F.IF.9 TOP: Graphing Quadratic Functions

3. ANS:
Function g has the larger maximum value. The maximum of function g is 11. The maximum of function f is 6.

Strategy: Determine the maximum for f from the graph. Determine the maximum for g by inputting the function rule in a graphing calculator and inspecting the graph.



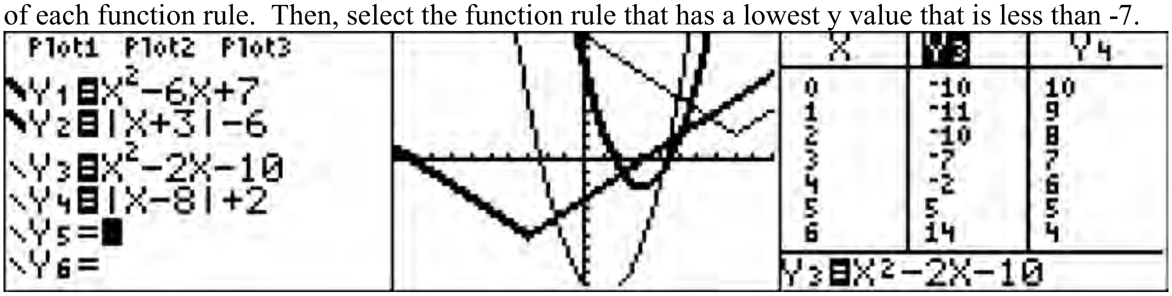
The table of values shows the maximum for g is 11.

Another way of finding the maximum for g is to use the axis of symmetry formula and the function rule, as follows: $x = \frac{-b}{2a} = \frac{-4}{2\left(-\frac{1}{2}\right)} = \frac{-4}{-1} = 4$

$$y = -\frac{1}{2}(4)^2 + 4(4) + 3 = -8 + 16 + 3 = 11$$

PTS: 2 REF: 081429a1 NAT: F.IF.9 TOP: Graphing Quadratic Functions

4. ANS: C
Strategy: The graph shows a parabola with a vertex at (3, -7), so the minima is at -7. Identify the lowest y-value of each function rule. Then, select the function rule that has a lowest y value that is less than -7.



The graph view of the four functions shows that the function $y = x^2 - 2x - 10$ has a y-value less than -7.

PTS: 2 REF: 011622ai NAT: F.IF.9 TOP: Comparing Functions

F.IF.3: Define Sequences as Recursive Functions

Understand the concept of a function and use function notation.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ?5= 1.*

Vocabulary

An **explicit formula** is one where you do not need to know the value of the term in front of the term that you are seeking. For example, if you want to know the 55th term in a series, an explicit formula could be used without knowing the value of the 54th term.

Example: The sequence 3, 11, 19, 27, ... begins with 3, and 8 is added each time to form the pattern. The sequence can be shown in a table as follows:

Term # (n)	1	2	3	4
$f(n)$	3	11	19	27

Explicit formulas for the sequence 3, 11, 19, 27, ... can be written as:

$$f(n) = 8n - 5 \qquad \text{or} \qquad f(n) = 3 + 8(n-1)$$

Using these **explicit formulas**, we can find the following values for any term, and we do not need to know the value of any other term, as shown below:

$f(n) = 8n - 5$	$f(n) = 3 + 8(n-1)$
$f(1) = 8(1) - 5 = 3$	$f(1) = 3 + 8(1-1) = 3 + 0 = 3$
$f(2) = 8(2) - 5 = 16 - 5 = 11$	$f(2) = 3 + 8(2-1) = 3 + 8 = 11$
$f(3) = 8(3) - 5 = 24 - 5 = 19$	$f(3) = 3 + 8(3-1) = 3 + 16 = 19$
$f(4) = 8(4) - 5 = 32 - 5 = 27$	$f(4) = 3 + 8(4-1) = 3 + 24 = 27$
$f(5) = 8(5) - 5 = 40 - 5 = 35$	$f(5) = 3 + 8(5-1) = 3 + 32 = 35$
$f(10) = 8(10) - 5 = 80 - 5 = 75$	$f(10) = 3 + 8(10-1) = 3 + 72 = 75$
$f(100) = 8(100) - 5 = 800 - 5 = 795$	$f(100) = 3 + 8(100-1) = 3 + 792 = 795$

Recursive formulas requires you to know the value of another term, usually the preceding term, to find the value of a specific term.

Example: Using the same sequence 3, 11, 19, 27, ... as above, a **recursive formula** for the sequence 3, 11, 19, 27, ... can be written as:

$$f(n+1) = f(n) + 8$$

This **recursive formula** tells us that the value of any term in the sequence is equal to the value of the term before it plus 8. A recursive formula must usually be anchored to a specific term in the sequence (usually the first term), so the recursive formula for the sequence 3, 11, 19, 27, ... could be anchored with the statement

$$f(1) = 3$$

Using this **recursive formula**, we can reconstruct the sequence as follows:

$f(1) = 3$

$f(2) = f(1) + 8 = 3 + 8 = 11$

$f(3) = f(2) + 8 = 11 + 8 = 19$

$f(4) = f(3) + 8 = 19 + 8 = 27$

$f(5) = f(4) + 8 = 27 + 8 = 35$

$f(10) = f(9) + 8 = ? + 8 = ?$

Observe that the recursive formula $f(n+1) = f(n) + 8$ includes two different values of the dependent variable, which in this example are $f(n)$ and $f(n+1)$, and we can only reconstruct our original sequence using this **recursive formula** if we know the term immediately preceding the term we are seeking.

Two Kinds of Sequences

arithmetic sequence *(A2T)* A set of numbers in which the common difference between each term and the preceding term is constant.

Example: In the **arithmetic sequence** 2, 5, 8, 11, 14, … the common difference between each term and the preceding term is 3. A table of values for this sequence is:

Term # (n)	1	2	3	4	5
$f(n)$	2	5	8	11	14

An **explicit formula** for this sequence is $f(n) = 3n - 1$

A **recursive formula** for this sequence is: $f(n+1) = f(n) + 3, \quad f(1) = 2$

geometric sequence *(A2T)* A set of terms in which each term is formed by multiplying the preceding term by a common nonzero constant.

Example: In the geometric sequence 2, 4, 8, 16, 32... the common ratio is 2. Each term is 2 times the preceding term. A table of values for this sequence is:

Term (n)	1	2	3	4	5
$f(n)$	2	4	8	16	32

An **explicit formula** for this sequence is $f(n) = 2^n$

A **recursive formula** for this sequence is: $f(n+1) = 2f(n), \quad f(1) = 2$

REGENTS PROBLEMS

1. If $f(1) = 3$ and $f(n) = -2f(n-1) + 1$, then $f(5) =$

   a. -5
   b. 11
   c. 21
   d. 43

2. A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the sunflower in n weeks?

   I. $f(n) = 2n + 3$

   II. $f(n) = 2n + 3(n - 1)$

   III. $f(n) = f(n - 1) + 2$ where $f(0) = 3$

   a. I and II
   b. II, only
   c. III, only
   d. I and III

3. If a sequence is defined recursively by $f(0) = 2$ and $f(n + 1) = -2f(n) + 3$ for $n \geq 0$, then $f(2)$ is equal to

   a. 1
   b. -11
   c. 5
   d. 17

4. Which recursively defined function has a first term equal to 10 and a common difference of 4?

   a. $f(1) = 10$
      $f(x) = f(x - 1) + 4$
   b. $f(1) = 4$
      $f(x) = f(x - 1) + 10$
   c. $f(1) = 10$
      $f(x) = 4f(x - 1)$
   d. $f(1) = 4$
      $f(x) = 10f(x - 1)$

5. Which recursively defined function represents the sequence $3, 7, 15, 31, \ldots$?

   a. $f(1) = 3,\ f(n + 1) = 2^{f(n)} + 3$
   b. $f(1) = 3,\ f(n + 1) = 2^{f(n)} - 1$
   c. $f(1) = 3,\ f(n + 1) = 2f(n) + 1$
   d. $f(1) = 3,\ f(n + 1) = 3f(n) - 2$

F.IF.3: Define Sequences as Functions
Answer Section

1. ANS: D
Strategy: Use the recursive formula: $f(1) = 3$ and $f(n) = -2f(n-1) + 1$ to find each term in the sequence.
$f(1) = 3$
$f(n) = -2f(n-1) + 1$
$f(2) = -2f(2-1) + 1 = -2f(1) + 1 = -2(3) + 1 = -6 + 1 = -5$
$f(3) = -2f(3-1) + 1 = -2f(2) + 1 = -2(-5) + 1 = 10 + 1 = 11$
$f(4) = -2f(4-1) + 1 = -2f(3) + 1 = -2(11) + 1 = -22 + 1 = -21$
$f(5) = -2f(5-1) + 1 = -2f(4) + 1 = -2(-21) + 1 = 42 + 1 = 43$

PTS: 2 REF: 081424a1 NAT: F.IF.3 TOP: Sequences

2. ANS: D
Strategy: If sunflower's height is modelled using a table, then the three formulas can be tested to see which one(s) produce results that agree with the table.

Weeks (n)	Height $f(n)$	$f(n) = 2n + 3$	$f(n) = 2n + 3(n-1)$	$f(n) = f(n-1) + 2$ where $f(0) = 3$
0	3	$f(0) = 2(0) + 3 = 3$	$f(0) =$ $2(0) + 3(0-1) =$ -3	$f(0) = 3$
1	5	$f(1) = 2(1) + 3 = 5$		$f(1) = f(0) + 2 = 3 + 2 = 5$
2	7	$f(2) = 2(2) + 3 = 7$		$f(2) = f(1) + 2 = 5 + 2 = 7$
3	9	$f(3) = 2(3) + 3 = 9$		$f(3) = f(2) + 2 = 7 + 2 = 9$

Formula I, $f(n) = 2n + 3$, is an explicit formula that *agrees* with the table.
Formula II is an explicit formula that *does not agree* with the table.
Formula III, $f(n) = f(n-1) + 2$ where $f(0) = 3$, is a recursive formula that agrees with the table.

PTS: 2 REF: 061421a1 NAT: F.IF.3 TOP: Sequences

3. ANS: C
Strategy: Use the recursive formula: $f(0) = 2$ and $f(n+1) = -2f(n) + 3$ to find each term in the sequence.
$f(0) = 2$
$f(1) = f(0+1) = -2f(n) + 3 = -2(2) + 3 = -4 + 3 = -1$
$f(2) = f(1+1) = -2f(n) + 3 = -2(-1) + 3 = 2 + 3 = 5$
Answer choice *c* corresponds to $f(2) = 5$.

PTS: 2 REF: 011520a1 NAT: F.IF.3 TOP: Sequences

4. ANS: A
Strategy: Eliminate wrong answers.

Choices *b* and *d* have first terms equal to 4, but the problem states that the first term is equal to 10. Therefore, eliminate choices *b* and *d*.

A common difference of 4 requires the addition or subtraction of 4 to find the next term in the sequence. Eliminate choice *c* because choice *c multiplies* the preceding term by 4.

Choice *a* is correct because the first term is 10 and 4 is added to each preceding term.

PTS: 2 REF: 081514ai NAT: F.IF.3 TOP: Sequences

5. ANS: C
Each choice has a first term equal to 3.
Each additional term is twice its preceding term plus 1.
Strategy: Eliminate wrong answers and check.
All choices have show the the first term equals three: $f(1) = 3$.
Eliminate $f(1) = 3,\ f(n+1) = 2^{f(n)} + 3$ and $f(1) = 3,\ f(n+1) = 2^{f(n)} - 1$ because they are exponential.
Eliminate $f(1) = 3,\ f(n+1) = 3f(n) - 2$ because each term is not three times its preceding term minus two.
Check $f(1) = 3,\ f(n+1) = 2f(n) + 1$ as follows:

$$f(1) = 3,\ f(n+1) = 2f(n) + 1$$
$$f(2) = 2(3) + 1 = 7$$
$$f(3) = 2(7) + 1 = 15$$
$$f(4) = 2(15) + 1 = 31$$

$f(1) = 3,\ f(n+1) = 2f(n) + 1$ produces the sequence 3, 7, 15, 31,.....

PTS: 2 REF: 011618ai NAT: F.LE.2 TOP: Sequences

F.BF.l: Model Explicit and Recursive Processes

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Vocabulary

An **explicit formula** is one where you do not need to know the value of the term in front of the term that you are seeking.

A **recursive formula** requires you to know the value of another term, usually the preceding term, to find the value of a specific term.

BIG IDEA

Sometimes it is necessary to write a function rule using algebraic notation based on information from a narrative, a table, or a diagram.

Example of Modeling a Sample Function.

What follows are two views of the same function. The challenge is to write the function rule.

Narrative View: The inside of a freeze is kept at a constant temperature of 15 degrees farenheit. When a quart of liquid water is placed in the freezer, its farenheit temperature drops by one-half every 20 minutes until it turns into ice and reaches a constant temperature of 15 degrees.

Table View: These tables shows what the temperatures of two different quarts of water would be after *m* minutes in the freezer.

Initial Temperature = 80 degrees

Minutes in Freezer (*m*)	0	20	40	60	80
Temperature *f(m)*	80	40	20	15	15

Initial Temperature = 120 degrees

Minutes in Freezer (*m*)	0	20	40	60	80
Temperature *f(m)*	120	60	30	15	15

Writing the Function Rule

The challenge is to write the function rule. In this problem, it is necessary to model two different functions: one for what happens before the water reaches 15 degrees, and one for what happens after the water reaches 15 degrees. The narrative and table views suggest that the temperature starts at 80 degrees, drops exponentially until the water reaches 15 degrees, then stays at a constant 15 degrees. The graph would go down quickly at first, then go down more slowly, then become a horzontal line.

The horizontal line can be modeled with the equation: $y = 15$.

The exponential part of the function can be modeled using the formula for exponential decay, $A = P(1 \pm r)^t$, together with information from the table and narrative views.

$$A = P(1 \pm r)^t$$

$$f(m) = I(1 - \frac{1}{2})^{\frac{m}{20}}$$

$$f(m) = I\left(\frac{1}{2}\right)^{\frac{m}{20}}$$

$$f(m) = 80\left(\frac{1}{2}\right)^{\frac{m}{20}}$$

$f(m)$ represents the temperature of the water after m minutes in the freezer.

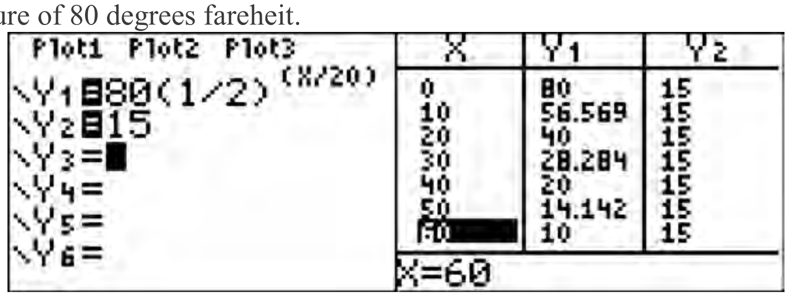
I represents the initial temperature of the water.

$(\frac{1}{2})$ represents the exponential rate of decay.

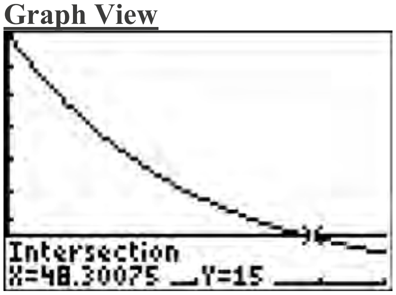
$\frac{m}{20}$ represents time.

The **range** of the function would be limited to $212 \geq f(m) \geq 15$

Check: We can check this answer by inputting both function rules in a graphing calculator for a quart of water with an initial temperature of 80 degrees fareheit.



Graph View



DIMS - Does It Make Sense? Yes, all four views of the function show that the water cools down quickly at first, then more slowly, then reaches a final temperature of 15 degrees. The graph view shows that it would take about 48 minutes for a quart of liquid water with an initial temperature of 80 degrees to reach a frozen temperature of 15 degrees.

Regents Problems

1. Krystal was given \$3000 when she turned 2 years old. Her parents invested it at a 2% interest rate compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Krystal had in the account when she turned 18?

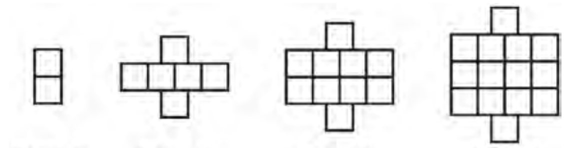
   a. $3000(1+0.02)^{16}$
   b. $3000(1-0.02)^{16}$
   c. $3000(1+0.02)^{18}$
   d. $3000(1-0.02)^{18}$

2. Rhonda deposited \$3000 in an account in the Merrick National Bank, earning 4.2% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find B, her account balance after t years.

3. Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75. Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals. Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

4. Alex is selling tickets to a school play. An adult ticket costs \$6.50 and a student ticket costs \$4.00. Alex sells x adult tickets and 12 student tickets. Write a function, $f(x)$, to represent how much money Alex collected from selling tickets.

5. A pattern of blocks is shown below.



If the pattern of blocks continues, which formula(s) could be used to determine the number of blocks in the *n*th term?

I	II	III
$a_n = n + 4$	$a_1 = 2$ $a_n = a_{n-1} + 4$	$a_n = 4n - 2$

a. I and II
b. I and III
c. II and III
d. III, only

F.BF.l: Model Explicit and Recursive Processes
Answer Section

1. ANS: A
Strategy 1: Use the formula for exponential growth to model the problem.

The formula for exponential **growth** is $y = a(1+r)^t$.
The formula for exponential **decay** is $y = a(1-r)^t$.
y = **final amount** after measuring growth/decay
a = **initial amount** before measuring growth/decay
r = growth/decay **rate** (usually a percent)
t = **number of time intervals** that have passed

The problem asks for the *right side expression for exponential growth*.
The problem states that $3,000 is the **initial amount**.
The problem states that the **growth factor** is 2%, which is written as .02 and added to 1.
The problem states that interest is compounded annually from age 2 through age 18, so the number of time intervals is 16 years.
The final expression for the right side of the exponential growth equation is written as $3000(1+0.02)^{16}$.

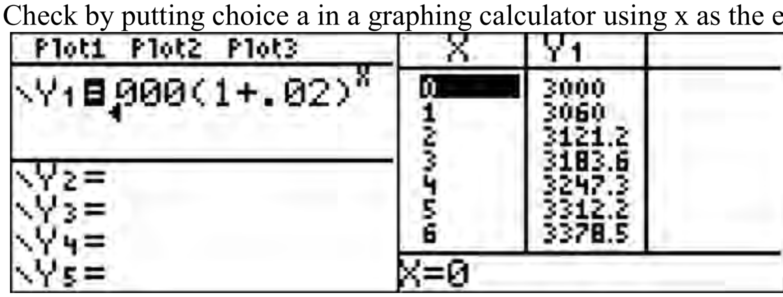
Strategy 2. Build a model and eliminate wrong answers.
Model the words using a table of values to see the pattern.

Krystal's Age	# Times Compounding	Amount
2	0	3000
3	1	3060
4	2	3121.2
5	3	3183.624
...	...	...
18	16	?

It is clear from the table that the number of times interest compounds is 2 less than Krystal's age. Eliminate answer choices *c* and *d*, because both show exponents of 18, which is Krystal's age, not the number of times the interest will compound.

The choices now are a and b. The table shows that the amounts are increasing, which is exponential growth, not exponential decay. Eliminate choice *b* because it shows exponential decay.

Check by putting choice a in a graphing calculator using x as the exponent.



Answer choice *a* creates the same table of values, and the amount of money on Krystal's 18th birthday will be $3000(1+0.02)^{16}$ dollars.

PTS: 2 REF: 011504a1 NAT: F.BF.1a TOP: Modeling Exponential Equations

2. ANS:
$B = 3000(1.042)^t$

Strategy: Use the formula for exponential growth to model the problem.

The formula for exponential **growth** is $y = a(1+r)^t$.
The formula for exponential **decay** is $y = a(1-r)^t$.
y = **final amount** after measuring growth/decay
a = **initial amount** before measuring growth/decay
r = growth/decay **rate** (usually a percent)
t = **number of time intervals** that have passed

The problem states that B should be used to represent the **final amount** after growth.
The problem states that $3,000 is the **initial amount**.
The problem states that the **growth factor** is 4.2%, which is added to 1 and written as 1.042
The problem states that interest is compounded annually, so the number of time intervals is t years.
The final equation is written as $B = 3000(1.042)^t$

PTS: 2 REF: 081426a1 NAT: F.BF.1a TOP: Modeling Exponential Equations

3. ANS:
63 weeks

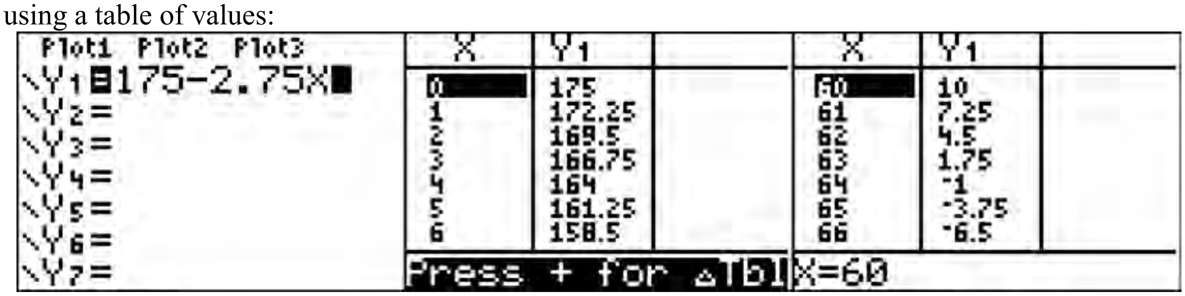
Strategy: Model the problem with a linear function.

$A(n) = \$175 - \$2.75n$
Each movie rental costs \$2.75
Let n represent the number of rentals.
Let $A(n)$ represent the amount of money on the rental card after n rentals.
Caitlin can rent a movie for 63 weeks in a row.

Explanation:
Caitlin has \$175.
Each movie rental costs \$2.75
\$175 divided by \$2.75 equals 63.6, so \$2.75 times 63.6 equals \$175.
Caitlin has enough money to rent 63 videos. After 63 weeks, Caitlin will not have enough money to rent another movie.
$A(63) = \$175 - \$2.75(63)$

$A(63) = \$175 - \$173.25$

$A(63) = \$1.75$
After 63 weeks, Caitlin will have \$1.75 on her rental card, which is not enough to rent another movie.

Check using a table of values:


PTS: 4 REF: 061435a1 NAT: F.BF.1a TOP: Modeling Linear Equations

4. ANS:
$f(x) = 6.50x + 4(12)$

Strategy: Translate the words into math.

\$6.50 per adult ticket plus \$4.00 per student ticket equals total money collected.
\$6.50 times x plus \$4.00 times 12 students equals total money collected
$\$6.50x + 4(12) = f(x)$

PTS: 2 REF: 061526AI NAT: F.BF.1a TOP: Modeling Linear Equations

5. ANS: C
Strategy: Examine the pattern, then test each formula and eliminate wrong choices.
Term 1 has 2 squares.
Term 2 has 6 squares.
Term 3 has 10 squares.
Term 4 has 14 squares

n	1	2	3	4
a_n	2	6	10	14

Formula	Equation	Term 1 = 2	Term 2 = 6	Term 3 = 10	Term 4 = 14
I	$a_n = n+4$	$a_n = n+4$ $a_1 = 1+4$ $a_1 = 5$ This is wrong, so eliminate choices a and b..			
II	$a_1 = 2$ $a_n = a_{n-1}+4$	$a_1 = 2$ correct	$a_n = a_{n-1}+4$ $a_2 = a_1+4$ $a_2 = 2+4$ $a_2 = 6$ correct	$a_n = a_{n-1}+4$ $a_3 = a_2+4$ $a_3 = 6+4$ $a_3 = 10$ correct	$a_n = a_{n-1}+4$ $a_4 = a_3+4$ $a_4 = 10+4$ $a_3 = 14$ correct
III	$a_n = 4n-2$	$a_n = 4n-2$ $a_1 = 4(1)-2$ $a_1 = 4-2$ $a_1 = 2$ correct	$a_n = 4n-2$ $a_1 = 4(1)-2$ $a_1 = 4-2$ $a_1 = 2$ correct	$a_n = 4n-2$ $a_1 = 4(1)-2$ $a_1 = 4-2$ $a_1 = 2$ correct	$a_n = 4n-2$ $a_1 = 4(1)-2$ $a_1 = 4-2$ $a_1 = 2$ correct

Choose answer choice *c* because Formulas II and III are both correct.

PTS: 2 REF: 061522AI NAT: F.BF.2 TOP: Sequences

F.BF.3: Build New Functions from Existing Functions.

F.BF.3: Transformations of Graphs of Functions

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

BIG IDEA

The graph of a function is changed when either $f(x)$ or x is multiplied by a scalar, or when a constant is added to or subtracted from either $f(x)$ or x. A graphing calculator can be used to explore the translations of graph views of functions.

Rules: $f(x) \Leftrightarrow f(x) \pm k$ moves the graph $\updownarrow$ up or down.

+k moves every point on the graph up k units.

$-k$ moves every point on the graph down k units

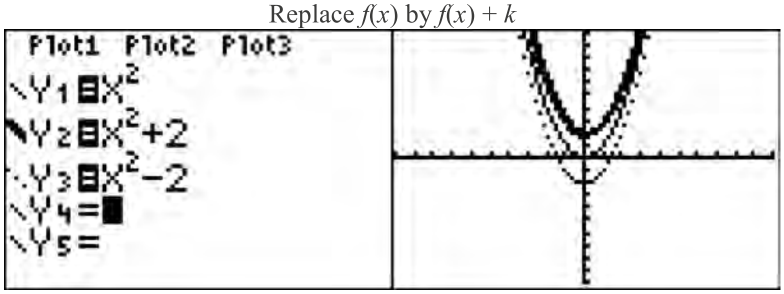
$f(x) \Leftrightarrow f(x \pm k)$ moves the graph $\leftrightarrow$ left or right.

+k moves every point on the graph left k units.

$-k$ moves every point on the graph right k units

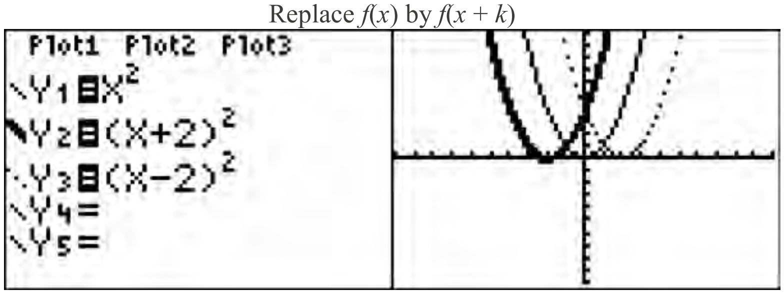
Examples:

Replace $f(x)$ by $f(x) + k$



The addition or subtraction of a constant **outside the parentheses** moves the graph up or down by the value of the constant.

Replace $f(x)$ by $f(x + k)$



The addtition or subtraction of a constant **inside the parentheses** moves the graph left or right by the value of the constant.

Rule: $f(x) \Leftrightarrow f(kx)$ changes the direction and width of a parabola

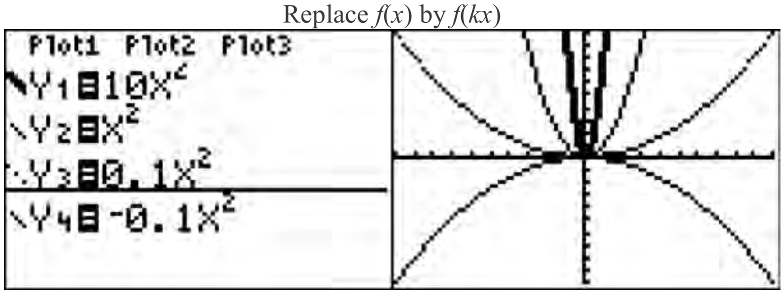
$-k$ inverts the parabola

If k is a fraction less than |1| , the parabola will become wider.

If k is a number larger than |1|, the parabola will become narrower.

Examples:

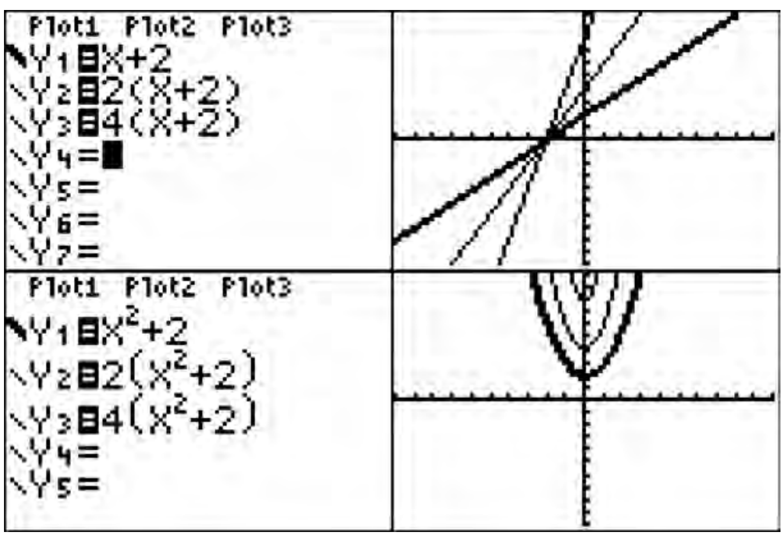
Replace *f*(*x*) by *f*(*kx*)



Changing the value of *a* in a quadratic affects the width and direction of a parabola. The bigger the absolute value of *a*, the narrower the parabola.

$f(x) \Leftrightarrow kf(x)$ changes the y intercept of the graph.

Replace *f*(*x*) by *k f*(*x*)

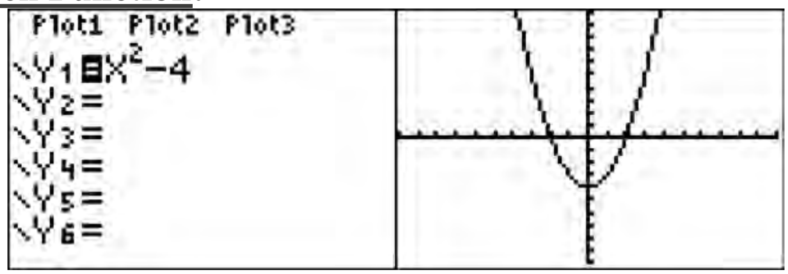


Even and Odd Functions

Even functions: must

1. have exponents that are all even numbers (divisible by 2)
2. reflect in the y-axis.

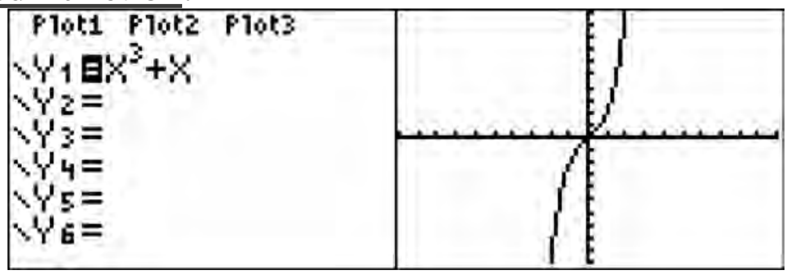
Example of an Even Function:



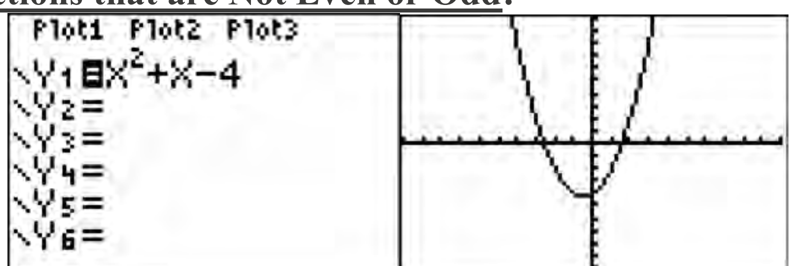
Odd functions: must

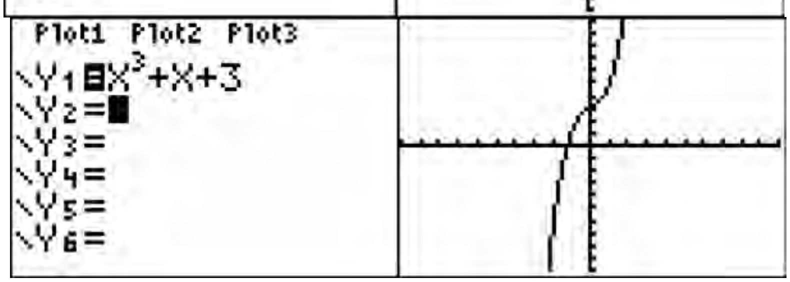
1. have exponents that are all odd numbers
2. reflect in the origin (0,0).

Example of an Odd Function:



Examples of Functions that are Not Even or Odd:





An Algebraic Test to Determine if a Function is Even, Odd, or Neither:

Evaluate the function for $f(-x)$.

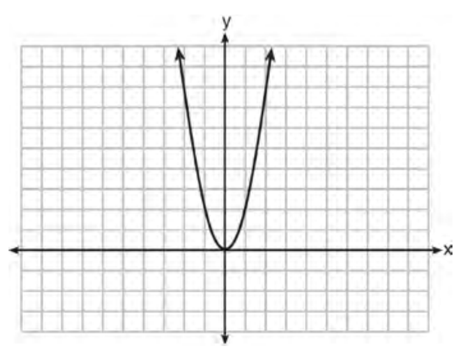
Even	Odd	Neither
$f(x) = x^2 + 4$	$f(x) = x^3 + x$	$f(x) = x^3 + x + 3$
$f(-x) = (-x)^2 + 4$	$f(-x) = (-x)^3 + (-x)$	$f(-x) = (-x)^3 + (-x) + 3$
$f(-x) = x^2 + 4$	$f(-x) = -x^3 - x$	$f(-x) = -x^3 - x + 3$
The function is **even** if $f(x)$ has exactly the same terms as $f(-x)$.	The function is **odd** if all the terms of $f(x)$ and $f(-x)$ are additive inverses.	The function is **neither** even or odd if the terms if all the terms are not the same or opposites.

REGENTS PROBLEMS

1. Given the graph of the line represented by the equation $f(x) = -2x + b$, if b is increased by 4 units, the graph of the new line would be shifted 4 units

a. right
b. up
c. left
d. down

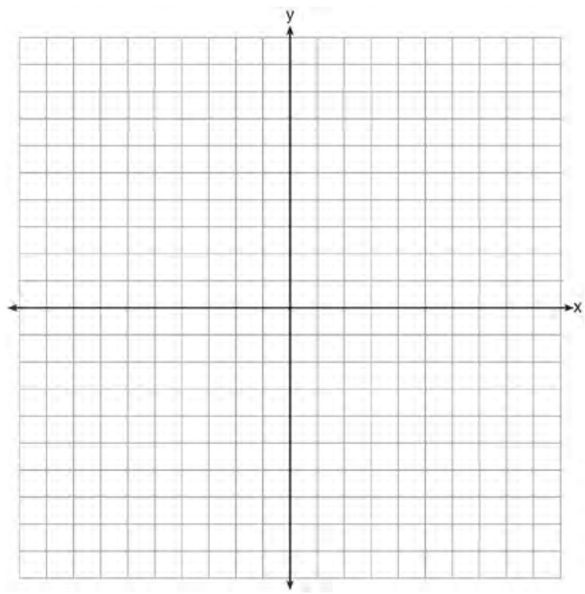
2. The graph of the equation $y = ax^2$ is shown below.



If a is multiplied by $-\frac{1}{2}$, the graph of the new equation is

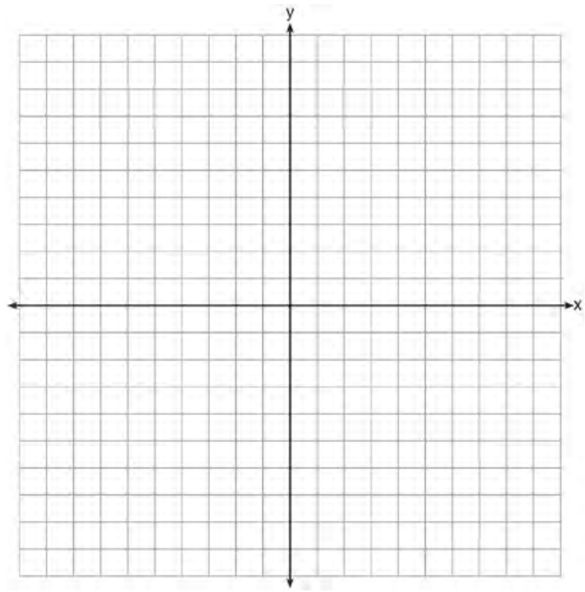
a. wider and opens downward
b. wider and opens upward
c. narrower and opens downward
d. narrower and opens upward

3. The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2,-1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x-2)$. Explain how you arrived at your answer. [The use of the set of axes below is optional.]



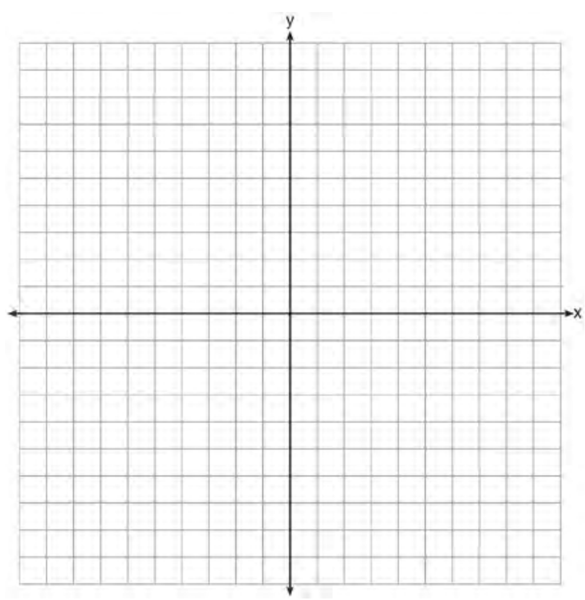
4. How does the graph of $f(x) = 3(x-2)^2 + 1$ compare to the graph of $g(x) = x^2$?
   a. The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
   b. The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.
   c. The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
   d. The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

5. On the axes below, graph $f(x) = |3x|$.



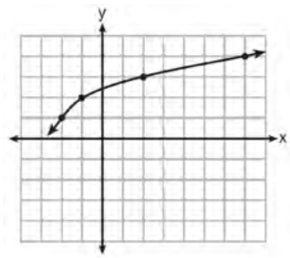
If $g(x) = f(x) - 2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?
If $h(x) = f(x - 4)$, how is the graph of $f(x)$ translated to form the graph of $h(x)$?

6. Graph the function $y = |x - 3|$ on the set of axes below.

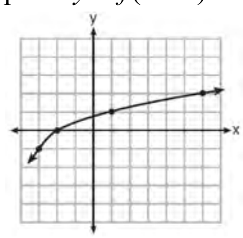


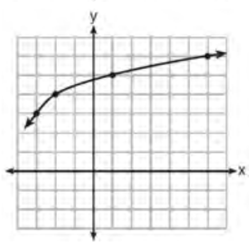
Explain how the graph of $y = |x - 3|$ has changed from the related graph $y = |x|$.

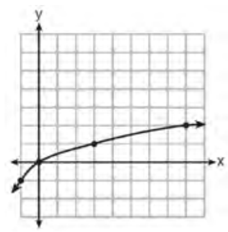
7. The graph of $y = f(x)$ is shown below.

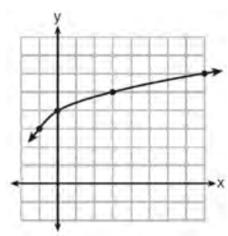


What is the graph of $y = f(x+1) - 2$?

a. 

b. 

c. 

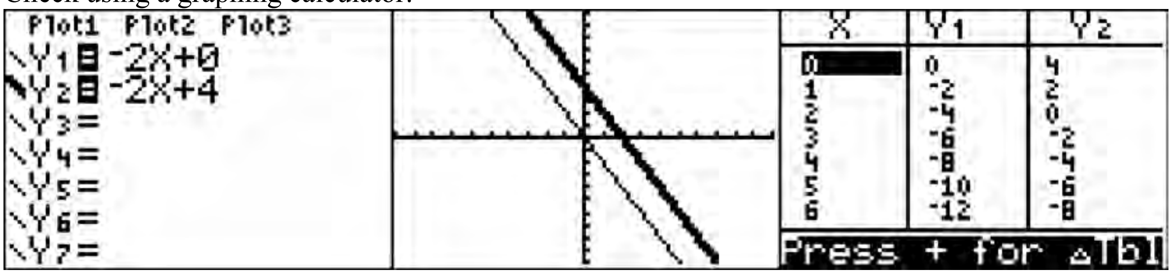
d. 

F.BF.3: Build New Functions from Existing Functions.
Answer Section

1. ANS: B
   Strategy: Use the characteristics of the slope intercept form of a line, which is $y = mx + b$, where y is the dependent variable, m is the slope, x is the dependent variable, and b is the y-inctercept.

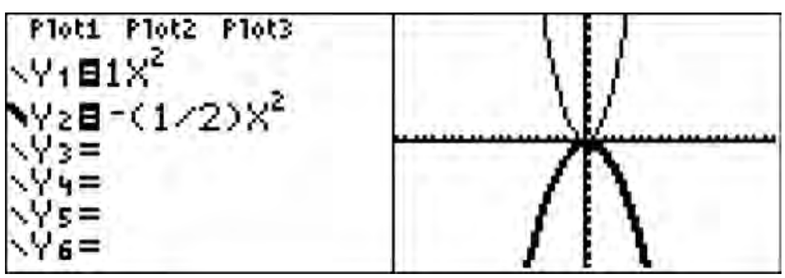
   If b (the y-intercept) is increased by four, the slope remains the same and the new line is shifted up 4 units.

   Check using a graphing calculator.



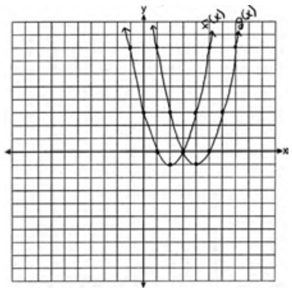
   PTS: 2 REF: 081501ai NAT: F.BF.3
   TOP: Transformations with Functions and Relations
2. ANS: A
   Strategy: Use the following general rules for quadratics, then check with a graphiong calculator.
   As the value of a approaches 0, the parabola gets wider.
   A positive value of a opens upward.
   A negative value of a opens downward.

   Check with graphing calculator:
   Assume $a = 1$, then $y_1 = 1x^2$
   If a is multiplied by $-\frac{1}{2}$, then $y_2 = -\frac{1}{2}x^2$.
   Input both equations in a graphing calculator, as follows:



   PTS: 2 REF: 081417a1 NAT: F.BF.3
   TOP: Transformations with Functions and Relations

3. ANS:



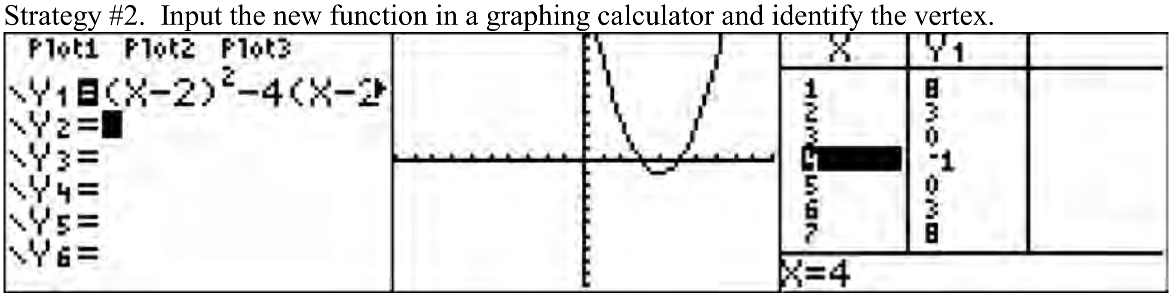
(4,–1). $f(x-2)$ is a horizontal shift two units to the right

Strategy 1: Compose a new function, find the axis of symmetry, solve for g(x) at axis of symmetry, as follows:

$f(x) = x^2 - 4x + 3$ and $g(x) = f(x-2)$ Therefore: $g(x) = (x-2)^2 - 4(x-2) + 3$ $g(x) = x^2 - 4x + 4 - 4x + 8 + 3$ $g(x) = x^2 - 8x + 15$	$axis\ of\ symmetry = \frac{-b}{2a} = \frac{-(-8)}{2(1)} = \frac{8}{2} = 4$ $g(x) = x^2 - 8x + 15$ $g(4) = (4)^2 - 8(4) + 15$ $g(4) = 16 - 32 + 15$ $g(4) = -1$

The coordinates of the vertex of *g(x)* are (4,-1)

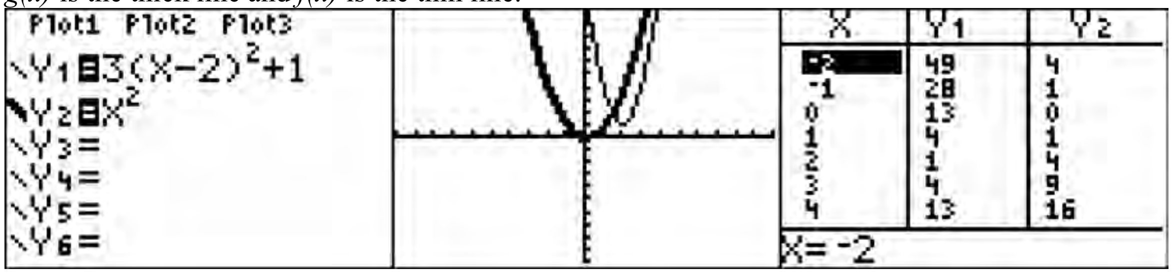
Strategy #2. Input the new function in a graphing calculator and identify the vertex.



PTS: 2 REF: 061428a1 NAT: F.BF.3
TOP: Transformations with Functions and Relations

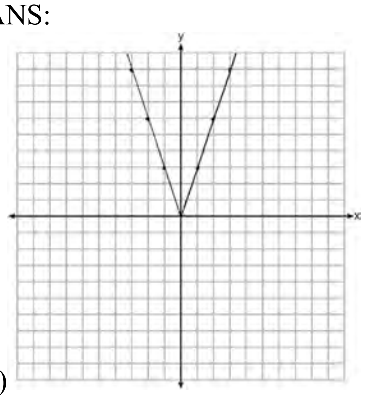
4. ANS: B
Strategy: Input both functions in a graphing calculator and compare them.

Let the graph of Y_1 be the graph of $f(x) = 3(x-2)^2 + 1$
Let the graph of Y_2 be the graph of $g(x) = x^2$
Input both functions in a graphing calculator.
g(x) is the thick line and *f(x)* is the thin line.



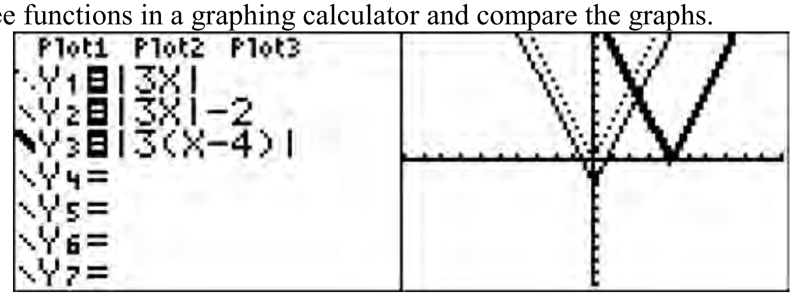
PTS: 2 REF: 011512a1 NAT: F.BF.3
TOP: Transformations with Functions and Relations

5. ANS:



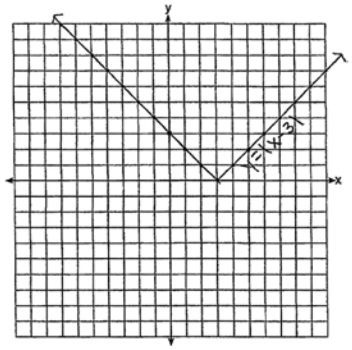
a)
b) If $g(x) = f(x) - 2$, the graph of $f(x)$ is translated 2 down to form the graph of $g(x)$.
c) If $h(x) = f(x-4)$, the graph of $f(x)$ translated 4 right to form the graph of $h(x)$.

Strategy: Input the three functions in a graphing calculator and compare the graphs.



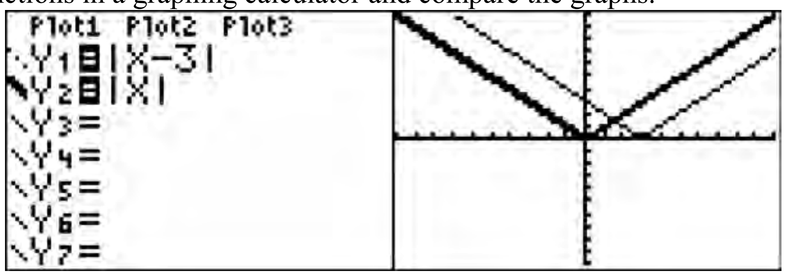
PTS: 4 REF: 081433a1 NAT: F.BF.3
TOP: Transformations with Functions and Relations

6. ANS:



The graph has shifted three units to the right.

Strategy: Input both functions in a graphing calculator and compare the graphs.



PTS: 2 REF: 061525AI NAT: F.BF.3
TOP: Transformations with Functions and Relations

7. ANS: A
Strategy: Identify the differences between the two function rules, then verify using the four points shown in the answer choices.
Function rules:
Difference #1: The term $f(x)$ becomes $f(x+1)$. This means the graph will move to the left 1 unit. The mapping of each x value can be expressed as $(x) \to (x-1)$
Difference #2: The term –2 is added to the function rule. This means the graph will move 2 units down. The mapping of each y value can be expressed as $\left(y\right) \to \left(y-2\right)$.
The 2 differences in the function rules mean that each point on the graph will move left 1 unit and down 2 units. Answer choice (a) shows this:

$y = f(x)$	(-2, 1)	(-1, 2)	(2, 3)	(7, 7)
$y = f(x+1) - 2$	(-3, -1)	(-2, 0)	(1, 1)	(6, 2)

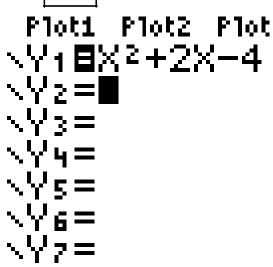
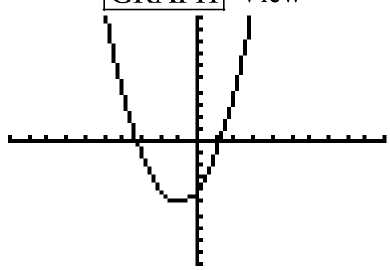
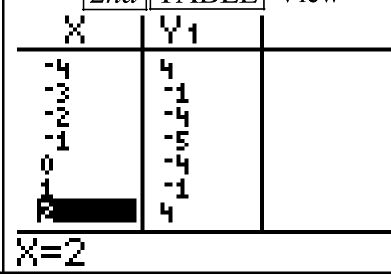
PTS: 2 REF: 011620ai NAT: F.BF.3 TOP: Graphing Radical Functions

F.LE.2: Construct a Function Rule from Other Views of a Function

Construct and compare linear, quadratic, and exponential models and solve problems.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

<u>BIG IDEAS</u>:
A function can be represented mathematically through four inter-related views. These are:
1. a function rule (equation)
2. a table of values
3. a graph.
4. words

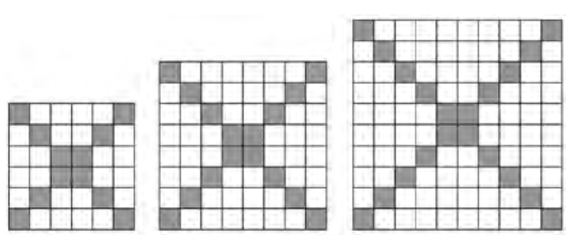
The TI-83+ graphing calculator allows you to input the function rule and access the graph and table of values, as shown below:

$\boxed{\text{Y=}}$ Function Rule View	$\boxed{\text{GRAPH}}$ View	$\boxed{\textit{2nd}}\boxed{\text{TABLE}}$ View
		

Students must be able to move from one view of a function to another. The problems in this set can generally be better understood by using different views of functions. For example, it is easier for some students to understand the first problem if the problem is modeled using a table of values

REGENTS PROBLEMS

1. The diagrams below represent the first three terms of a sequence.

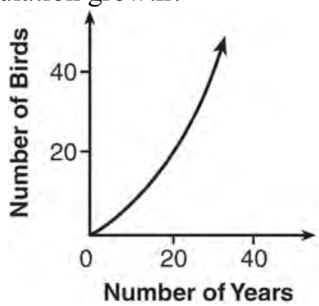


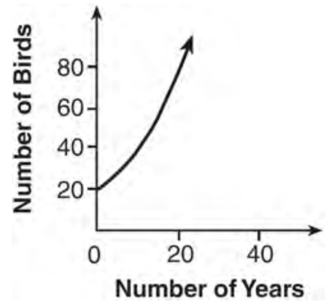
Term 1 Term 2 Term 3

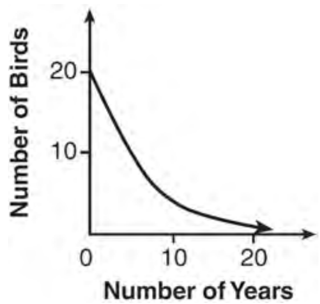
Assuming the pattern continues, which formula determines a_n, the number of shaded squares in the nth term?

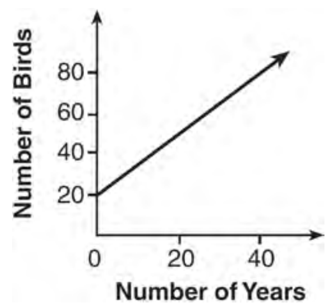
a. $a_n = 4n + 12$
b. $a_n = 4n + 8$
c. $a_n = 4n + 4$
d. $a_n = 4n + 2$

2. A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

a. 

c. 

b. 

d. 

3. The table below represents the function F.

x	3	4	6	7	8
$F(x)$	9	17	65	129	257

The equation that represents this function is

a. $F(x) = 3^x$

b. $F(x) = 3x$

c. $F(x) = 2^x + 1$

d. $F(x) = 2x + 3$

4. The country of Benin in West Africa has a population of 9.05 million people. The population is growing at a rate of 3.1% each year. Which function can be used to find the population 7 years from now?

a. $f(t) = (9.05 \times 10^6)(1 - 0.31)^7$
c. $f(t) = (9.05 \times 10^6)(1 + 0.031)^7$
b. $f(t) = (9.05 \times 10^6)(1 + 0.31)^7$
d. $f(t) = (9.05 \times 10^6)(1 - 0.031)^7$

5. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1, which is an equation for the nth term of this sequence?

a. $a_n = 8n + 10$
c. $a_n = 16n + 10$
b. $a_n = 8n - 14$
d. $a_n = 16n - 38$

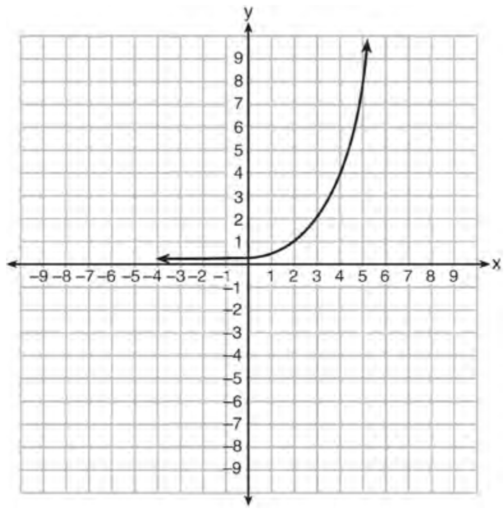
6. A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

Which function would accurately model the technician's data?

a. $f(t) = 25^t$
c. $f(t) = 25t$
b. $f(t) = 25^{t+1}$
d. $f(t) = 25(t + 1)$

7. Write an exponential equation for the graph shown below.



Explain how you determined the equation.

8. Which function is shown in the table below?

x	f(x)
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

a. $f(x) = 3x$

b. $f(x) = x + 3$

c. $f(x) = -x^3$

d. $f(x) = 3^x$

F.LE.2: Construct a Function Rule from Other Views of a Function
Answer Section

1. ANS: B
Strategy: Examine the pattern, then test each formula and eliminate wrong choices.
Term 1 has 12 shaded squares.
Term 2 has 16 shaded squares.
Term 3 has 20 shaded squares.

Choice	Equation	Term 1 = 12	Term 2 = 16	Term 3 = 20
a	$a_n = 4n + 12$	= 16 (eliminate)		
b	$a_n = 4n + 8$	= 12 (correct)	= 16 (correct)	= 20 (correct)
c	$a_n = 4n + 4$	= 8 (eliminate)		
d	$a_n = 4n + 2$	= 6 (eliminate)		

PTS: 2 REF: 061424a1 NAT: F.LE.2 TOP: Sequences

2. ANS: C
Strategy: Build a second model of the problem using a table of values.

If a population starts with 20 birds and doubles every ten years, the following table of values can be created:

Number of Years	Population of Birds
0	20
10	40
20	80
30	160
40	320

Choice a can be eliminated because it shows 20 birds after 20 years.
Choice b can be eliminated because it shows 0 birds after 20 years.
Choice c looks good because it shows 80 birds after 20 years.
Choice d can be eliminated because it shows 40 birds after 20 years.

PTS: 2 REF: 081410a1 NAT: F.LE.2 TOP: Families of Functions
KEY: bimodalgraph

3. ANS: C
Strategy: Test each function to see if it fits the table:

Choice	Equation	(3,9)	(6,65)	(8,257)
a	$F(x) = 3^x$	$F(3) = 3^3 = 27$ (eliminate)		
b	$F(x) = 3x$	$F(3) = 3(3) = 9$ (correct)	$F(6) = 3(6) = 18$ (eliminate)	
c	$F(x) = 2^x + 1$	$F(3) = 2^3 + 1 = 9$ (correct)	$F(6) = 2^6 + 1 = 65$ (correct)	$F(8) = 2^8 + 1 = 257$ (correct)
d	$F(x) = 2x + 3$	$F(3) = 2(3) + 3 = 9$ (correct)	$F(6) = 2(6) + 3 = 15$ (eliminate)	

PTS: 2 REF: 061415a1 NAT: F.LE.2 TOP: Modeling Exponential Equations

4. ANS: C
Strategy: Use the formula for exponential growth: $A = P(1+r)^t$, where
A represents the amount after growth, which in this problem will be $f(t)$.
P represents the initial amount, which in this problem will be 9.05×10^6.
r represents the rate of growth expressed as a decimal, which in this problem will be 0.031 per year.
t represents the number of growth cycles, which in this problem will be 7
Use the exponential growth formula and substitution to write:

$$A = P(1+r)^t$$

$$f(t) = \left(9.05 \times 10^6\right)(1+0.031)^7$$

Answer choice *c* is correct.

PTS: 2 REF: 081507ai NAT: F.LE.2 TOP: Modeling Exponential Functions

5. ANS: B
Strategy: Build the sequence in a table, then test each equation choice and eliminate wrong answers.

a_1	a_2	a_3	a_4	a_5
		10		26

The a_4 term must be half way between 10 and 26, so it must be 18.
The common difference is 8, so we can fill in the rest of the table as follows:

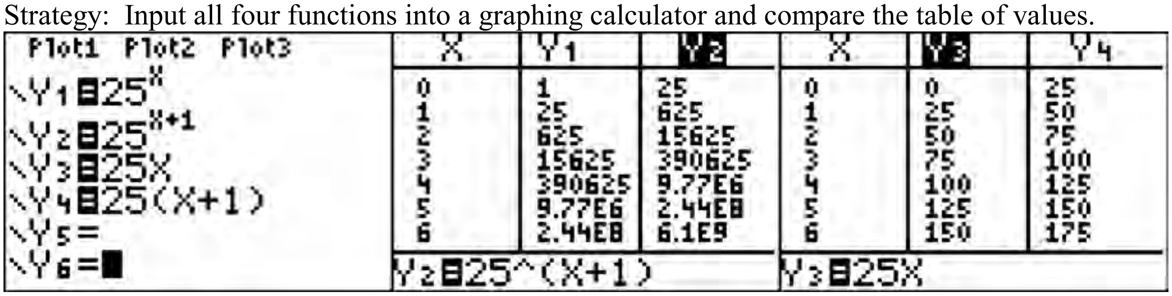
a_1	a_2	a_3	a_4	a_5
-6	2	10	18	26

The first term in the sequence is -6.

Choice	Equation	Term a_1 = -6	Term a_3 = 10	Term a_5 = 26
a	$a_n = 8n + 10$	= 18 (eliminate)		
b	$a_n = 8n - 14$	= -6 (correct)	= 10 (correct)	= 26 (correct)
c	$a_n = 16n + 10$	= 26 (eliminate)		
d	$a_n = 16n - 38$	= -12 (eliminate)		

PTS: 2 REF: 081416a1 NAT: F.LE.2 TOP: Sequences

6. ANS: B
Strategy: Input all four functions into a graphing calculator and compare the table of values.



Answer choice *b* produces a table of values that agrees with the table of values in the problem.

PTS: 2 REF: 061513AI NAT: F.LE.2
TOP: Modeling Linear and Exponential Equations

7. ANS:
$y = 0.25(2)^x$.
Strategy: Input the four integral values from the graph into a graphing calculator and use exponental regression to find the equation.

Alternative Strategy: Use the standard form of an exponential equation, which is $y = ab^x$.
Substitute the integral pairs of (2,1) and (3,2) from the graph into the standard form of an exponential equation and obtain the following: $1 = ab^2$ and $2 = ab^3$.
Therefore, $2ab^2 = ab^3$

$$2 = \frac{ab^3}{ab^2}$$

$$2 = b$$

Accordingly, the equation for the graph can now be written as $y = a \cdot 2^x$.
Substitute the integral pair (4,4) from the graph into the new equation and solve for *a*, as follows:

$$y = a \cdot 2^x$$

$$4 = a \cdot 2^4$$

$$4 = a \cdot 16$$

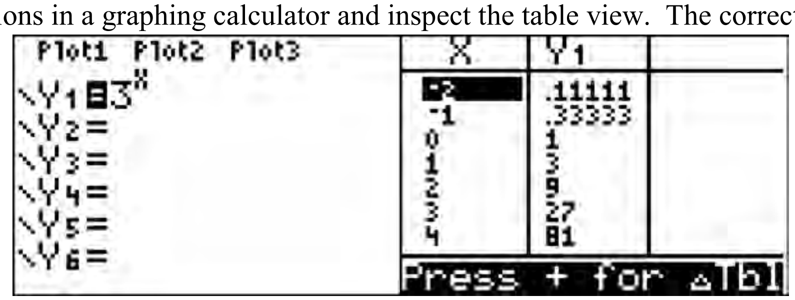
$$\frac{4}{16} = a$$

$$\frac{1}{4} = a$$

The graph of the equation can now be written as $y = \frac{1}{4}(2)^x$

PTS: 2 REF: 011532a1 NAT: F.LE.2 TOP: Modeling Exponential Equations

8. ANS: D
Strategy: Put the functions in a graphing calculator and inspect the table view. The correct answer is $f(x) = 3^x$.



PTS: 2 REF: 011616ai NAT: F.LE.2 TOP: Families of Functions

F.IF.5: Use Sensible Domains and Ranges

Interpret functions that arise in applications in terms of the context.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Vocabulary

The **domain of x** and the **range of y**.

The coordinate plane consists of two perpendicular number lines, which are commonly referred to as the x-axis and the y-axis. Each number line represents **the set of real numbers**.

The Set of Real Numbers

§ **Counting numbers** $\{1,2,3,...\}$
§ **Whole numbers** $\{0,1,2,3,...\}$
§ **Integers** are whole numbers and their opposites $\{...-3, -2, -1, 0, 1, 2, 3, ...\}$.
§ **Rational numbers** (all number that can be expressed as a ratio of two integers)
  Rational begins with the word ratio. A ratio is a comparison of two numbers using division.
  § A ratio can be expressed as a fraction.
  § All fractions are rational numbers.
  § All repeating or terminating decimals.
§ **Irrational numbers** (all numbers that cannot be expressed as ratios of integers)
  § Never ending, never repeating decimals, such as π, e, and the square roots of all prime numbers.

Big Ideas

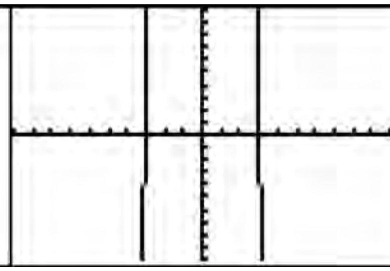
A functions maps an element of the **domain** onto one and only one element of the **range**.

Many functions make sense only when a subset of all the **Real Numbers** are used as inputs. This subset of the **Real Numbers** that makes sense is known as the **domain** of the function.
Example: If a vendor makes \$2.00 profit on each sandwich sold, total profits might be modeled by the function $P(s) = 2s$, where $P(s)$ represents total profits and s represents the number of sandwiches sold. It would not make sense to use the entire set of real numbers as inputs for this function.
- It **would not make sense** to say that the vendor sold -3 sandwiches or to use any other negative numbers.
- It **would not make sense** to say the vendor sold π, sandwiches, e sandwiches, or $\sqrt{7}$ sandwiches.
- It **would make sense** to say that the vendor sold 0, 1, 2, or any whole number of sandwiches.

Thus, the **domain** of $P(s) = 2s$ can be restricted to a subset of the Real Number system, which can be described as either the set of **whole numbers** or by listing the set $\{0,1,2,3,...\}$. The **range** of a function can also be limited to a well-defined subset of the Real Numbers on the y-axis.

Domains and **ranges** can be either **continuous** or **discrete**.

NOTE: The **window function on a graphing calculator** allows us to set specific **continuous** intervals for the domain and range of the graph of a function.

These screenshots show **inappropriate domain and range** settings for the first Regents Problem in this lesson.		
These screenshots show **proper domain and range** settings for the first Regents Problem in this lesson.		

REGENTS PROBLEMS

1. The function $h(t) = -16t^2 + 144$ represents the height, $h(t)$, in feet, of an object from the ground at t seconds after it is dropped. A realistic domain for this function is

a. $-3 \le t \le 3$
b. $0 \le t \le 3$
c. $0 \le h(t) \le 144$
d. all real numbers

2. Officials in a town use a function, C, to analyze traffic patterns. $C(n)$ represents the rate of traffic through an intersection where n is the number of observed vehicles in a specified time interval. What would be the most appropriate domain for the function?

a. $\{\dots -2, -1, 0, 1, 2, 3, \dots\}$
b. $\{-2, -1, 0, 1, 2, 3\}$
c. $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$
d. $\{0, 1, 2, 3, \dots\}$

3. Which domain would be the most appropriate set to use for a function that predicts the number of household online-devices in terms of the number of people in the household?
   a. integers
   b. whole numbers
   c. irrational numbers
   d. rational numbers

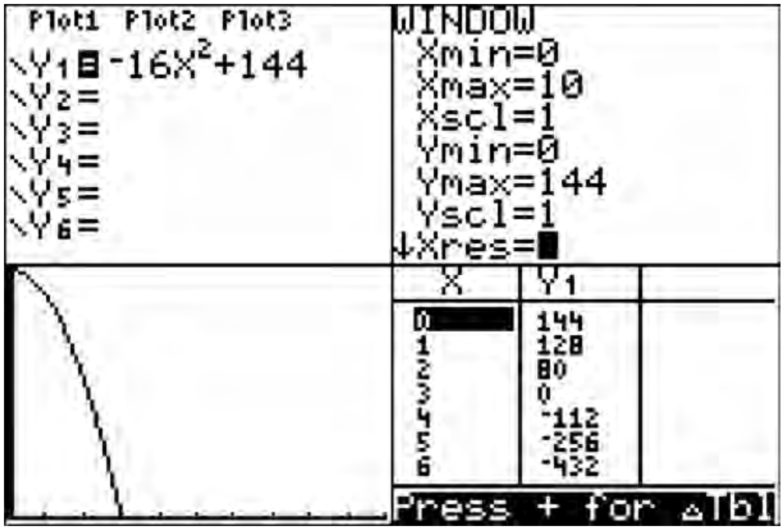
4. A construction company uses the function $f(p)$, where p is the number of people working on a project, to model the amount of money it spends to complete a project. A reasonable domain for this function would be
   a. positive integers
   b. positive real numbers
   c. both positive and negative integers
   d. both positive and negative real numbers

5. Let f be a function such that $f(x) = 2x - 4$ is defined on the domain $2 \le x \le 6$. The range of this function is
   a. $0 \le y \le 8$
   b. $0 \le y < \infty$
   c. $2 \le y \le 6$
   d. $-\infty < y < \infty$

6. The range of the function defined as $y = 5^x$ is
   a. $y < 0$
   b. $y > 0$
   c. $y \le 0$
   d. $y \ge 0$

F.IF.5: Use Sensible Domains and Ranges
Answer Section

1. ANS: B
Strategy: Input the function into a graphing calculator and examine it to determine a realistic range. First, transform $h(t) = -16t^2 + 144$ to $Y_1 = -16x^2 + 144$ for input.



The graph and table of values show that it takes 3 seconds for the object to reach the ground. Therefore, a realistic domain for this function is $0 \le t \le 3$.
$t = 0$ represents the time when the object is dropped.
$t = 3$ represents the time when the object hits the ground.
Answer choice *b* is correct.

PTS: 2 REF: 081423a1 NAT: F.IF.5 TOP: Domain and Range

2. ANS: D
Strategy: Examine each answer choice and eliminate wrong answers.

Eliminate answer choices *a* and *b* because *negative numbers* of cars observed do not make sense.
Eliminate answer choice *c* because *fractional numbers* of cars observed do not make sense.
Choose answer choice *d* because it is the only choice that makes sense. The number of cars observed must be either zero or some counting number.

PTS: 2 REF: 061402a1 NAT: F.IF.5 TOP: Domain and Range

3. ANS: B
Strategy: Eliminate wrong answers.

Eliminate answer choice *a* because the set of integers contains negative numbers, which do not make sense when counting the number of appliances in a household.
Choose answer choice *b* because the set of whole numbers is defined as $\{0, 1, 2, 3, ...\}$. This does make sense when counting the number of appliances in a household.
Eliminate answer choice *c* because the set of irrational numbers includes numbers like π and $\sqrt{7}$, which do not make sense when counting the number of appliances in a household.
Eliminate answer choice *d* because the set of rational numbers includes fractions such as $\frac{3}{4}$ and $\frac{15}{23}$, which do not make sense when counting the number of appliances in a household.

PTS: 2 REF: 011506a1 NAT: F.IF.5 TOP: Domain and Range

4. ANS: A
Strategy: Eliminate wrong answers. The number of people must be counting numbers, since it makes no sense to have a half a person or a quarter person.
The **positive integers** are 1, 2, 3, 4, ..., which makes sense.
Positive real numbers should be eliminated because positive real numbers include fractions, and fractions make no sense for the number of workers.
Both positive and negative integers should be eliminated because it makes no sense to have negative numbers of workers.
Both positive and negative real numbers should also be eliminated because it makes no sense to have negative numbers of workers.
The correct choice is **positive integers**.

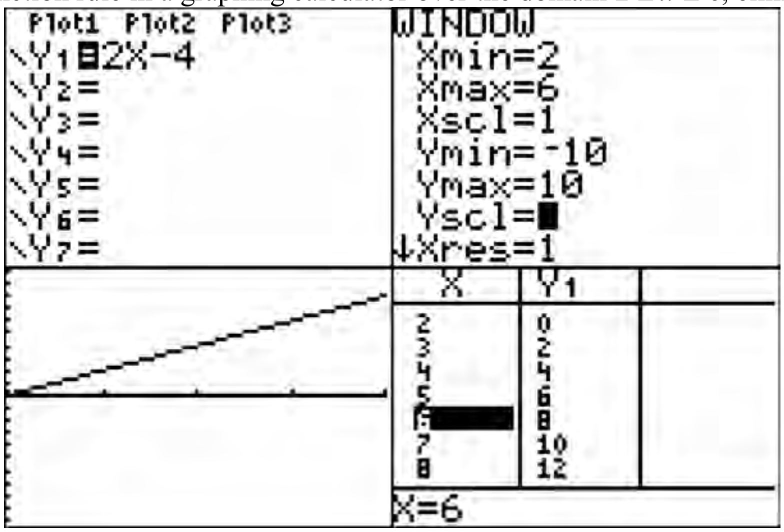
PTS: 2 REF: 011615ai NAT: F.IF.5 TOP: Domain and Range

5. ANS: A

$f(2) = 0$

$f(6) = 8$

Strategy: Inspect the function rule in a graphing calculator over the domain $2 \le x \le 6$, eliminate wrong answers.

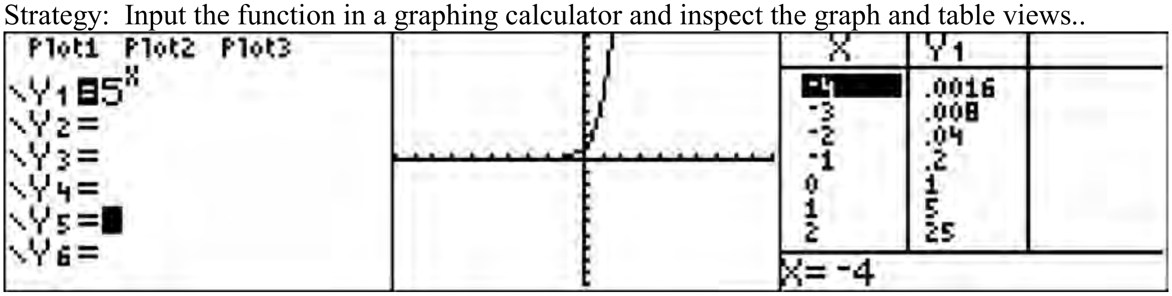


Choose answer choice a because the table of values and the graph clearly show that $f(2) = 0$ and $f(6) = 8$, and all values of y between $x = 2$ and $x = 6$ are between 0 and 8.
Eliminate answer choice *b* because infinity is clearly bigger than 8.
Eliminate answer choice *c* because these are the domain of x, not the range of y.
Eliminate answer choice *d* because negative infinity is clearly less than 0.

PTS: 2 REF: 081411a1 NAT: F.IF.2 TOP: Domain and Range

6. ANS: B

Strategy: Input the function in a graphing calculator and inspect the graph and table views..



The value of y approaches zero, but never reachers zero, as the value of x decreases.
The the range of $y = 5^x$ is $y > 0$.

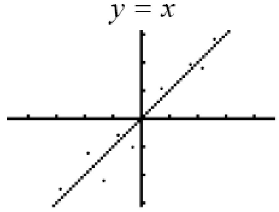
PTS: 2 REF: 011619ai NAT: F.IF.2 TOP: Domain and Range
KEY: real domain, exponential

F.LE.1: Model Families of Functions

Construct and compare linear, quadratic, and exponential models and solve problems.
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Families of Functions

$y = x$

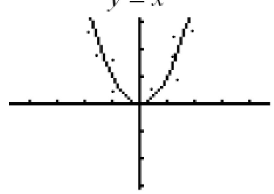


If the graph is a straight line, the function is in the family of **linear functions.**

All **first degree functions** are linear functions, except those lines that are vertical.

All linear functions can be expressed as $y = mx + b$, where m is a constant defined slope and b is the y-intercept.

$y = x^2$



If the graph is a parabola, the function is in the family of **quadratic functions**.

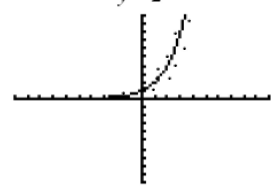
All **quadratic functions** have an exponent of 2 or can be factored into a single factor with an exponent of 2.

Examples:

$x^2 + 6x + 9 = (x+3)^2$

$x^{16} + 6x^8 + 9 = \left(x^8 + 3\right)^2$

$y = 2^x$



If the graph is a curve that approached a horizontal limit on one end and gets steeper on the other end, the function is in the family of **exponential functions**.

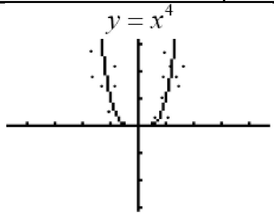
An **exponential function** is a function that contains a variable for an exponent.

Example: $y = 2^x$

Exponential growth and decay can be modeled using the general formula

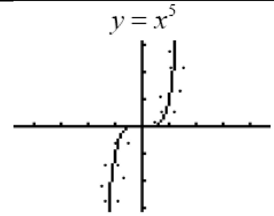
$A = P(1+r)^t$

$y = x^4$



NOTE: All functions in the form of $y = ax^n$, where $a \neq 0$ and $n > 1$ and n is an ***even number***, take the form of parabolas. The larger the value of n, the wider the flat part at the bottom/top.

$y = x^5$



NOTE: All functions in the form of $y = ax^n$, where $a \neq 0$ and $n > 1$ and n is an ***odd number***, take the form of hyperbolas. These are not quadratic functions.

REGENTS PROBLEMS

1. Which situation could be modeled by using a linear function?
   a. a bank account balance that grows at a rate of 5% per year, compounded annually
   b. a population of bacteria that doubles every 4.5 hours
   c. the cost of cell phone service that charges a base amount plus 20 cents per minute
   d. the concentration of medicine in a person's body that decays by a factor of one-third every hour

2. The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

   Which type of function best models the given data?
   a. linear function with a negative rate of change
   b. linear function with a positive rate of change
   c. exponential decay function
   d. exponential growth function

3. Grisham is considering the three situations below.
   I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.
   II. The value of a car depreciates at a rate of 15% per year after it is purchased.
   III. The amount of bacteria in a culture triples every two days during an experiment.
   Which of the statements describes a situation with an equal difference over an equal interval?
   a. I, only
   b. II, only
   c. I and III
   d. II and III

4. Which table of values represents a linear relationship?

a.

x	f(x)
−1	−3
0	−2
1	1
2	6
3	13

b.

x	f(x)
−1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

c.

x	f(x)
−1	−3
0	−1
1	1
2	3
3	5

d.

x	f(x)
−1	−1
0	0
1	1
2	8
3	27

5. The function, $t(x)$, is shown in the table below.

x	t(x)
–3	10
–1	7.5
1	5
3	2.5
5	0

   Determine whether $t(x)$ is linear or exponential. Explain your answer.

F.LE.1: Model Families of Functions
Answer Section

1. ANS: C
   Strategy: Eliminate wrong answers.

   a) Eliminate answer choice *a* because it describes exponential growth of money in a bank account.
   b) Eliminate answer choice *b* because is describes exponential growth of bacteria.
   c) Choose answer choice *c* because it can be modeled using the slope intercept formula as follows:

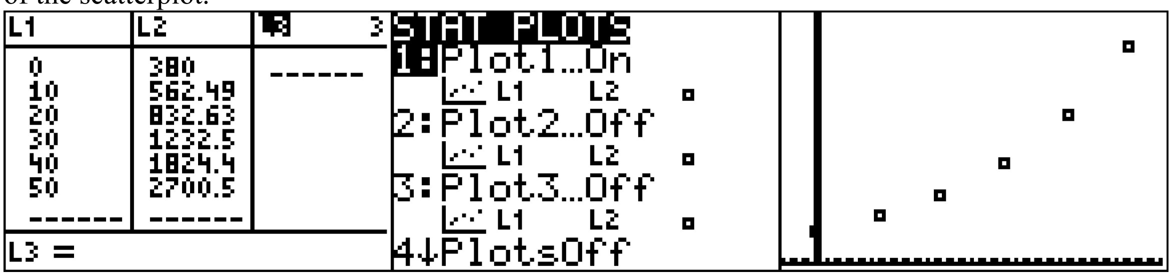
   $$y = mx + b$$

   $$\text{cost of cell phone service} = \$0.20 \times \text{number of minutes plus the base cost}$$

   d) Eliminate answer choice *d* because it describes exponential decay of medicine in the body.

   PTS: 2 REF: 081412a1 NAT: F.LE.1b TOP: Families of Functions

2. ANS: D
   Strategy: Input the table into the stats editor of a graphing calculator, then plot the points and examine the shape of the scatterplot.



   The data in this table creates a scatterplot that appears to model an exponential growth function.

   DIMS? Does It Make Sense? Yes. Savings accounts are excellent exemplars of exponential growth.

   PTS: 2 REF: 061406a1 NAT: F.LE.1c TOP: Modeling Exponential Equations

3. ANS: A
Interpreting the Question: Equal differences over equal intervals suggests a constant rate of change, which would be a linear relationship.
Strategy: Model each situation with a function rule, then select the linear functions.
I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.
This can be modeled with the **linear** function $h = 3.5d$, where h represents the height of the sunflower and d represents the number of days. Since this function is linear, it represents a situation with an equal difference over an equal interval.
II. The value of a car depreciates at a rate of 15% per year after it is purchased.
This can be modeled with the **exponential decay** function $V = P(1 - .15)^t$, where V represents the value of the car, P represents its price when purchased, -.15 represents the annual depreciation rate, and t represents the number of years after purchase. This is an exponential decay function, so it does not represent a situation with an equal difference over an equal interval.
III. The amount of bacteria in a culture triples every two days during an experiment.
This can be modeled with the **exponential growth** function $A = B(3)^{\frac{d}{2}}$, where A represents the amount of bacteria, B represents starting amount of bacteria, 3 represents the growth rate, and $\frac{d}{2}$ represents the number of growth cyles. This is an exponential growth function, so it does not represent a situation with an equal difference over an equal interval.
The only choice that represents a situation with an equal difference over an equal interval is the first situation.

PTS: 2 REF: 011623ai NAT: F.LE.1 TOP: Families of Functions

4. ANS: C
Strategy: Use $\frac{\Delta Y}{\Delta X}$ (the slope formula) to determine which table represents a constant rate of change. A linear function will have a constant rate of change.

Answer Choice	First set of coordinates	Second set of coordinates
a eliminate because *slope is not constant*	(1,1) and (2,6) $slope = \frac{6-1}{2-1} = 5$	(2,6) and (3, 13) $slope = \frac{13-6}{3-2} = 7$
b eliminate because *slope is not constant*	(1,2) and (2,4) $slope = \frac{4-2}{2-1} = 2$	(2,4) and (3, 8) $slope = \frac{8-4}{3-2} = 4$
c choose because ***slope is constant***	(1,1) and (2,3) $slope = \frac{3-1}{2-1} = 2$	(2,3) and (3, 5) $slope = \frac{5-3}{3-2} = 2$
d eliminate because *slope is not constant*	(1,1) and (2,8) $slope = \frac{8-1}{2-1} = 7$	(2,8) and (3, 27) $slope = \frac{27-8}{3-2} = 19$

PTS: 2 REF: 011505a1 NAT: F.LE.1b TOP: Families of Functions

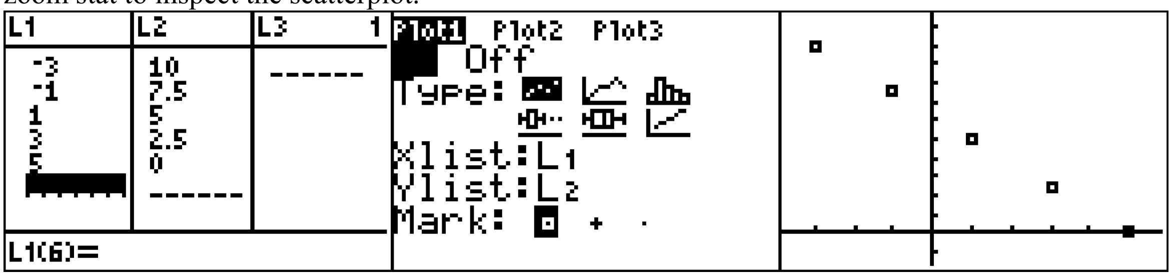
5. ANS:

Strategy #1. Calculate the change in x and the change in y for each ordered pair in the table. If the ratio of $\frac{\Delta y}{\Delta x}$ is constant, the function is linear.

Change in x	x	t(x)	Change in y
	-3	10	
+2<			>-2.5
	-1	7.5	
+2<			>-2.5
	1	5	
+2<			>-2.5
	3	2.5	
+2<			>-2.5
	5	0	

This table shows a linear function, because the ratio of $\frac{\Delta y}{\Delta x}$ can always be expressed as $\frac{-2.5}{2}$.

Strategy #2. Input values from the table into the stats editor of a graphing calculator, turn stats plot on, then use zoom stat to inspect the scatterplot.



The scatterplot shows a linear relationship.

PTS: 2 REF: 011625ai NAT: F.LE.1 TOP: Families of Functions

F.LE.3: Compare Families of Functions

Linear, Quadratic, & Exponential Models

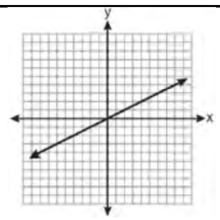
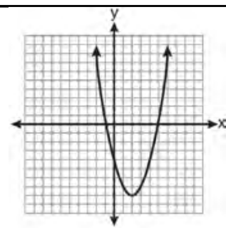
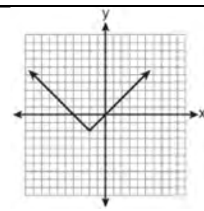
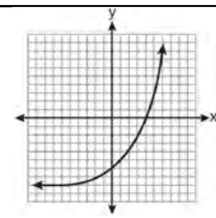
Construct and compare linear, quadratic, and exponential models and solve problems.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Big Idea

A quantity increasing exponentially will eventually exceeds a quantity increasing linearly or quadratically,

Use a graphing calculator and different views of functions to compare linear, quadratic, and exponential models and solve problems.

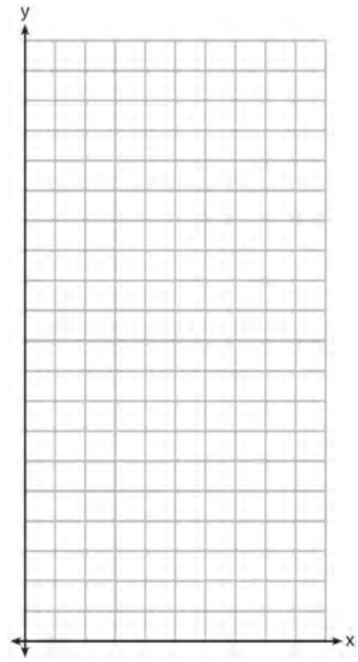
 **Linear Functions** …look like straight lines that are *not* vertical. The rate of change is a constant.	 **Quadratic Functions** …look like parabolas that open up or down.	 **Absolute Value Functions** …look like v-shapes.	 **Exponential Functions** …look like one-sided curves. The rate of change is exponential.

Regents Problems

1. If $f(x) = 3^x$ and $g(x) = 2x + 5$, at which value of x is $f(x) < g(x)$?

a. -1 c. -3
b. 2 d. 4

2. Graph $f(x) = x^2$ and $g(x) = 2^x$ for $x \geq 0$ on the set of axes below.



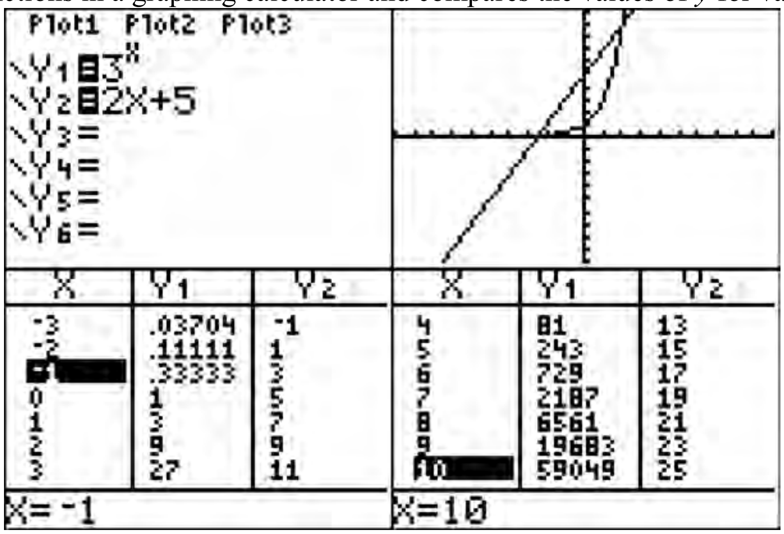
State which function, $f(x)$ or $g(x)$, has a greater value when $x = 20$. Justify your reasoning.

3. Alicia has invented a new app for smart phones that two companies are interested in purchasing for a 2-year contract. Company A is offering her \$10,000 for the first month and will increase the amount each month by \$5000. Company B is offering \$500 for the first month and will double their payment each month from the previous month. Monthly payments are made at the end of each month. For which monthly payment will company B's payment first exceed company A's payment?

a. 6
b. 7
c. 8
d. 9

F.LE.3: Compare Families of Functions
Answer Section

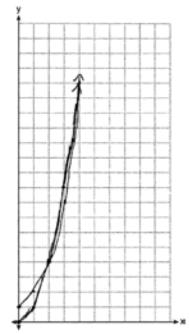
1. ANS: A
   Strategy: Input both functions in a graphing calculator and compares the values of y for various values of x.



The table of values shows:
When $x = -1$, $f(x) < g(x)$
When $x = 2$, $f(x) = g(x)$
When $x = -3$, $f(x) > g(x)$
When $x = 4$, $f(x) > g(x)$

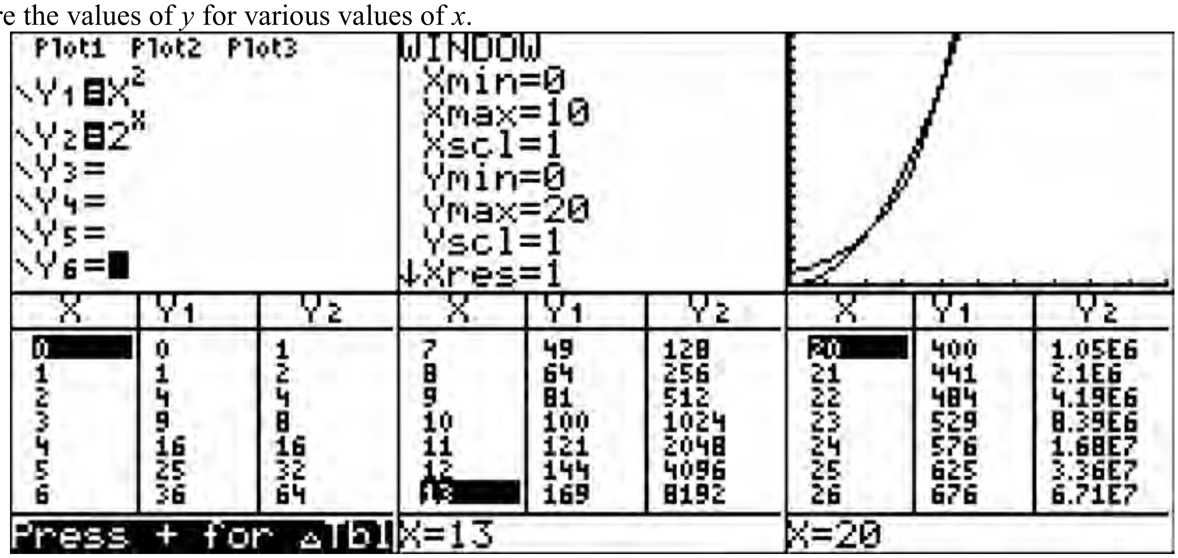
PTS: 2 REF: 061515AI NAT: F.LE.3 TOP: Families of Functions

2. ANS:



$g(x)$ has a greater value: $2^{20} > 20^2$

Strategy: Input both functions in a graphing calculator, use the table of values to create the paper graph, and to compare the values of y for various values of x.



The table of values shows that when $x = 20$, $g(x) > f(x)$.

DIMS? Does It Make Sense? Yes. $2^{20} > 20^2$

PTS: 4 REF: 081533ai NAT: F.LE.3
TOP: Comparing Quadratic and Exponential Functions

3. ANS: C

Strategy: Build a table of values for the integer values of the domain $6 \le x \le 9$ to compare both offers.

x	$A = 5000x + 10000$	$B = 500(2)^{x-1}$
6	40,000	16,000
7	45,000	32,000
8	50,000	64,000
9	55,000	128,000

Offer B is greater than offer A when $x = 8$.

PTS: 2 REF: 081518ai NAT: F.LE.3
TOP: Comparing Linear and Exponential Functions

F.IF.7b: Root, Piecewise, Step, & Absolute Value Functions

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

NOTE: All of the functions in this lesson require special consideration for the domain of the independent variable (the x-axis).

ROOT FUNCTIONS

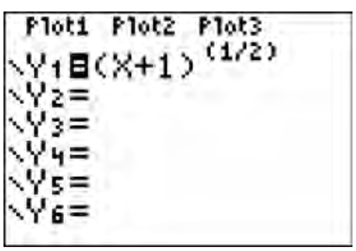
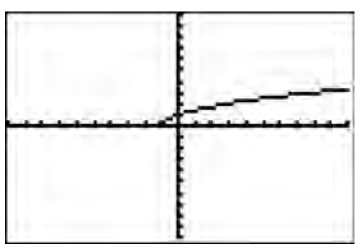
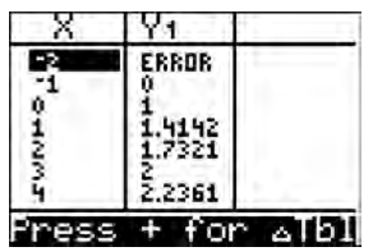
Root functions are associated with equations involving square roots, cube roots, or nth roots. The easiest way to graph a root function is to use the three views of a function that are associated with a graphing calculator.

STEP 1. Input the root function in the y-editor of the calculator. (Note: The use of rational exponents is recommended, i.e. $\sqrt{x} = x^{(1/2)}$, $\sqrt[3]{x} = x^{(1/3)}$, etc.).

STEP 2. Look at the graph of the function.

STEP 3. Use the table of values to transfer coordinate pairs to graph paper.

Example: Graph the root function $f(x) = \sqrt{x+1}$

STEP 1 Input the function rule in the y-editor of your graphing calculator	STEP 2. Look at the graph view of the function.	STEP 3. Select coordinate pairs from the table view to create your own graph.
		

PIECEWISE FUNCTIONS

A **piecewise function** is a function that is defined by two or more *sub* functions, with each sub function applying to a certain interval on the x-axis. Each *sub* function may also be referred to as a *piece* of the overall **piecewise function**, hence the name piecewise.

Example. The following is a piecewise function:

$$f(x) = \begin{cases} 2x+1, & -3 \le x < 3 \\ 4, & 3 \le x \le 7 \end{cases}$$

This example of a piecewise function has two "pieces," or sub functions.

a. Over the interval $-3 \le x < 1$, the sub function is $f(x) = 2x + 1$

b. Over the interval $1 \le x \le 7$, the sub function is $f(x) = 4$.

A table of values for this function might look like this, reflecting two pieces.

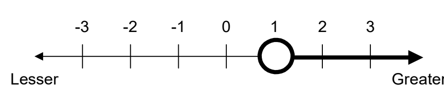
x	$f(x) = 2x + 1$	$f(x) = 4$		x	$f(x) = 2x + 1$	$f(x) = 4$
-3	-5	na		3	na	4
-2	-3	na		4	na	4
-1	-1	na		5	na	4
0	1	na		6	na	4
1	3	4		7	na	4
2	na	4		8	na	na

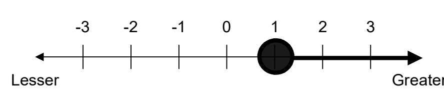
Continuity

Piecewise functions are often discontinuous, which means that the graph will not appear as a single line. In the above table, the piecewise function is discontinuous when $x = 3$. This is because $x = 3$ is not included in the first piece of the piecewise function. Because piecewise functions are often discontinuous, care must be taken to use proper inequalities notation when graphing.

Using Line Segments to Define Pieces

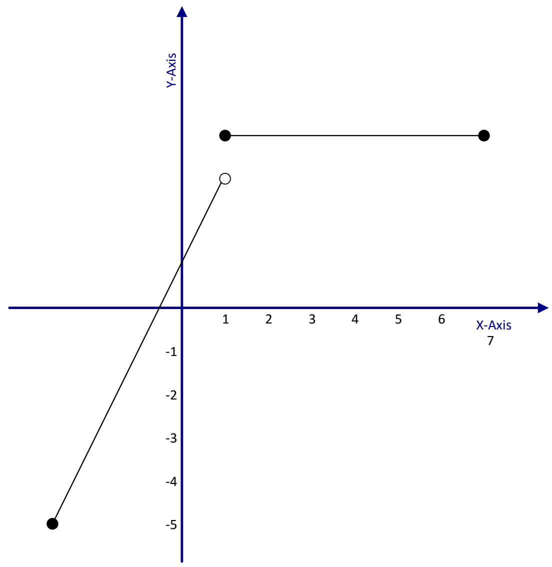
If the circle at the beginning or end of a solution set (graph) is empty, that value *is not included* in the solution set. If the circle is filled in, that value *is included* in the solution set.

The number 1 is not included in the this solution set: 

The number 1 is included in this following solution set: 

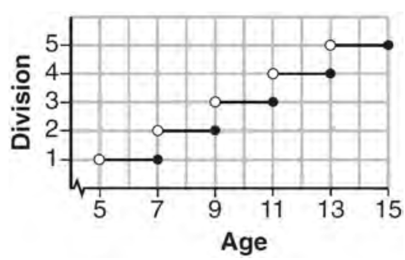
The graph of this piecewise function looks like this:

The graph of $f(x) = \begin{cases} 2x+1, & -3 \le x < 3 \\ 4, & 3 \le x \le 7 \end{cases}$ appears below:



STEP FUNCTIONS

A step function is typically a piecewise function with many pieces that resemble stair steps.



Each step correspondends to a specific domain. The function rule for the graph above is:

$$f(x) = \begin{vmatrix} 1, & 5 < x \le 7 \\ 2, & 7 < x \le 9 \\ 3, & 9 < x \le 11 \\ 4, & 11 < x \le 13 \\ 5, & 13 < x \le 15 \end{vmatrix}$$

ABSOLUTE VALUE FUNCTIONS

Using a Graphing Calculator To Solve and Graph Absolute Value Functions:
Absolute value functions may be solved in a graphing calculator by moving all terms to one side of the inequality and reducing the other side to zero. The inequality is then entered into the graphing calculator's $\boxed{Y=}$ feature. Once input, the calculator's $\boxed{\text{2nd}}\boxed{\text{TABLE}}$ and $\boxed{\text{GRAPH}}$ features may be accessed and manipulated using the ($\boxed{\text{2nd}}\boxed{\text{TBL SET}}$) and graph $\boxed{\text{WINDOW}}$ features.

Example: *Given*: $|x+1|-3>6$

First, move everything to one side of the inequality, leaving the other side zero.

$|x+1|-3>6$

$|x+1|-9>0$

or $0<|x+1|-9$

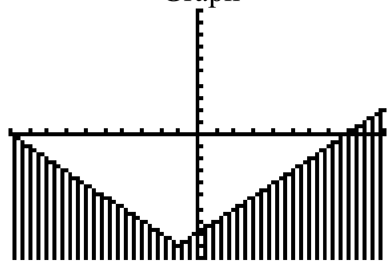
Y= Input

Pay particular attention to setting the inequality sign on the far left of the input screen.

```
  Plot1  Plot2  Plot3
◣Y1■abs(X+1)-9
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

NOTE: The abs entry is found in the graphing calculator's catalog.

Graph



You can see from the graph that the solution boundaries are -10 *and* $+8$. Test $x=0$ to confirm the answers $x<10$ *and* $x>8$, which are the parts of the graph that lie **above** the x-axis. Typically, you would graph only the x-axis on the Regents Math B Exam.

NOTE: The table of values in the graphing calculator provides an excellent opportunity to reinforce the idea that absolute values cannot have negative values. Any value of x that results in a negative value of y cannot be part of the solution set of an absolute value inequality.

Table of Values

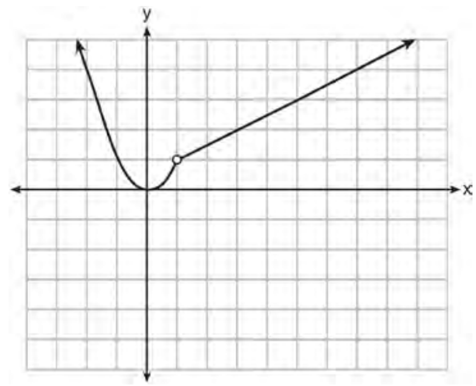
X	Y1	
3	-5	
4	-4	
5	-3	
6	-2	
7	-1	
8	0	
9	1	

X=4

NOTE: The table of values in the graphing calculator provides an excellent opportunity to reinforce the idea that absolute values cannot have negative values.

REGENTS PROBLEMS

1. A function is graphed on the set of axes below.



Which function is related to the graph?

a. $f(x) = \begin{cases} x^2, x < 1 \\ x - 2, x > 1 \end{cases}$

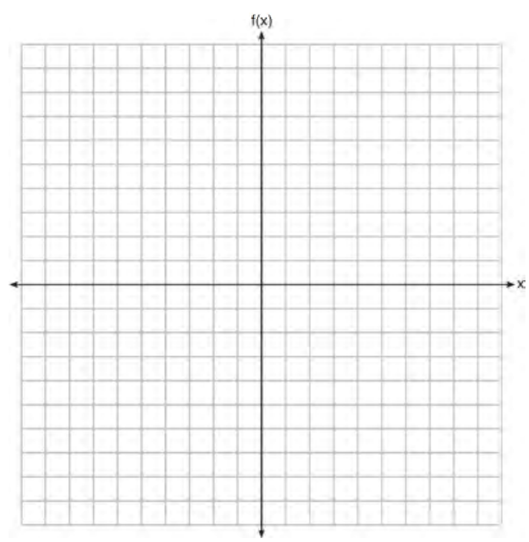
b. $f(x) = \begin{cases} x^2, x < 1 \\ \frac{1}{2}x + \frac{1}{2}, x > 1 \end{cases}$

c. $f(x) = \begin{cases} x^2, x < 1 \\ 2x - 7, x > 1 \end{cases}$

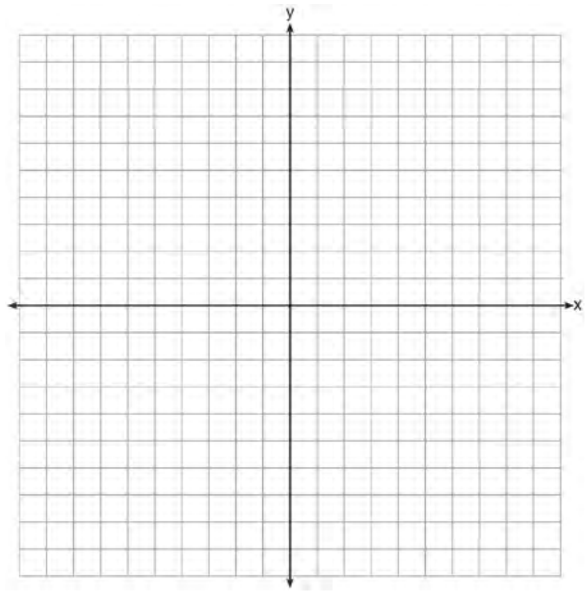
d. $f(x) = \begin{cases} x^2, x < 1 \\ \frac{3}{2}x - \frac{9}{2}, x > 1 \end{cases}$

2. Graph the following function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -3 \le x < 1 \\ 4, & 1 \le x \le 8 \end{cases}$$

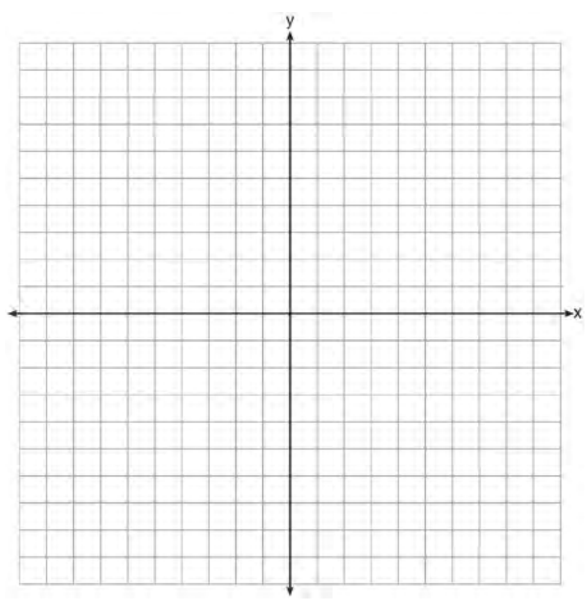


3. On the set of axes below, graph the function $y = |x + 1|$.

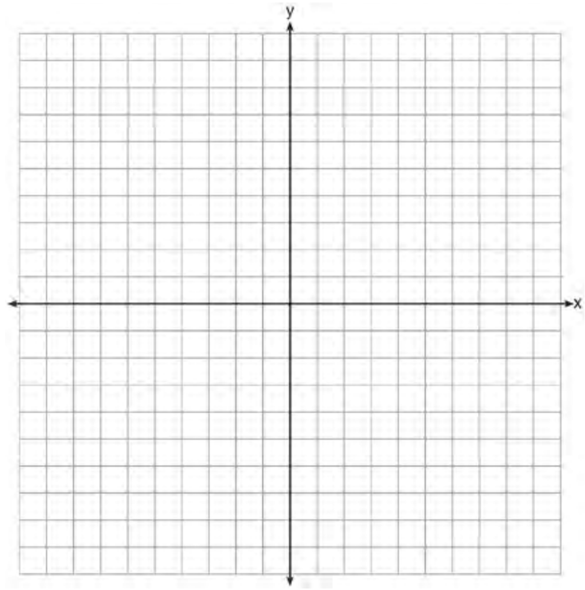


State the range of the function. State the domain over which the function is increasing.

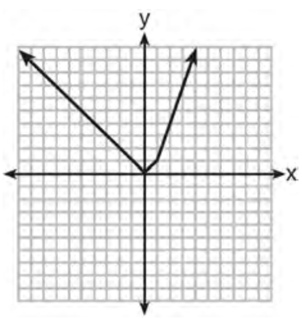
4. On the set of axes below, graph the function represented by $y = \sqrt[3]{x-2}$ for the domain $-6 \le x \le 10$.

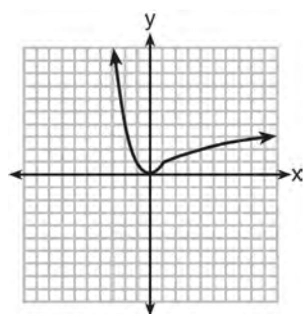


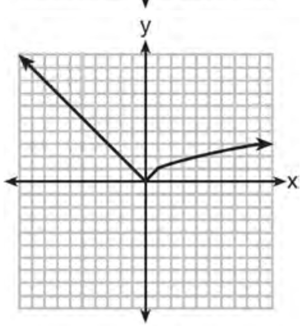
5. Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.

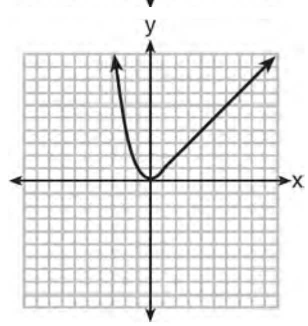


6. Which graph represents $f(x) = \begin{cases} |x| & x < 1 \\ \sqrt{x} & x \geq 1 \end{cases}$?

a. 

c. 

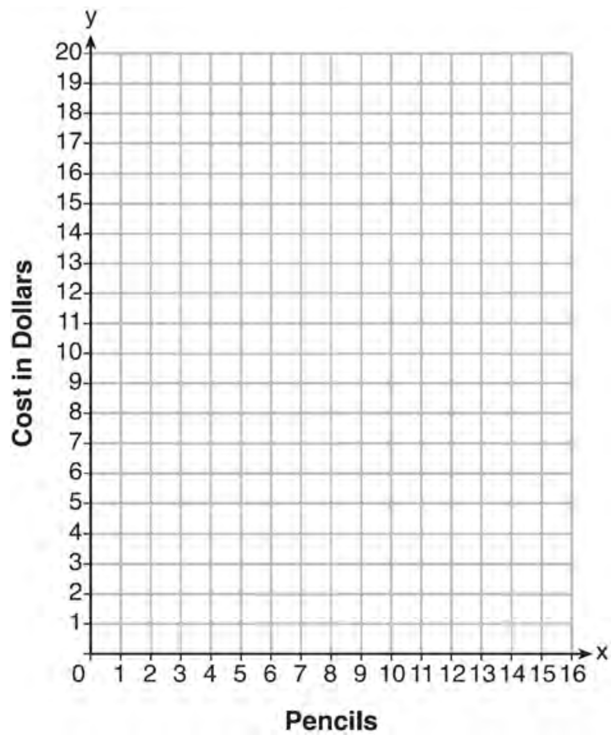
b. 

d. 

7. At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is \$1.75. If a customer purchases 10 or more pencils, the cost of each pencil is \$1.25. Let c be a function for which $c(x)$ is the cost of purchasing x pencils, where x is a whole number.

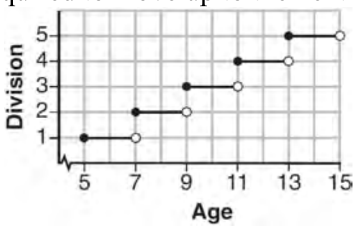
$$c(x) = \begin{cases} 1.75x, \text{ if } 0 \le \text{x} \le 9 \\ 1.25x, \text{ if } x \ge 10 \end{cases}$$

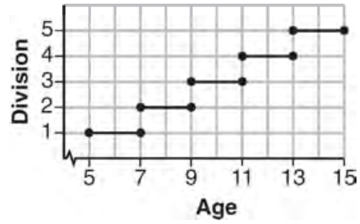
Create a graph of c on the axes below.

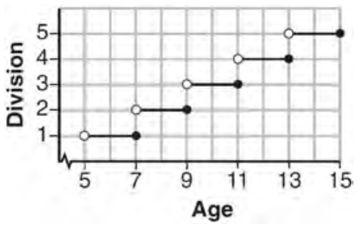


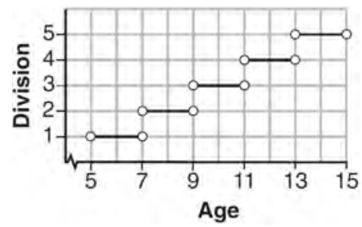
A customer brings 8 pencils to the cashier. The cashier suggests that the total cost to purchase 10 pencils would be less expensive. State whether the cashier is correct or incorrect. Justify your answer.

8. Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?

a. 

c. 

b. 

d. 

9. The equation to determine the weekly earnings of an employee at The Hamburger Shack is given by $w(x)$, where x is the number of hours worked.

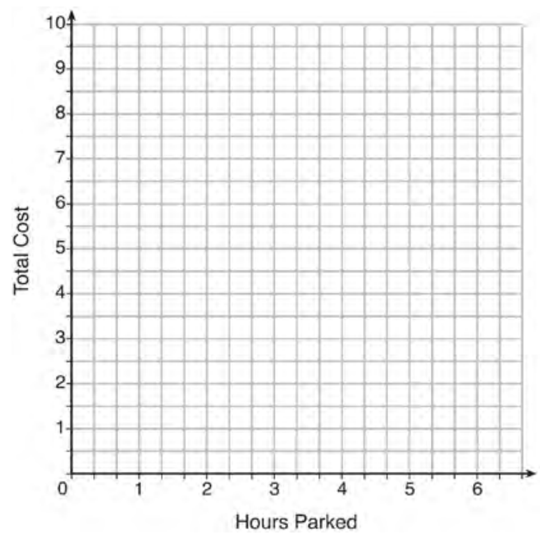
$$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$$

Determine the difference in salary, *in dollars*, for an employee who works 52 hours versus one who works 38 hours. Determine the number of hours an employee must work in order to earn \$445. Explain how you arrived at this answer.

10. The table below lists the total cost for parking for a period of time on a street in Albany, N.Y. The total cost is for any length of time up to and including the hours parked. For example, parking for up to and including 1 hour would cost \$1.25; parking for 3.5 hours would cost \$5.75.

Hours Parked	Total Cost
1	1.25
2	2.50
3	4.00
4	5.75
5	7.75
6	10.00

Graph the step function that represents the cost for the number of hours parked.



Explain how the cost per hour to park changes over the six-hour period.

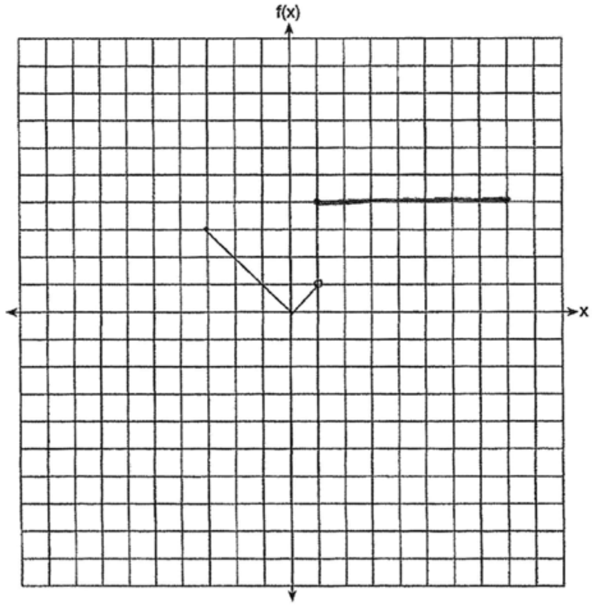
F.IF.7b: Graph Root, Piecewise, Step, & Absolute Value Functions
Answer Section

1. ANS: B
   Strategy: Since $f(x) = x^2, x < 1$ is included in every answer choice, concentrate on the linear functions for $x > 1$.

   The linear equation has a slope of $\frac{rise}{run} = \frac{1}{2}$. The only linear function that has a slope of $\frac{1}{2}$ is $f(x) = \frac{1}{2}x + \frac{1}{2}$, which is answer choice b.

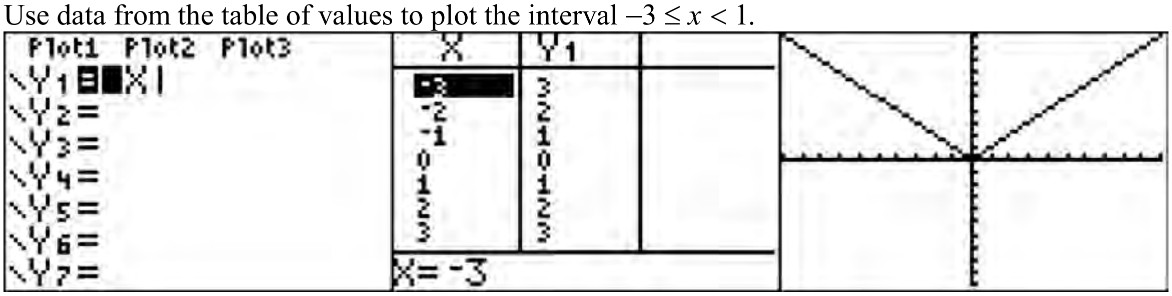
   PTS: 2 REF: 081422a1 NAT: F.IF.7b TOP: Graphing Piecewise-Defined Functions

2. ANS:

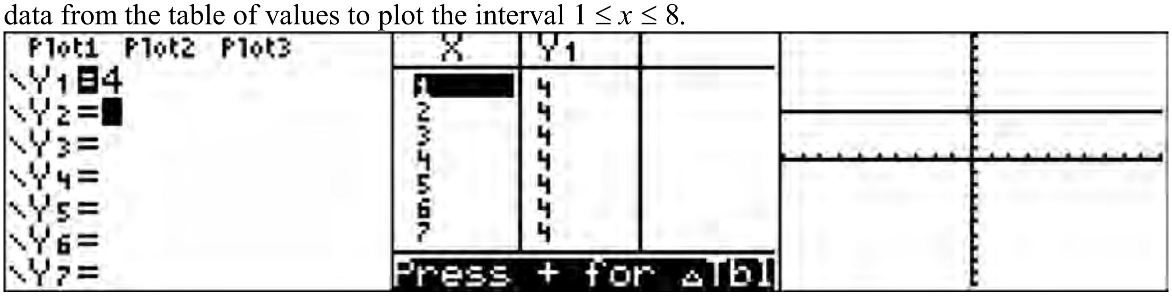


Strategy: Use a graphing calculator and graph the function in sections, paying careful attention to open and closed circles at the end of each function segment.

STEP 1. Graph $f(x) = |x|$ over the interval $-3 \le x < 1$. Use a closed dot for $\left(-3,3\right)$ and an open dot for $(1,1)$. Use data from the table of values to plot the interval $-3 \le x < 1$.



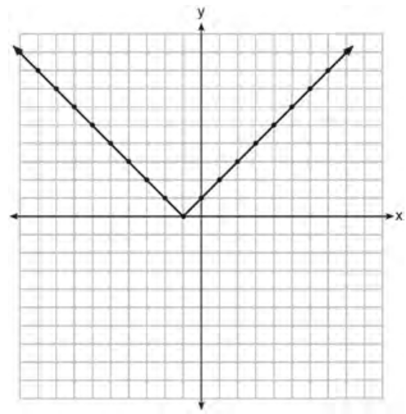
STEP 2: Graph $f(x) = 4$ over the interval $1 \le x \le 8$. Use a closed dot for $\left(1,4\right)$ and a closed dot for $(8,4)$. Use data from the table of values to plot the interval $1 \le x \le 8$.



Do not connect the two graph segments.

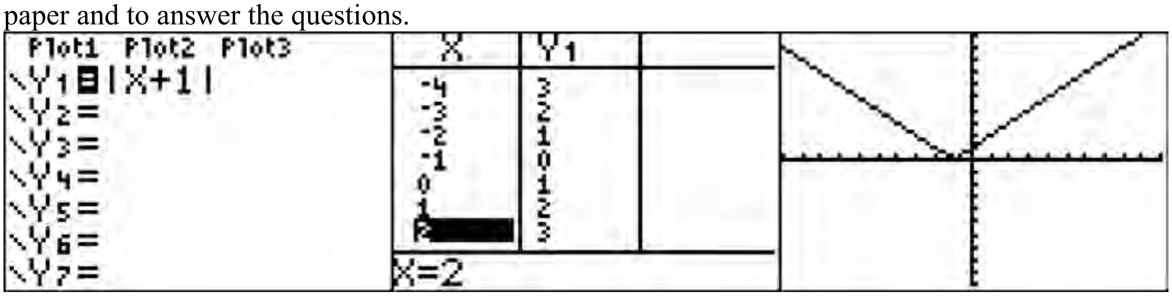
PTS: 2 REF: 011530a1 NAT: F.IF.7b TOP: Graphing Piecewise-Defined Functions

3. ANS:



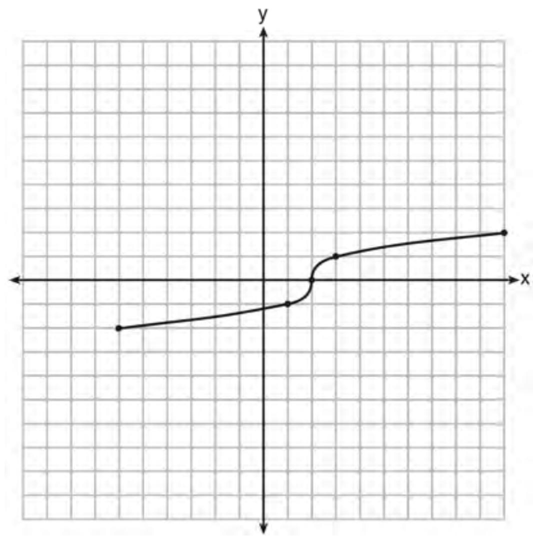
The range is $y \geq 0$.
The function is increasing for $x > -1$.

Strategy: Input the function in a graphing calculator and use the table and graph views to complete the graph on paper and to answer the questions.



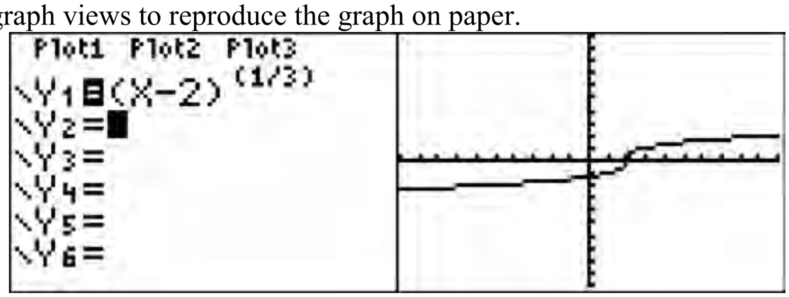
PTS: 4 REF: fall1310a1 NAT: F.IF.7b TOP: Graphing Absolute Value Functions

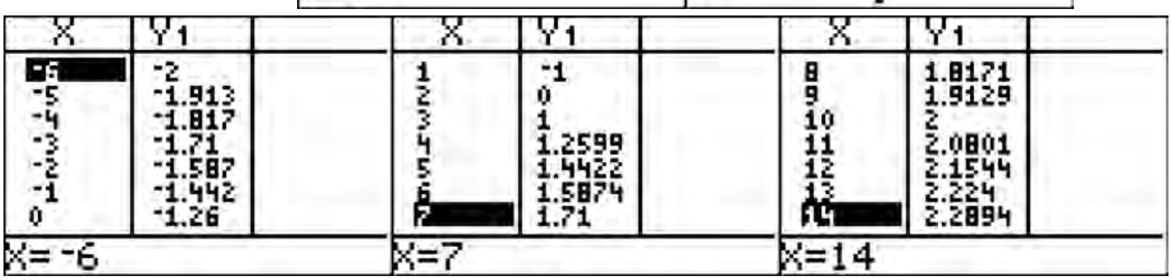
4. ANS:



Strategy: Input the function in a graphing calculator, then use the graph and table views to construct the graph on paper. Limit the domain of the graph to $-6 \le x \le 10$.

STEP 1: Use exponential notation to input the function into the graphing calculator, where $\sqrt[3]{x-2} = (x-2)^{(1/3)}$. Then use the table and graph views to reproduce the graph on paper.





X	Y1
-6	-2
-5	-1.913
-4	-1.817
-3	-1.71
-2	-1.587
-1	-1.442
0	-1.26

X= -6

X	Y1
1	-1
2	0
3	1
4	1.2599
5	1.4422
6	1.5874
7	1.71

X=7

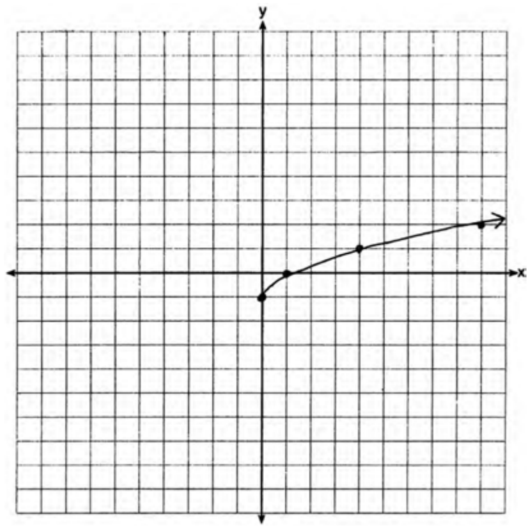
X	Y1
8	1.8171
9	1.9129
10	2
11	2.0801
12	2.1544
13	2.224
14	2.2894

X=14

STEP 2: Limit the domain of the function to $-6 \le x \le 10$. Used closed dots to show the ends of the function at coordinates (-6, -2) and for (10, 2).

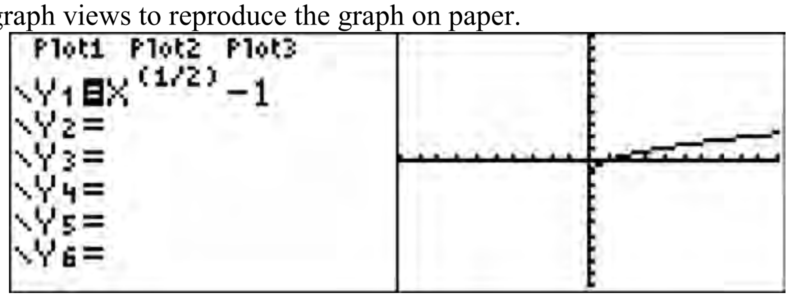
PTS: 2 REF: fall1304a1 NAT: F.IF.7b TOP: Graphing Root Functions

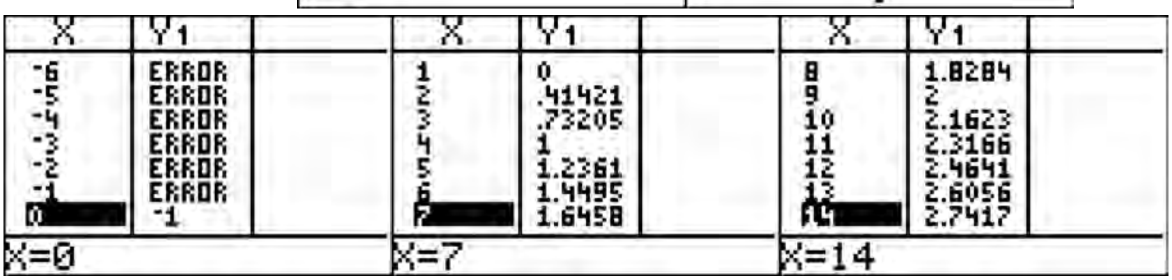
5. ANS:



Strategy: Input the function in a graphing calculator, then use the graph and table views to construct the graph on paper.

STEP 1: Use exponential notation to input the function into the graphing calculator, where $\sqrt{x}-1=x^{(1/2)}-1$. Then use the table and graph views to reproduce the graph on paper.





X	Y1	X	Y1	X	Y1
-6	ERROR	1	0	8	1.8284
-5	ERROR	2	.41421	9	2
-4	ERROR	3	.73205	10	2.1623
-3	ERROR	4	1	11	2.3166
-2	ERROR	5	1.2361	12	2.4641
-1	ERROR	6	1.4495	13	2.6056
0	-1	7	1.6458	14	2.7417
X=0		X=7		X=14	

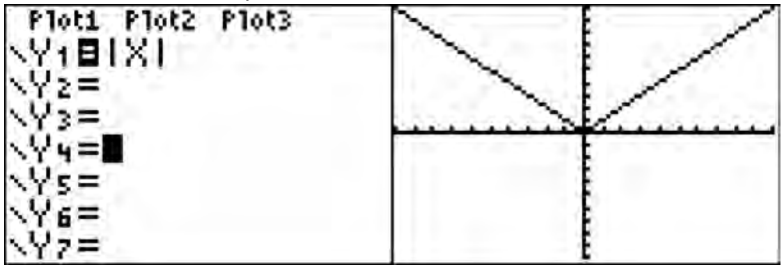
Note: Do nopt plot coordinates with errors. Focus on plotting coordinates with integer values and estimate the graph between the points with integer values when drawing the graph.

STEP 2: Limit the domain of the function to $-6 \le x \le 10$. Used closed dots to show the ends of the function at coordinates (-6, -2) and for (10, 2).

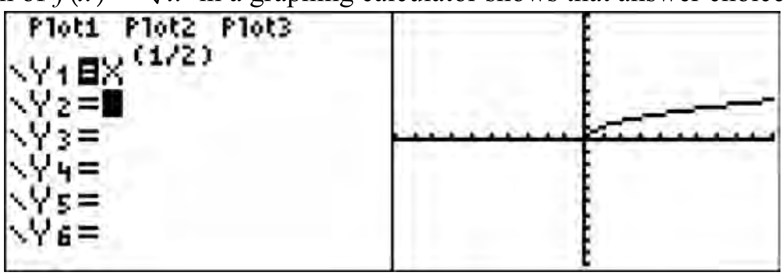
PTS: 2 REF: 061425a1 NAT: F.IF.7b TOP: Graphing Root Functions

6. ANS: B
Strategy: Eliminate wrong answers.

The left half of each graph corresonds to $f(x) = |x|$ over the domain $x < 1$. The graph of $f(x) = |x|$ should not curve because x is of the first degree. Answer choices *c* and *d* should be eliminated because they have curves over the domain $x < 1$. A quick look at the graph of
$f(x) = |x|$ in a graphing calculator shows why answer choices *c* and *d* should be eliminated.

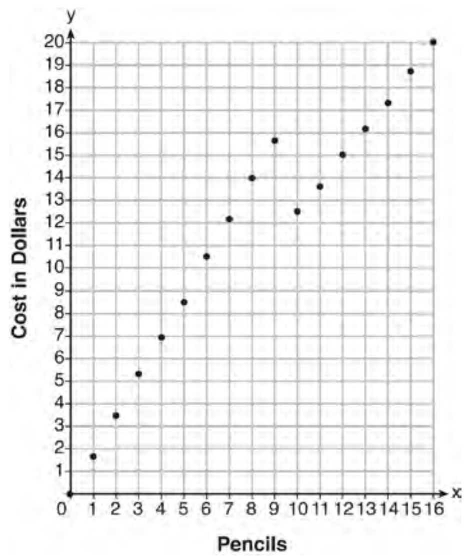


The graph of $f(x) = \sqrt{x}$ over the domain $x \ge 1$ should be not be a straight line because the degree of x is not 1. A quick look at the graph of $f(x) = \sqrt{x}$ in a graphing calculator shows that answer choice *b* is correct.



PTS: 2 REF: 081516ai NAT: F.IF.7 TOP: Graphing Piecewise-Defined Functions
KEY: bimodalgraph

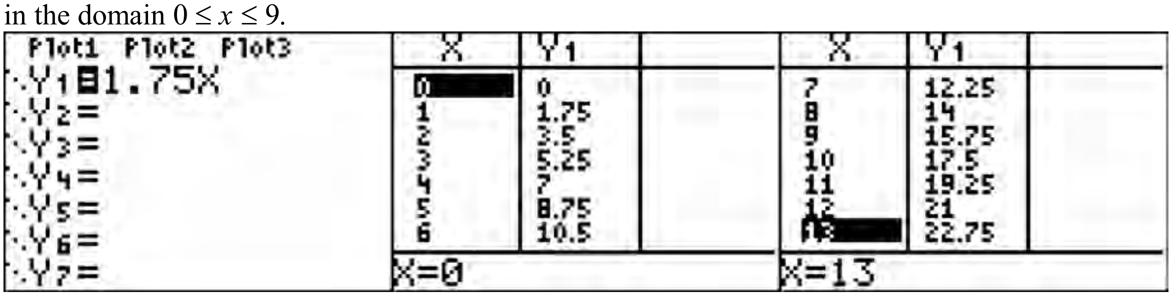
7. ANS:



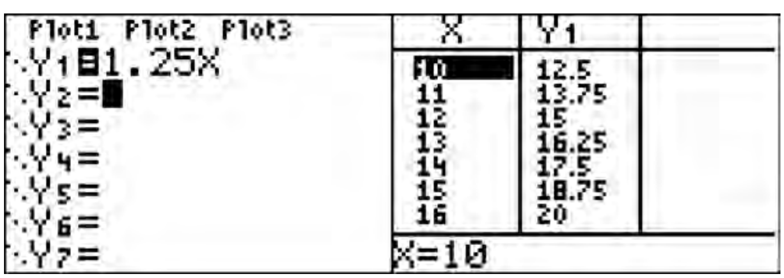
he cashier is correct. 8 pencils cost \$14 and 10 pencils cost \$12.50.

Strategy: Use a graphing calculator and graph the function in two sections. Note that the domain of the function is whole numbers. You cannot buy a part of a pencil. This means that the graph of the function will consist of points and not lines. After completing the graph, answer the questions presented in the problem.

STEP 1: Graph the section of the function represented by $c(x) = 1.75x$. Plot closed dots for each whole number in the domain $0 \le x \le 9$.



STEP 2: Graph the section of the function represented by $c(x) = 1.25x$. Plot closed dots for each whole number in the domain $x > 10$.



STEP 3: Answer the questions presented in the problem.
The data tables and the graph show that it would be cheaper to purchase 10 pencils that to purchase 8 pencils.

PTS: 4 REF: fall1312a1 NAT: F.IF.7b TOP: Graphing Piecewise-Defined Functions

8. ANS: A
Strategy: Focus on whether the line segments should begin and end with closed or open circles. A closed circle is included. An open circle is not included.

PTS: 2 REF: 061507AI NAT: F.IF.7b TOP: Graphing Step Functions
KEY: bimodalgraph

9. ANS:
a) The difference in salary, *in dollars*, for an employee who works 52 hours versus one who works 38 hours, is \$200.
b) An employee must work 43 hours in order to earn \$445. See work below.

Strategy: Part a: Use the piecewise function to first determine the salaries of 1) an employee who works 52 hours, and 2) an employee who works 38 hours. Then, find the difference of the two salaries.

Working 38 Hours	Working 52 Hours
$x = 38$	$x = 52$
$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$	$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$
$w(38) = \begin{cases} 10(38), & 0 \le x \le 40 \\ \text{not applicable}, & x > 40 \end{cases}$	$w(52) = \begin{cases} \text{not applicable}, & 0 \le x \le 40 \\ 15(52-40)+400, & x > 40 \end{cases}$
$w(38) = \{10(38), \quad 0 \le x \le 40$	$w(52) = \{15(52-40)+400, \quad x > 40$
$w(38) = 380$	$w(52) = \{15(12)+400, \quad x > 40$
	$w(52) = \{180+400, \quad x > 40$
	$w(52) = 580$

The difference between the values of $w(38)$ and $w(52)$ is \$200.

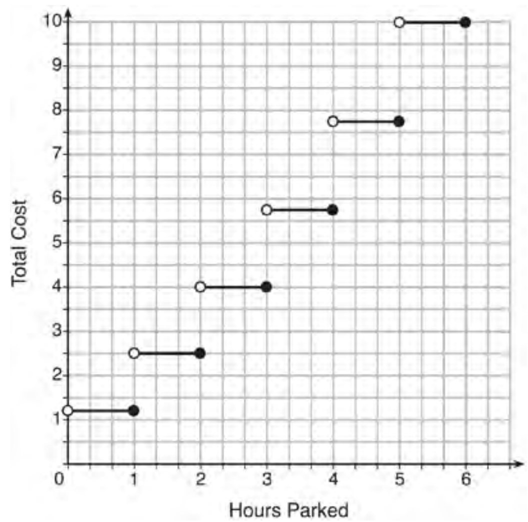
Strategy: Part b: The employee must work more than 40 hours, and compensation for hours worked in excess of 40 hours is found in the second formula and is equal to \$15 per hour. The compensation worked in excess of 40 hours is \$445 – \$400 = \$45, so

$$\frac{45 \text{ dollars}}{15 \text{ dollars/hour}} = 3 \text{ hours}$$

The employee must work a total of 43 hours. The employee receives \$400 for the first 40 hours and \$45 for the 3 hours in excess of 40 hours.

PTS: 4 REF: 061534AI NAT: F.IF.2 TOP: Functional Notation

10. ANS:



The cost per houir to park gets bigger over the six hour period.

Strategy: Graph this step function by hand using information from the table. This function has too many sections to easily input into a graphing calculator.

STEP 1. Graph the section for the domain $0 < x \le 1$. The table shows that this interval corresponds to a cost of \$1.25 on the y-axis. Use an open dot at (0, 1.25) and a closed dot at (1, 1.25). Connect the two dots with a solid line.
STEP 2. Graph the section for the domain $1 < x \le 2$. The table shows that this interval corresponds to a cost of \$2.50 on the y-axis. Use an open dot at (1, 2.50) and a closed dot at (2, 2.50). Connect the two dots with a solid line.
STEP 3. Graph the section for the domain $2 < x \le 3$. The table shows that this interval corresponds to a cost of \$4.00 on the y-axis. Use an open dot at (2, 4.00) and a closed dot at (3, 4.00). Connect the two dots with a solid line.
STEP 4. Graph the section for the domain $3 < x \le 4$. The table shows that this interval corresponds to a cost of \$5.75 on the y-axis. Use an open dot at (3, 4.75) and a closed dot at (4, 4.75). Connect the two dots with a solid line.
STEP 5. Graph the section for the domain $4 < x \le 5$. The table shows that this interval corresponds to a cost of \$7.75 on the y-axis. Use an open dot at (4, 7.75) and a closed dot at (5, 7.75). Connect the two dots with a solid line.
STEP 6. Graph the section for the domain $5 < x \le 6$. The table shows that this interval corresponds to a cost of \$10.00 on the y-axis. Use an open dot at (5, 10.00) and a closed dot at (6, 10.00). Connect the two dots with a solid line.

STEP 7: Answer the question based on the graph and the table.

PTS: 4 REF: fall1311a1 NAT: F.IF.7b TOP: Graphing Step Functions

POLYNOMIALS

A.APR.1: Arithmetic Operations on Polynomials

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Vocabulary

- **Polynomial:** A monomial or the sum of two or more monomials whose exponents are positive.
  Example: $5a^2 + ba - 3$
- **Monomial:** A polynomial with one term; it is a number, a variable, or the product of a number (the coefficient) and one or more variables
  Examples: $-\frac{1}{4}$, x^2, $4a^2b$, -1.2, $m^2n^3p^4$
- **Binomial:** An algebraic expression consisting of two terms
  Example $(5a + 6)$
- **Trinomial:** A polynomial with exactly three terms.
  Example $(a^2 + 2a - 3)$
- **Like Terms:** Like terms must have **exactly the same base and the same exponent**. Their coefficients may be different. Real numbers are like terms.
  Example: Given the expression
  $1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2$,
  the following are like terms:
  $1x^2$ and $3x^2$
  2y and 7y
  4x has no other like terms in the expression
  $5x^3$ and $8x^3$
  $6y^2$ and $9y^2$

Like terms in the same expression can be combined by adding their coefficients.

$1x^2$ and $3x^2 = 4x^2$
2y and 7y =9y
4x has no other like terms in the expression = 4x
$5x^3$ and $8x^3 = 13x^3$
$6y^2$ and $9y^2 = 15y^2$

$$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2 = 4x^2 + 9y + 4x + 13x^3 + 15y^2$$

Adding and Subtracting Polynomials: To add or subtract polynomials, arrange the polynomials one above the other with like terms in the same columns. Then, add or subtract the coefficients of the like terms in each column and write a new expression.

Addition Example	**Subtraction** Example
Add: $(3r^4 - 9r^3 - 8) + (4r^4 + 8r^3 - 8)$	Subtract: $(3r^4 - 9r^3 - 8) - (4r^4 + 8r^3 - 8)$
$3r^4$ $-9r^3$ -8	$3r^4$ $-9r^3$ -8
$4r^4$ $+8r^3$ -8	$-(4r^4)$ $-(+8r^3)$ $-(-8)$
$7r^4$ $-r^3$ -16	$-1r^4$ $-17r^3$ $+0$

Multiplying Polynomials: To multiply two polynomials, multiply each term in the first polynomial by each term in the second polynomial, then combine like terms.

Example:

Multiply: $\left(-8r^2 - 9r + 7\right)\left(-5r + 1\right)$

STEP 1: Multiply the first term in the first polynomial by each term in the second polynomial, as follows:

$$-8r^2\left(-5r+1\right)$$
$$-8r^2\left(-5r\right)+-8r^2\left(1\right)$$
$$\boxed{40r^3-8r^2}$$

STEP 2. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$-9r\left(-5r+1\right)$$
$$-9r\left(-5r\right)+-9r\left(1\right)$$
$$\boxed{45r^2-9r}$$

STEP 3. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$7\left(-5r+1\right)$$
$$7\left(-5r\right)+7\left(1\right)$$
$$\boxed{-35r+7}$$

STEP 4. Combine like terms from each step.

$$40r^3 \ -8r^2 + 45r^2 \ -9r \ -35r \ +7$$
$$\boxed{40r^3 \quad +37r^2 \quad -44r \quad +7}$$

REGENTS PROBLEMS

1. If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals
   a. $-5x^2 - 11x + 13$
   b. $5x^2 + 11x - 13$
   c. $-5x^2 - x + 1$
   d. $5x^2 - x + 1$

2. A company produces x units of a product per month, where $C(x)$ represents the total cost and $R(x)$ represents the total revenue for the month. The functions are modeled by $C(x) = 300x + 250$ and $R(x) = -0.5x^2 + 800x - 100$. The profit is the difference between revenue and cost where $P(x) = R(x) - C(x)$. What is the total profit, $P(x)$, for the month?

a. $P(x) = -0.5x^2 + 500x - 150$
b. $P(x) = -0.5x^2 + 500x - 350$
c. $P(x) = -0.5x^2 - 500x + 350$
d. $P(x) = -0.5x^2 + 500x + 350$

3. Which trinomial is equivalent to $3(x-2)^2 - 2(x-1)$?

a. $3x^2 - 2x - 10$
b. $3x^2 - 2x - 14$
c. $3x^2 - 14x + 10$
d. $3x^2 - 14x + 14$

4. Subtract $5x^2 + 2x - 11$ from $3x^2 + 8x - 7$. Express the result as a trinomial.

5. Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by $2x-6$ and the width is represented by $3x-5$, then the paper has a total area represented by

a. $5x-11$
b. $6x^2-28x+30$
c. $10x-22$
d. $6x^2-6x-11$

6. Express the product of $2x^2+7x-10$ and $x+5$ in standard form.

7. If the difference $(3x^2-2x+5)-(x^2+3x-2)$ is multiplied by $\frac{1}{2}x^2$, what is the result, written in standard form?

8. When $(2x-3)^2$ is subtracted from $5x^2$, the result is

a. $x^2-12x-9$
b. $x^2-12x+9$
c. $x^2+12x-9$
d. $x^2+12x+9$

A.APR.1: Arithmetic Operations on Polynomials
Answer Section

1. ANS: B
   Strategy: To subtract, change the signs of the subtrahend and add.

Given:	Change the signs and add:
$3x^2+5x-6$ $\underline{-\left(-2x^2-6x+7\right)}$	$3x^2+5x-6$ $\underline{+2x^2+6x-7}$ $5x^2+11x-13$

   PTS: 2 REF: 061403a1 NAT: A.APR.1 TOP: Addition and Subtraction of Polynomials
   KEY: subtraction

2. ANS: B
   Strategy: Substitute $R(x)$ and $C(x)$ into $P(x)=R(x)-C(x)$.

   Given: $P(x)=R(x)-C(x)$

   $R(x)=-0.5x^2+800x-100$

   $C(x)=300x+250$

   Therefore: $P(x)=\left(-0.5x^2+800x-100\right)-(300x+250)$

   $P(x)=-0.5x^2+800x-100-300x-250$

   $P(x)=-0.5x^2+500x-350$

   PTS: 2 REF: 081406a1 NAT: A.APR.1 TOP: Addition and Subtraction of Polynomials
   KEY: subtraction

3. ANS: D
   Strategy: Expand and simplify the expression $3(x-2)^2-2(x-1)$

   STEP 1 Expand the expression.

$$3(x^2-4x+4)-2(x-1)$$

$$3x^2-12x+12-2x+2$$

   STEP 2: Simplify the expanded expression by combining like terms.

$$3x^2-12x+12-2x+2$$

$$3x^2-14x+14$$

   PTS: 2 REF: 081524ai NAT: A.APR.1 TOP: Operations with Polynomials
   KEY: mixed

4. ANS:

Strategy: To subtract, change the signs of the subtrahend and add.

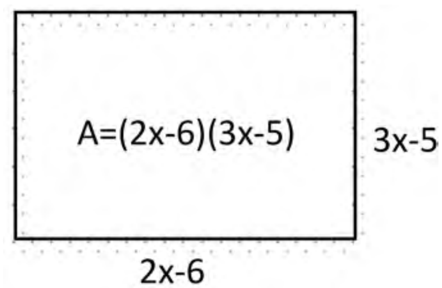
Given: $3x^2+8x-7$ $\underline{-\left(5x^2+2x-11\right)}$	Change the signs and add: $\begin{array}{r} 3x^2+8x-7 \\ \underline{-5x^2-2x+11} \\ -2x^2+6x+4 \end{array}$

PTS: 2 REF: 011528a1 NAT: A.APR.1 TOP: Addition and Subtraction of Polynomials

KEY: subtraction

5. ANS: B

Strategy: Draw a picture and use the area formula for a rectange: $A = lw$.



$$A = (2x-6)(3x-5)$$

$$A = 6x^2 - 10x - 18x + 30$$

$$A = 6x^2 - 28x + 30$$

PTS: 2 REF: 011510a1 NAT: A.APR.1 TOP: Multiplication of Polynomials

6. ANS:

$2x^3 + 17x^2 + 25x - 50$

Strategy: Use the distribution property to multiply polynomials, then simplify.

STEP 1. Use the distributive property

$$(2x^2 + 7x - 10)(x + 5)$$

$$2x^3 + 10x^2 + 7x^2 + 35x - 10x - 50$$

$$2x^3 + 17x^2 + 25x - 50$$

STEP 2. Simplify by combining like terms.

$$2x^3 + 10x^2 + 7x^2 + 35x - 10x - 50$$

$$2x^3 + 17x^2 + 25x - 50$$

PTS: 2 REF: 081428a1 NAT: A.APR.1 TOP: Multiplication of Polynomials

7. ANS:
$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$

Strategy. First, find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$, the use the distributive property to multiply the difference by $\frac{1}{2}x^2$. Simplify as necessary.

STEP 1. Find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$. To subtract polynomials, change the signs of the subtrahend and add.

Given: $(3x^2 - 2x + 5)$ $\underline{-(x^2 + 3x - 2)}$	Change the signs and add: $3x^2 - 2x + 5$ $\underline{-x^2 - 3x + 2}$ $2x^2 - 5x + 7$

STEP 2. Multiply $2x^2 - 5x + 7$ by $\frac{1}{2}x^2$.

$$\frac{1}{2}x^2\left(2x^2 - 5x + 7\right)$$

$$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$$

PTS: 2 REF: 061528AI NAT: A.APR.1 TOP: Operations with Polynomials
KEY: multiplication

8. ANS: C
Strategy: Expand the binomial, then subtract it from $5x^{2.}$

$$5x^2 - (2x - 3)^2$$

$$5x^2 - (2x - 3)(2x - 3)$$

$$5x^2 - (4x^2 - 6x - 6x + 9)$$

$$5x^2 - (4x^2 - 12x + 9)$$

$$5x^2 - 4x^2 + 12x - 9$$

$$x^2 + 12x - 9$$

PTS: 2 REF: 011610ai NAT: A.APR.1 TOP: Operations with Polynomials
KEY: multiplication

A.SSE.2: Factor Polynomials

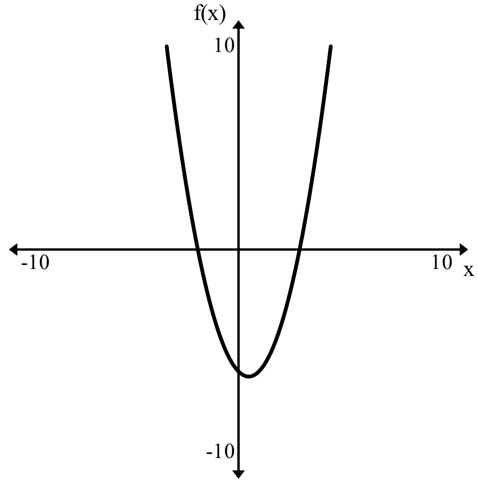
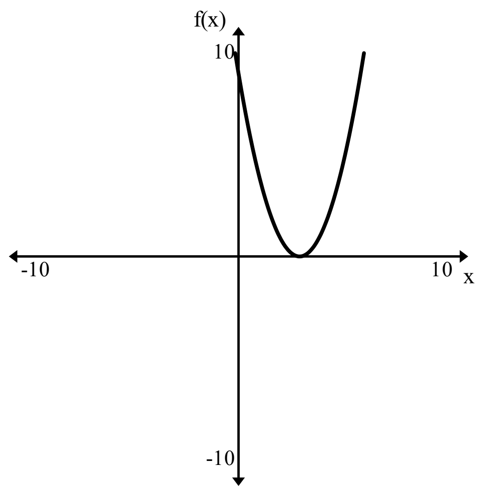
Interpret the structure of expressions.

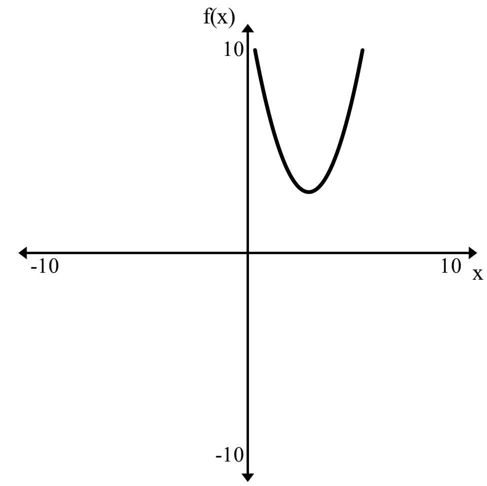
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Big Idea:

Factoring polynomials is one of four general methods taught in the Regents mathematics curriculum for finding the roots of a quadratic equation. The other three methods are the quadratic formula, completing the square and graphing.

- The roots of a quadratic equation can found using the **factoring** method when the discriminant's value is equal to either zero or a perfect square.

Using the Discriminant to Predict Types of Roots	
$b^2 - 4ac > 0$ Two real roots: The parabola intersects the x-axis in two places.	
$b^2 - 4ac = 0$ One real root: The parabola intersects with the x-axis at only one point.	

$b^2 - 4ac < 0$ No real roots: The parabola does not intersect the x-axis and the roots are imaginary numbers.	

<u>Prime Factoring of a Monomial</u>:

$204x^2 = 2\left(102x^2\right) = 2 \bullet 2\left(51x^2\right) = 2 \bullet 2 \bullet 3\left(17x^2\right) = 2^2 \bullet 3 \bullet 17 \bullet x^2$

<u>Factoring Binomials</u>: *NOTE: This is the inverse of the distributive property.*

$3\left(x+2\right) = 3x+6$

$2x^2 + 6x = 2x\left(x+3\right)$

<u>Special Case: Factoring the Difference of Perfect Squares</u>.

General Rule	Examples
$\left(a^2 - b^2\right) = \left(a+b\right)\left(a-b\right)$	$x^2 - 4 = \left(x+2\right)\left(x-2\right)$
	$x^4 - 9 = \left(x^2+3\right)\left(x^2-3\right)$

<u>Special Case: Factoring the Sum and Difference of Perfect Cubes</u>.

General Rule	Examples
$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$	$x^3 + 27 = (x+3)\left(x^2 - 3x + 3^2\right)$
$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$	$x^3 - 125 = (x-5)\left(x^2 + 5x + 5^2\right)$

How to Factor Trinomials.
Given a trinomial in the form $ax^2+bx+c=0$ whose discriminant equals zero or a perfect square, it may be factored as follows:

STEP 1. The product of these two numbers must equal c.

$$ax^2+bx+c=0=\left(\square\, x \underline{\quad\quad} \square\right)\left(\square\, x \underline{\quad\quad} \square\right)$$

STEP 2. The signs of these two numbers are determined by the signs of b and c.

Inners

$$ax^2+bx+c=0=\left(\square\, x \underline{\quad\quad} \square\right)\left(\square\, x \underline{\quad\quad} \square\right)$$

Outers

STEP 3. The product of the outer numbers plus the product of the inner numbers must sum to b.

Modeling:

$$x^2-5x+6=(x-2)(x-3)$$
$$2x^2-8x+6=(2x-2)(x-3)$$
$$4x^2-10x+6=(2x-2)(2x-3)$$

Turning Factors into Roots. Students frequently do not understand why each factor of a binomial or trinomial can be set to equal zero, thus leading to the roots of the equation. Recall that the standard form of a quadratic equation is $ax^2+bx+c=0$ and only the left side of the equation is factored. Thus, the left side of the equation equals zero.

For all numbers	$a\cdot 0=0$		
and if	$a\cdot b=0$	$b\neq 0$	(NOTE: substitute any two factors)
Therefore	$a=0$		

REGENTS PROBLEMS

1. Which expression is equivalent to $x^4 - 12x^2 + 36$?
   a. $(x^2 - 6)(x^2 - 6)$
   b. $(x^2 + 6)(x^2 + 6)$
   c. $(6 - x^2)(6 + x^2)$
   d. $(x^2 + 6)(x^2 - 6)$

2. When factored completely, the expression $p^4 - 81$ is equivalent to
   a. $(p^2 + 9)(p^2 - 9)$
   b. $(p^2 - 9)(p^2 - 9)$
   c. $(p^2 + 9)(p + 3)(p - 3)$
   d. $(p + 3)(p - 3)(p + 3)(p - 3)$

3. Four expressions are shown below.

   I $2(2x^2 - 2x - 60)$
   II $4(x^2 - x - 30)$
   III $4(x + 6)(x - 5)$
   IV $4x(x - 1) - 120$

   The expression $4x^2 - 4x - 120$ is equivalent to
   a. I and II, only
   b. II and IV, only
   c. I, II, and IV
   d. II, III, and IV

4. If the area of a rectangle is expressed as $x^4 - 9y^2$, then the product of the length and the width of the rectangle could be expressed as

a. $(x-3y)(x+3y)$	c. $(x^2-3y)(x^2-3y)$
b. $(x^2-3y)(x^2+3y)$	d. $(x^4+y)(x-9y)$

5. Factor the expression $x^4 + 6x^2 - 7$ completely.

6. If Lylah completes the square for $f(x) = x^2 - 12x + 7$ in order to find the minimum, she must write $f(x)$ in the general form $f(x) = (x-a)^2 + b$. What is the value of a for $f(x)$?

a. 6	c. 12
b. –6	d. –12

7. When factored completely, $x^3 - 13x^2 - 30x$ is

a. $x(x+3)(x-10)$	c. $x(x+2)(x-15)$
b. $x(x-3)(x-10)$	d. $x(x-2)(x+15)$

A.SSE.2: Factor Polynomials
Answer Section

1. ANS: A
Strategy 1. Factor $x^4 - 12x^2 + 36$

$x^4 - 12x^2 + 36$

$\left(x^2 - \_\_\_\right)\left(x^2 - \_\_\_\right)$

The factors of 36 are:

1 and 36,

2 and 18,

3 and 12,

4 and 9,

6 and 6 (use these because they sum to 12)

$\left(x^2 - 6\right)\left(x^2 - 6\right)$

Strategy 2. Work backwards using the distributive property to check each answer choice.

a $(x^2 - 6)(x^2 - 6)$ $x^4 - 6x^2 - 6x^2 + 36$ $x^4 - 12x^2 + 36$ (correct)	c $(6 - x^2)(6 + x^2)$ $36 + 6x^2 - 6x^2 - x^4$ $36 - x^4$ (wrong)
b $(x^2 + 6)(x^2 + 6)$ $x^4 + 6x^2 + 6x^2 + 36$ $x^4 + 12x^2 + 36$ (wrong)	d $(x^2 + 6)(x^2 - 6)$ $x^4 - 6x^2 + 6x^2 - 36$ $x^4 - 36$ (wrong)

PTS: 2 REF: 081415a1 NAT: A.SSE.2 TOP: Factoring Polynomials

2. ANS: C
Strategy: Use difference of perfect squares.

STEP 1. Factor $p^4 - 81$
$p^4 - 81$
$\left(p^2+9\right)\left(p^2-9\right)$
STEP 2. Factor $p^2 - 9$
$\left(p^2+9\right)\left(p^2-9\right)$
$\left(p^2+9\right)\left(p+3\right)\left(p-3\right)$

PTS: 2 REF: 011522a1 NAT: A.SSE.2 TOP: Factoring Polynomials

3. ANS: C
Strategy: Use the distributive property to expand each expression, then match the expanded expressions to the answer choices.

I	III
$2(2x^2-2x-60)$ $4x^2-4x-120$ *yes*	$4(x+6)(x-5)$ $(4x+24)(x-5)$ $4x^2-20x+24x-120$ $4x^2+4x-120$ *no*
II $4(x^2-x-30)$ $4x^2-4x-120$ *yes*	**IV** $4x(x-1)-120$ $4x^2-4x-120$ *yes*

Answer choice *c* is correct.

PTS: 2 REF: 081509ai NAT: A.SSE.2 TOP: Factoring Polynomials

4. ANS: B
Strategy: Use the distributive property to work backwards from the answer choices.

a.	c.
$(x-3y)(x+3y)$ $x^2+3xy-3xy-9y^2$ x^2-9y^2 (wrong)	$(x^2-3y)(x^2-3y)$ $x^4-3x^2y-3x^2y+9y^2$ $x^4-6x^2y+9y^2$ (wrong)
b. $(x^2-3y)(x^2+3y)$ $x^4+3x^2y-3x^2y-9y^2$ x^4-9y^2 (correct)	d. $(x^4+y)(x-9y)$ $x^5-9x^4y+xy-9y^2$ (wrong)

PTS: 2 REF: 061503AI NAT: A.SSE.2 TOP: Factoring Polynomials

5. ANS:
$\left(x^2+7\right)(x+1)(x-1)$

Strategy: Factor the trinomial, then factor the perfect square.

STEP 1. Factor the trinomial x^4+6x^2-7.

$$x^4+6x^2-7$$
$$\left(x^2+\_\_\_\right)\left(x^2-\_\_\_\right)$$
The factors of 7 are 1 and 7.
$$\left(x^2+7\right)\left(x^2-1\right)$$

STEP 2. Factor the perfect square.

$$\left(x^2+7\right)\left(x^2-1\right)$$
$$\left(x^2+7\right)(x+1)(x-1)$$

PTS: 2 REF: 061431a1 NAT: A.SSE.2 TOP: Factoring Polynomials

6. ANS: A

Strategy: Transform $f(x) = x^2 - 12x + 7$ into the form of $f(x) = (x - a)^2 + b$ and find the value of a.

$$x^2 - 12x + 7 = f(x)$$

$$x^2 - 12x + 7 = 0$$

$$x^2 - 12x = -7$$

$$x^2 - 12x + \left(\frac{-12}{2}\right)^2 = -7 + \left(\frac{-12}{2}\right)^2$$

$$x^2 - 12x + (-6)^2 = -7 + (-6)^2$$

$$(x-6)^2 = -7 + 36$$

$$(x-6)^2 = +29$$

$$(x-6)^2 - 29 = 0$$

$$f(x) = (x-6)^2 - 29$$

If $-a = -6$, then $a = 6$.

PTS: 2 REF: 081520ai NAT: A.SSE.3 TOP: Solving Quadratics
KEY: completing the square

7. ANS: C

$$x^3 - 13x^2 - 30x$$

$$x(x^2 - 13x - 30)$$

$$x(x+2)(x-15)$$

PTS: 2 REF: 011612ai NAT: A.SSE.2 TOP: Factoring Polynomials

A.APR.3: Find Zeros of Polynomials

Understand the relationship between zeros and factors of polynomials.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Vocabulary

Multiplication Property of Zero: The **multiplication property of zero** says that if the product of two numbers or expressions is zero, then one or both of the numbers or expressions must equal zero. More simply, if $x \cdot y = 0$, then either $x = 0$ or $y = 0$, or, x and y both equal zero.

Factor: A **factor** is:

1) a whole number that is a **divisor** of another number, or
2) an algebraic expression that is a **divisor** of another algebraic expression.

Examples:

- 1, 2, 3, 4, 6, and 12 all divide the number 12, so 1, 2, 3, 4, 6, and 12 are all factors of 12.
- $(x-3)$ and $(x+2)$ will divide the trinomial expression $x^2 - x - 6$, so $(x-3)$ and $(x+2)$ are both factors of the $x^2 - x - 6$.

Zeros: A **zero** of an equation is a **solution** or **root** of the equation. The words **zero**, **solution**, and **root** all mean the same thing. The zeros of a polynomial expression are found by finding the value of x when the value of y is 0. This done by making and solving an equation with the value of the polynomial expression equal to zero.

Example:

- The **zeros** of the trinomial expression $x^2 + 2x - 24$ can be found by writing and then factoring the equation:

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

After factoring the equation, use the **multiplication property of zero** to find the zeros, as follows:

$$(x+6)(x-4) = 0$$

$$\therefore\ x+6=0\ \ and/or\ \ x-4=0$$

$$If\ \ x+6=0,\ then\ \ x=-6$$

$$If\ \ x-4=0,\ then\ \ x=+4$$

The zeros of the expression $x^2 + 2x - 24 = 0$ are -6 and +4.

Check: You can check this by substituting both -6 or +4 into the expression, as follows:

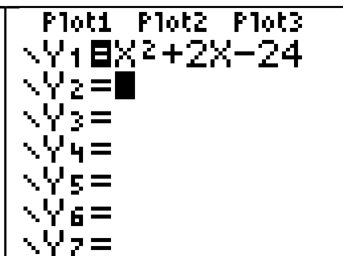
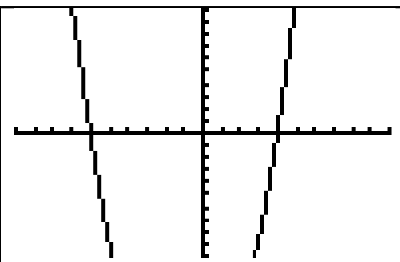
Check for -6

$$x^2+2x-24$$
$$(-6)^2+2(-6)-24$$
$$36-12-24$$
$$0$$

Check for +4

$$x^2+2x-24$$
$$(4)^2+2(4)-24$$
$$16+8-24$$
$$0$$

x-axis intercepts: The zeros of an expression can also be understood as the **x-axis intercepts** of the graph of the equation when $f(x)=0$. This is because the coordinates of the x-axis intercepts, by definition, have y-values equal to zero, and is the same as writing an equation where the expression is equal to zero.

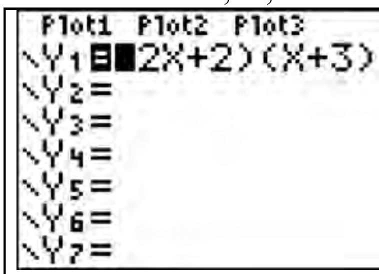
The roots of $x^2+2x-24=0$ are $x=-6$ and $x=4$. These are the x coordinate values of the x-axis intercepts.		

Starting with Factors and Finding Zeros
Remember that the **factors** of an expression are *related to* the **zeros** of the expression by the **multiplication property of zero**. Thus, if you know the **factors**, it is easy to find the **zeros**.

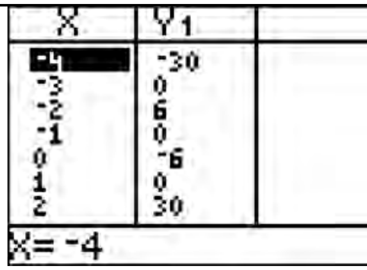
Example: The factors of an expression are $(2x+2)$, $(x+3)$ and $\left(x-1\right)$.
The zeros are found as follows using the multiplication property of zero:

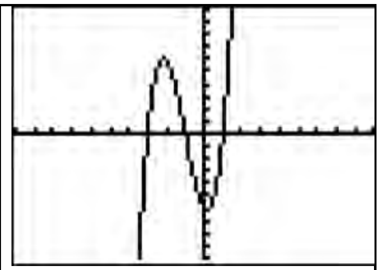
$$(2x+2)(x+3)(x-1)=0$$
$$\therefore\ 2x+2=0\ \ and\ \ x=-1$$
$$and\,/\,or\ x+3=0\ \ and\ \ x=-3$$
$$and\,/\,or\ x-1=0\ \ and\ \ x=1$$

The zeros are -3, -1, and +1.



Be sure to input all factors.





Starting with Zeros and Finding Factors
If you know the **zeros** of an expression, you can work backwards using the **multiplication property of zero** to find the **factors** of the expression. For example, if you inspect the graph of an equation and find that it has **x-intercepts** at $x = 3$ and $x = -2$, you can write:

$$x=3$$
$$\therefore\ (x-3)=0$$
$$and$$
$$x=-2$$
$$\therefore\ (x+2)=0$$

The equation of the graph has **factors** of $(x-3)$ and $(x+2)$, so you can write the equation:

$$(x-3)(x+2)=0$$

which simplifies to

$$x^2+2x-3x-6=f(x)$$
$$x^2-x-6=f(x)$$

With practice, you can probably move back and forth between the **zeros** of an expression and the **factors** of an expression with ease.

REGENTS PROBLEMS

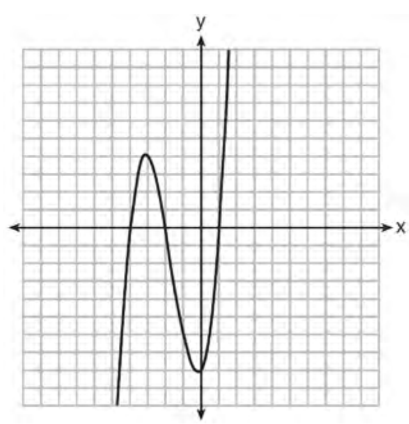
1. What are the zeros of the function $f(x) = x^2 - 13x - 30$?

   a. -10 and 3
   b. 10 and -3
   c. -15 and 2
   d. 15 and -2

2. The zeros of the function $f(x) = 3x^2 - 3x - 6$ are

   a. -1 and -2
   b. 1 and -2
   c. 1 and 2
   d. -1 and 2

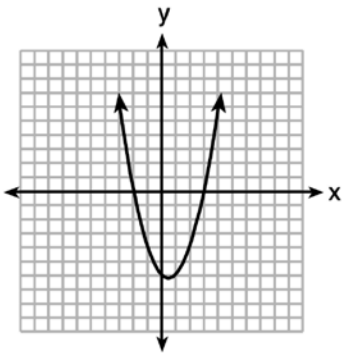
3. The graph of $f(x)$ is shown below.

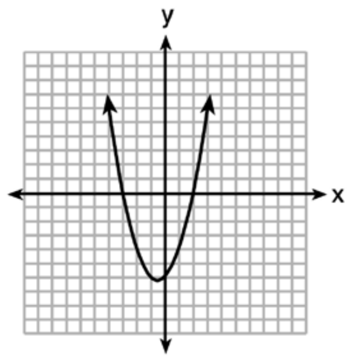


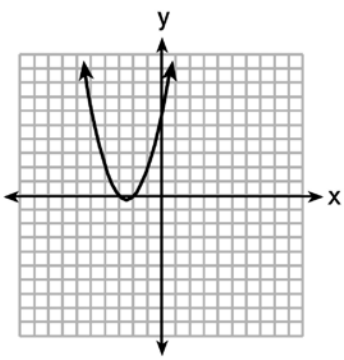
   Which function could represent the graph of $f(x)$?

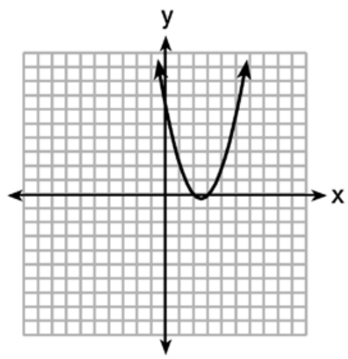
   a. $f(x) = (x + 2)(x^2 + 3x - 4)$
   b. $f(x) = (x - 2)(x^2 + 3x - 4)$
   c. $f(x) = (x + 2)(x^2 + 3x + 4)$
   d. $f(x) = (x - 2)(x^2 + 3x + 4)$

4. The graphs below represent functions defined by polynomials. For which function are the zeros of the polynomials 2 and -3?

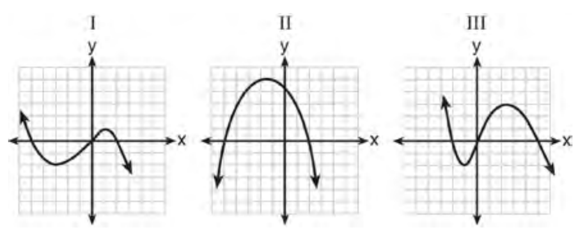
a. 

c. 

b. 

d. 

5. A polynomial function contains the factors x, $x-2$, and $x+5$. Which graph(s) below could represent the graph of this function?



a. I, only
b. II, only
c. I and III
d. I, II, and III

6. For which function defined by a polynomial are the zeros of the polynomial −4 and −6?

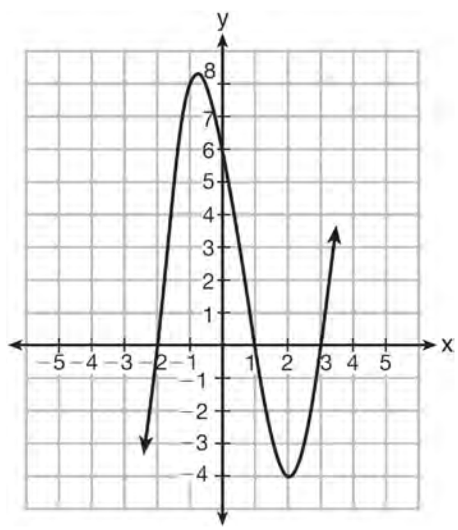
a. $y = x^2 - 10x - 24$
c. $y = x^2 + 10x - 24$
b. $y = x^2 + 10x + 24$
d. $y = x^2 - 10x + 24$

7. Which equation(s) represent the graph below?

I $y = (x + 2)(x^2 - 4x - 12)$

II $y = (x - 3)(x^2 + x - 2)$

III $y = (x - 1)(x^2 - 5x - 6)$



a. I, only
c. I and II
b. II, only
d. II and III

8. Keith determines the zeros of the function $f(x)$ to be −6 and 5. What could be Keith's function?

a. $f(x) = (x + 5)(x + 6)$
c. $f(x) = (x - 5)(x + 6)$
b. $f(x) = (x + 5)(x - 6)$
d. $f(x) = (x - 5)(x - 6)$

A.APR.3: Find Zeros of Polynomials
Answer Section

1. ANS: D

   Strategy: Find the factors of $f(x) = x^2 - 13x - 30$, then convert the factors to zeros.

   STEP 1. Find the factors of $f(x) = x^2 - 13x - 30$.

   $f(x) = x^2 - 13x - 30$

   $f(x) = (x - \_\_\_)(x + \_\_\_)$

   The factors of 30 are

   1 and 30

   2 and 15 (*use these*)

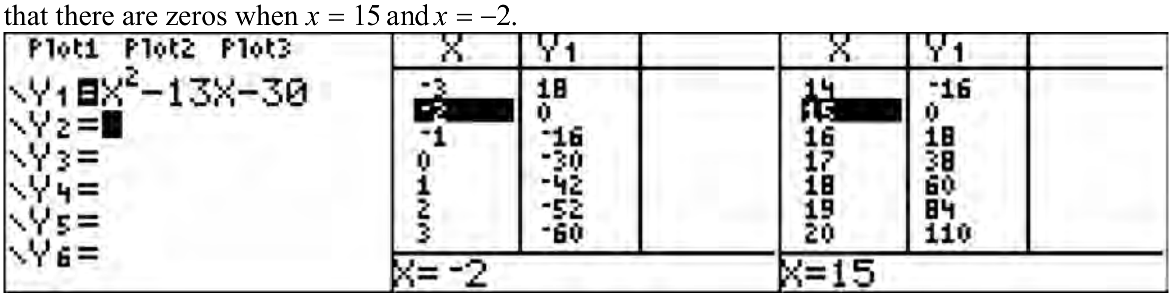
   $f(x) = (x - 15)(x + 2)$

   STEP 2. Convert the factors to zeros.

   If the factors are $(x - 15)$ and $(x + 2)$,

   then the zeros are at $x = 15$ and $x = -2$.

   DIMS? Does It Make Sense? Yes. Check by inputting $f(x) = x^2 - 13x - 30$ into a graphing calculator and verify that there are zeros when $x = 15$ and $x = -2$.



   PTS: 2 REF: 061510AI NAT: A.APR.3a TOP: Zeros of Polynomials

2. ANS: D

Strategy 1. Factor, then use the multiplication property of zero to find zeros.

$$3x^2 - 3x - 6 = 0$$
$$3(x^2 - x - 2) = 0$$
$$3(x-2)(x+1) = 0$$
$$x = 2, -1$$

Strategy 2. Use the quadratic formula.
$a = 3, b = -3,$ and $c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-6)}}{2(3)}$$

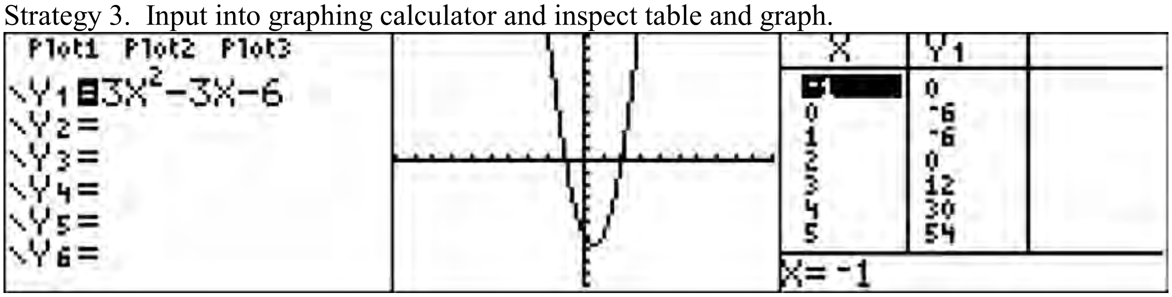
$$x = \frac{3 \pm \sqrt{9 + 72}}{6}$$

$$x = \frac{3 \pm \sqrt{81}}{6}$$

$$x = \frac{3 \pm 9}{6}$$

$$x = \frac{12}{6} = 2 \text{ and } x = \frac{-6}{6} = -1$$

Strategy 3. Input into graphing calculator and inspect table and graph.



PTS: 2 REF: 081513ai NAT: A.SSE.3 TOP: Solving Quadratics

3. ANS: A
Strategy:
STEP 1. Identify the zeros and convert them into factors.
The graph has zeros at -4, -2, and 1. Convert these zeros of the function into the following factors: (x+4)(x+2)(x-1). The functiuon rule is f(x)= (x+4)(x+2)(x-1)

STEP 2. Eliminate wrong answers. Choices b and d can be eliminated because (x-2) is not a factor.

b.	d.
$f(x)=(x-2)(x^2+3x-4)$ $(x-2)$ is not a factor. (Wrong Choice)	$f(x)=(x-2)(x^2+3x+4)$ $(x-2)$ is not a factor. (Wrong Choice)

STEP 3. Choose between remaining choices by factoring the trinomials.

a.	c.
$f(x)=(x+2)(x^2+3x-4)$ $f(x)=(x+2)(x+4)(x-1)$ Contains all three factors. (Correct Choice)	$f(x)=(x+2)(x^2+3x+4)$ (x^2+3x+4) cannot be factored into $(x+4)(x-1)$ (Wrong Choice)

PTS: 2 REF: 081504ai NAT: A.APR.3 TOP: Zeros of Polynomials

4. ANS: C
Strategy: Look for the coordinates of the x-intercepts (where the graph crosses the x-axis). The zeros are the x-values of those coordinates.

Answer c is the correct choice. The coordinates of the x-intercepts of the graph are (2, 0) and (-3, 0). The zeros of the polynomial are 2 and -3.

PTS: 2 REF: spr1302a1 NAT: A.APR.3 TOP: Zeros of Polynomials
KEY: bimodalgraph

5. ANS: A

Stategy 1. Convert the factors to zeros, then find the graph(s) with the corresponding zeros.

STEP 1. Convert the factors to zeros.

A factor of $x-0$ equates to a zero of the polynomial at x=0.

A factor of $x-2$ equates to a zero of the polynomial at x=2.

A factor of $x+5$ equates to a zero of the polynomial at x=-5.

STEP 2. Find the zeros of the graphs.

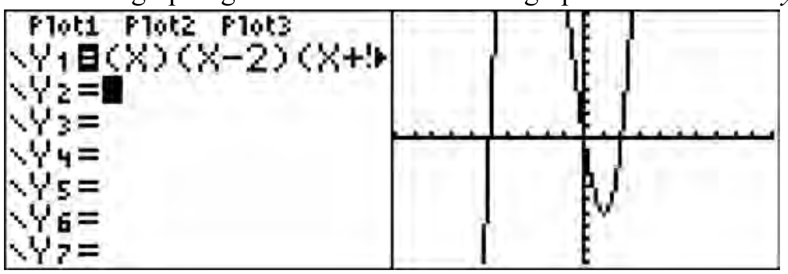
Graph I has zeros at -5, 0, and 2.

Graph II has zeros at -5 and 2.

Graph III has zeros at -2, 0, and 5.

Answer choice *a* is correct.

Strategy 2: Input the factors into a graphing calculator and view the graph of the function $y=(x)(x-2)(x+5)$.

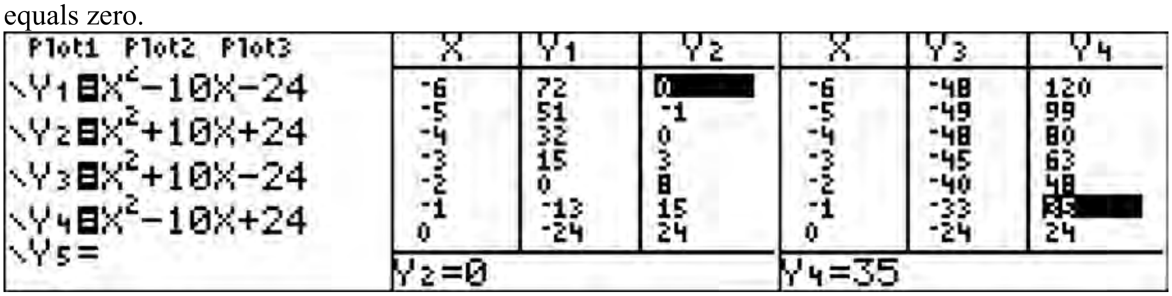


Note: This graph has the same zeros as graph I, but the end behaviors of the graph are reversed. This graph is a reflection in the x-axis of graph I and the reversal is caused by a change in the sign of the leading coefficient in the expansion of $y=(x)(x-2)(x+5)$. It makes no difference in answering this problem. The zeros are the same and the correct answer choice is answer choice *a*.

PTS: 2 REF: 011524a1 NAT: A.APR.3 TOP: Zeros of Polynomials

6. ANS: B

Strategy. Input each function in a graphing calculator and look at the table views to find the values of x when y equals zero.



Answer choice b, enterred as Y_2, has zeros at $x=-4$ and $x=-6$.

PTS: 2 REF: spr1303a1 NAT: A.APR.3 TOP: Zeros of Polynomials

7. ANS: B
Strategy: Factor the trinomials in each equation, then convert the factors into zeros and select the equations that have zeros at -2, 1, and 3.

STEP 1.

I	II	III
$y=(x+2)(x^2-4x-12)$	$y=(x-3)(x^2+x-2)$	$y=(x-1)(x^2-5x-6)$
$y=(x+2)(x-6)(x+2)$	$y=(x-3)(x+2)(x-1)$	$y=(x-1)(x-6)(x+1)$
Zeros at -2, 6, and -2	Zeros at 3, -2, and 1	Zeros at 1, 6, and -1
(Wrong Choice)	(Correct Choice)	(Wrong Choice)

The correct answer choice is *b*.

PTS: 2 REF: 061512AI NAT: A.APR.3 TOP: Zeros of Polynomials

8. ANS: C
Strategy: Convert the zeros to factors.

If the zeros of $f(x)$ are -6 and 5, then the factors of $f(x)$ are $(x+6)$ and $(x-5)$.
Therefore, the function can be written as $f(x)=(x+6)(x-5)$.
The correct answer choice is *c*.

PTS: 2 REF: 061412a1 NAT: A.SSE.3a TOP: Solving Quadratics

QUADRATICS

A.SSE.3a: Factor Quadratic Expressions

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Factoring by Grouping (This method works with any factorable trinomial).

1. Start with a factorable trinomial:
$12x^2+23x+10$
b^2-4ac
$(23)^2-4(12)(10)$
$529-480$
49 - a perfect square #

2. Identify the values of *a*, *b*, and *c*
a=12
b=23
c=10

3. Multiply *a* times *c*.
ac=120 $\quad |ac|=120$

4. Find the factors of $|ac|$

1 120	6 20
2 60	$\boxed{8 \quad 15}$
3 40	10 12
4 30	12 10
5 24	

5. Box the set of factors in step 4 whose sum or difference equals $|b|$

6. Assign a positive or negative value to each factor. Write the signed factors below.
$+\ 8\ +\ 15\ =\ \underset{b}{23}$

7. Replace the middle term of the trinomial with two new terms.
$12x^2+8x+15x+10$

8. Group the new polynomial into two binomials using parentheses..
$(12x^2+8x)+(15x+10)$

9. Factor each binomial. (Note that the factors in parenthesis will always be identical.)
$4x(3x+2)+(15x+10)$
$4x(3x+2)+??(3x+2)$
$4x(3x+2)+5(3x+2)$

10. Extract the common factor and add the remaining terms as a second factor.
$(3x+2)$
$(3x+2)(4x+5)$

11. Check. Use the distributive property of multiplication to make sure that your binomials in Step 10 return you to the trinomial that you started with in Step 1. If so, put a check mark here.
$(3x+2)(4x+5)$
$12x^2+15x+8x+10$
$12x^2+23x+10$

REGENTS PROBLEMS

1. Which equation has the same solutions as $2x^2 + x - 3 = 0$
   a. $(2x - 1)(x + 3) = 0$
   b. $(2x + 1)(x - 3) = 0$
   c. $(2x - 3)(x + 1) = 0$
   d. $(2x + 3)(x - 1) = 0$

2. In the equation $x^2 + 10x + 24 = (x + a)(x + b)$, b is an integer. Find algebraically *all* possible values of b.

3. Solve $8m^2 + 20m = 12$ for m by factoring.

4. The zeros of the function $f(x) = 2x^2 - 4x - 6$ are
   a. 3 and −1
   b. 3 and 1
   c. −3 and 1
   d. −3 and −1

A.SSE.3a: Factor Quadratic Expressions
Answer Section

1. ANS: D

Strategy 1: Factor by grouping.

$$2x^2 + x - 3 = 0$$
$$|ac| = 6$$
Factors of 6 are
1 and 6
2 and 3 (use these)
$$2x^2 + 3x - 2x - 3 = 0$$
$$\left(2x^2 + 3x\right) - (2x + 3) = 0$$
$$x(2x - 3) - 1(2x + 3) = 0$$
$$(x - 1)(2x + 3) = 0$$

Answer choice *d* is correct

Strategy 2: Work backwards by using the disrtibutive property to expand all answer choices and match the expanded trinomials to the function $2x^2 + x - 3 = 0$.

a. $(2x-1)(x+3)=0$ $2x^2+6x-x-3$ $2x^2+5x-3$ (Wrong Choice)	c. $(2x-3)(x+1)=0$ $2x^2+2x-3x-3$ $2x^2-x-3$ (Wrong Choice)
b. $(2x+1)(x-3)=0$ $2x^2-6x+x-3=0$ $2x^2-5x-3=0$ (Wrong Choice)	d. $(2x+3)(x-1)=0$ $2x^2-2x+3x-3=0$ $2x^2+x-3=0$ (Correct Choice)

PTS: 2 REF: 011503a1 NAT: A.SSE.3a TOP: Solving Quadratics

2. ANS:
6 and 4

Strategy: Factor the trinomial $x^2 + 10x + 24$ into two binomials.

$x^2 + 10x + 24$

$(x + \_\_\_)(x + \_\_\_)$

The factors of 24 are:

1 and 24

2 and 12

3 and 8

4 and 6 (use these)

$(x + 4)(x + 6)$

Possible values for *a* and *c* are 4 and 6.

PTS: 2 REF: 081425a1 NAT: A.SSE.3a TOP: Solving Quadratics

3. ANS:
$m = \frac{1}{2}$ and $m = -3$

Strategy: Factor by grouping.

$$8m^2 + 20m = 12$$
$$8m^2 + 20m - 12 = 0$$
$$|ac| = 96$$

The factors of 96 are:

1 and 96

2 and 48

3 and 32

4 and 24 (use these)

$$8m^2 + 24m - 4m - 12 = 0$$
$$\left(8m^2 + 24m\right) - (4m + 12) = 0$$
$$8m(m+3) - 4(m+3) = 0$$
$$(8m-4)(m+3) = 0$$

Use the multiplication property of zero to solve for m.

$8m - 4 = 0$ $8m = 4$ $m = \frac{4}{8}$ $m = \frac{1}{2}$	$m + 3 = 0$ $m = -3$

PTS: 2 REF: fall1305a1 NAT: A.SSE.3a TOP: Solving Quadratics

4. ANS: A

Strategy #1: Solve by factoring:

$$f(x) = 2x^2 - 4x - 6$$

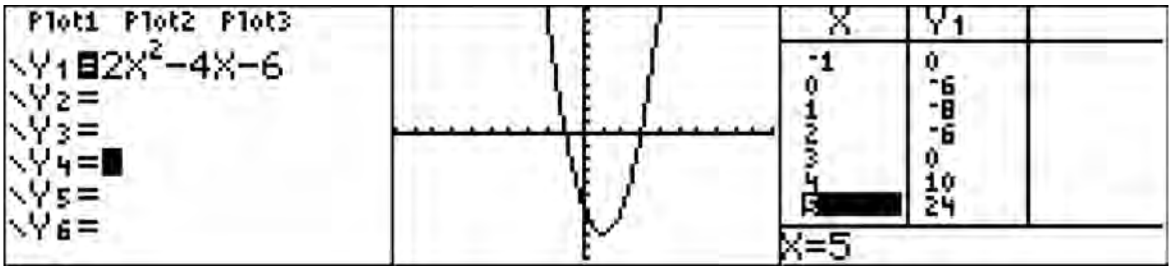
$$0 = 2x^2 - 4x - 6$$

$$0 = 2(x^2 - 2x - 3)$$

$$0 = 2(x - 3)(x + 1)$$

$$x = 3 \;\; and \;\; x = -1$$

Strategy #2: Solve by inputing equation into graphing calculator, the use the graph and table views to identify the zeros of the function.



The graph and table views show the zeros to be at -1 and 3.

PTS: 2 REF: 011609ai NAT: A.SSE.3 TOP: Solving Quadratics
KEY: zeros of polynomials

A.REI.4: Use Appropriate Strategies to Solve Quadratics

Solve equations and inequalities in one variable.

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and completing the square can be used to solve any quadratic.

Summary of the Completing the Square Method of Solving Quadratics

completing the square algorithm A process used to change an expression of the form $ax^2 + bx + c$ into a perfect square binomial by adding a suitable constant.

Source: NYSED Mathematics Glossary

<table>
<tr><th colspan="2">PROCEDURE TO FIND THE ZEROS AND EXTREMES OF A QUADRATIC</th></tr>
<tr><td colspan="2">Start with any quadratic equation of the general form $ax^2 + bx + c = 0$</td></tr>
<tr><td colspan="2">STEP 1
Isolate all terms with x^2 and x on one side of the equation. If $a \neq 1$, divide every term in the equation by a to get one expression in the form of $x^2 + bx$</td></tr>
<tr><td colspan="2">STEP 2
Complete the Square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.</td></tr>
<tr><td colspan="2">STEP 3
Factor the side containing $x^2 + bx + \left(\frac{b}{2}\right)^2$ into a binomial expression of the form
$\left(x + \frac{b}{2}\right)^2$</td></tr>
<tr><td>STEP 4a
(solving for roots and zeros only)
Take the square root of both sides of the equation and simplify,</td><td>STEP 4b
(solving for maxima and minima only)
Multiply both sides of the equation by a.
Move all terms to left side of equation.
Solve the factor in parenthesis for axis of symmetry and x-value of the vertex..
The number not in parentheses is the y-value of the vertes.</td></tr>
</table>

PROCEDURE:	EXAMPLE A	EXAMPLE B
Start with any quadratic equation of the general form $ax^2+bx+c=n$	$x^2+2x+3=4$	$5x^2+2x+3=4$
STEP 1) **Isolate all terms with x^2 and x on one side of the equation.** **If $a\neq 1$, divide every term in the equation by a to get one expression in the form of x^2+bx**	$x^2+2x=1$	$5x^2+2x=1$ $\frac{5x^2}{5}+\frac{2x}{5}=\frac{1}{5}$ $x^2+\frac{2}{5}x=\frac{1}{5}$
STEP 2) **Complete the Square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.**	$b=2,\ \frac{b}{2}=\frac{2}{2}=1,\ \left(\frac{b}{2}\right)^2=(1)^2$ $x^2+2x+(1)^2=1+(1)^2$ $x^2+2x+(1)^2=2$	$b=\frac{2}{5},\ \frac{b}{2}=\frac{1}{5},\ \left(\frac{b}{2}\right)^2=\left(\frac{1}{5}\right)^2$ $x^2+\frac{2}{5}x+\left(\frac{1}{5}\right)^2=\frac{1}{5}+\left(\frac{1}{5}\right)^2$
STEP 3) **Factor the side containing $x^2+bx+\left(\frac{b}{2}\right)^2$ into a binomial expression of the form $\left(x+\frac{b}{2}\right)^2$**	$(x+1)^2=2$	$\left(x+\frac{1}{5}\right)^2=\frac{1}{5}+\left(\frac{1}{5}\right)^2$ $\left(x+\frac{1}{5}\right)^2=\frac{5}{25}+\frac{1}{25}$ $\left(x+\frac{1}{5}\right)^2=\frac{6}{25}$
STEP 4a) **Take the square roots of both sides of the equation and simplify.**	$\sqrt{(x+1)^2}=\sqrt{2}$ $x+1=\pm\sqrt{2}$ $x=\boxed{-1\pm\sqrt{2}}$	$\sqrt{\left(x+\frac{1}{5}\right)^2}=\sqrt{\frac{6}{25}}$ $x+\frac{1}{5}=\pm\frac{\sqrt{6}}{5}$ $x=-\frac{1}{5}\pm\frac{\sqrt{6}}{5}=\boxed{\frac{1\pm\sqrt{6}}{5}}$

STEP 4b **Multiply both sides of the equation by *a*. Move all terms to left side of equation. Solve the factor in parenthesis for axis of symmety and x-value of the vertex. The number not in parentheses is the y-value of the vertes.**	$1(x+1)^2 = 1(2)$ $(x+1)^2 = 2$ $(x+1)^2 - 2 = 0$ vertex form. -1 is the axis of symmetry -2 is the y value of the vertex The vertex is at $(-1,-2)$ $(\boldsymbol{x}+1)^2 = 2$	$5\left(x+\frac{1}{5}\right)^2 = 5\left(\frac{6}{25}\right)$ $5\left(x+\frac{1}{5}\right)^2 = \frac{6}{5}$ $5\left(x+\frac{1}{5}\right)^2 - \frac{6}{5} = 0$ vertex form. $-\frac{1}{5}$ is the axis of symmetry $-\frac{6}{5}$ is the y value of the vertex The vertex is at $\left(-\frac{1}{5},-\frac{6}{5}\right)$

Derivation of the Quadratic Formula:

Given $ax^2 + bx + c = 0$

STEP 1. Isolate the variables

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$\frac{ax^2}{a} + \frac{bx}{a} = \frac{-c}{a}$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

STEP 2. Complete the Square

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

STEP 3. Factor the trinomial.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{4a}{4a}\right)\frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

STEP 4. Take the square roots of both expressions.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Regents Problems

1. Which equation has the same solutions as $x^2 + 6x - 7 = 0$?

   a. $(x+3)^2 = 2$
   b. $(x-3)^2 = 2$
   c. $(x-3)^2 = 16$
   d. $(x+3)^2 = 16$

2. Which equation has the same solution as $x^2 - 6x - 12 = 0$?

   a. $(x+3)^2 = 21$
   b. $(x-3)^2 = 21$
   c. $(x+3)^2 = 3$
   d. $(x-3)^2 = 3$

3. When directed to solve a quadratic equation by completing the square, Sam arrived at the equation $\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$. Which equation could have been the original equation given to Sam?

   a. $x^2 + 5x + 7 = 0$
   b. $x^2 + 5x + 3 = 0$
   c. $x^2 - 5x + 7 = 0$
   d. $x^2 - 5x + 3 = 0$

4. A student is asked to solve the equation $4(3x-1)^2 - 17 = 83$. The student's solution to the problem starts as

   $4(3x-1)^2 = 100$

   $(3x-1)^2 = 25$

   A correct next step in the solution of the problem is

   a. $3x - 1 = \pm 5$
   b. $3x - 1 = \pm 25$
   c. $9x^2 - 1 = 25$
   d. $9x^2 - 6x + 1 = 5$

5. The solution of the equation $(x+3)^2 = 7$ is

a. $3 \pm \sqrt{7}$
b. $7 \pm \sqrt{3}$
c. $-3 \pm \sqrt{7}$
d. $-7 \pm \sqrt{3}$

6. How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.

7. If $4x^2 - 100 = 0$, the roots of the equation are

a. -25 and 25
b. -25, only
c. -5 and 5
d. -5, only

8. If the quadratic formula is used to find the roots of the equation $x^2 - 6x - 19 = 0$, the correct roots are

a. $3 \pm 2\sqrt{7}$
b. $-3 \pm 2\sqrt{7}$
c. $3 \pm 4\sqrt{14}$
d. $-3 \pm 4\sqrt{14}$

9. What are the roots of the equation $x^2 + 4x - 16 = 0$?

a. $2 \pm 2\sqrt{5}$
b. $-2 \pm 2\sqrt{5}$
c. $2 \pm 4\sqrt{5}$
d. $-2 \pm 4\sqrt{5}$

10. What are the solutions to the equation $x^2 - 8x = 24$?

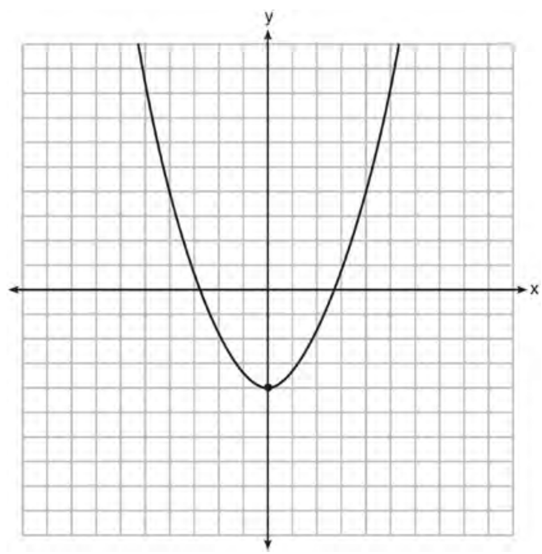
a. $x = 4 \pm 2\sqrt{10}$

b. $x = -4 \pm 2\sqrt{10}$

c. $x = 4 \pm 2\sqrt{2}$

d. $x = -4 \pm 2\sqrt{2}$

11. Ryker is given the graph of the function $y = \frac{1}{2}x^2 - 4$. He wants to find the zeros of the function, but is unable to read them exactly from the graph.



Find the zeros in simplest radical form.

12. Solve the equation $4x^2 - 12x = 7$ algebraically for x.

13. A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below.

$$x^2 + 6x = 13$$

The next step in the student's process was $x^2 + 6x + c = 13 + c$. State the value of c that creates a perfect square trinomial. Explain how the value of c is determined.

14. Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$. Solve for x when $m(x) = 0$.

15. Solve the equation for y: $(y - 3)^2 = 4y - 12$

16. When solving the equation $x^2 - 8x - 7 = 0$ by completing the square, which equation is a step in the process?

a. $(x - 4)^2 = 9$
b. $(x - 4)^2 = 23$
c. $(x - 8)^2 = 9$
d. $(x - 8)^2 = 23$

17. Which equation is equivalent to $y - 34 = x(x - 12)$?

a. $y = (x - 17)(x + 2)$
b. $y = (x - 17)(x - 2)$
c. $y = (x - 6)^2 + 2$
d. $y = (x - 6)^2 - 2$

A.REI.4: Use Appropriate Strategies to Solve Quadratics
Answer Section

1. ANS: D
   Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2 + 6x - 7 = 0$. Equivalent equations expressed in different terms will have the same solutions.

a.	c.
$(x+3)^2 = 2$ $(x+3)(x+3) = 2$ $x^2 + 6x + 9 = 2$ $x^2 + 6x + 7 = 0$ (Wrong Choice)	$(x-3)^2 = 16$ $(x-3)(x-3) = 16$ $x^2 - 6x + 9 = 16$ $x^2 - 6x - 7 = 0$ (Wrong Choice)
b.	**d.**
$(x-3)^2 = 2$ $(x-3)(x-3) = 2$ $x^2 - 6x + 9 = 2$ $x^2 - 6x + 7 = 0$ (Wrong Choice)	$(x+3)^2 = 16$ $(x+3)(x+3) = 16$ $x^2 + 6x + 9 = 16$ $x^2 + 6x - 7 = 0$ (Correct Choice)

PTS: 2 REF: 011517a1 NAT: A.REI.4a TOP: Solving Quadratics
KEY: completing the square

2. ANS: B
Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2 - 6x - 12 = 0$. Equivalent equations expressed in different terms will have the same solutions.

a.	c.
$(x+3)^2 = 21$ $(x+3)(x+3) = 21$ $x^2 + 6x + 9 = 21$ $x^2 + 6x - 12 = 0$ (Wrong Choice)	$(x+3)^2 = 3$ $(x+3)(x+3) = 3$ $x^2 + 6x + 9 = 3$ $x^2 + 6x + 6 = 0$ (Wrong Choice)
b. $(x-3)^2 = 21$ $(x-3)(x-3) = 21$ $x^2 - 6x + 9 = 21$ $x^2 - 6x - 12 = 0$ (Correct Choice)	d. $(x-3)^2 = 3$ $(x-3)(x-3) = 3$ $x^2 - 6x + 9 = 3$ $x^2 - 6x + 6 = 0$ (Wrong Choice)

PTS: 2 REF: 061408a1 NAT: A.REI.4a TOP: Solving Quadratics
KEY: completing the square

3. ANS: D
Strategy: Assume that Sam's equation is correct, then expand the square in his equation and simplify.

$$x^2 - 5x + 3 = 0$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$$
$$\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = \frac{13}{4}$$
$$x^2 - 5x + \frac{25}{4} = \frac{13}{4}$$
$$x^2 - 5x = \frac{13}{4} - \frac{25}{4}$$
$$x^2 - 5x = \frac{-12}{4}$$
$$x^2 - 5x = -3$$
$$x^2 - 5x + 3 = 0$$

PTS: 2 REF: 061518AI NAT: A.REI.4a TOP: Solving Quadratics
KEY: completing the square

4. ANS: A
Strategy: The next step should be to take the square roots of both expressions.

$$(3x-1)^2 = 25$$
$$\sqrt{(3x-1)^2} = \sqrt{25}$$
$$3x-1 = \pm 5$$

The correct answer choice is *a*.

PTS: 2 REF: 061521AI NAT: A.REI.4a TOP: Solving Quadratics
KEY: completing the square

5. ANS: C
Strategy 1: Solve using root operations.

$$(x+3)^2 = 7$$
$$\sqrt{(x+3)^2} = \sqrt{7}$$
$$x+3 = \pm\sqrt{7}$$
$$x = -3 \pm \sqrt{7}$$

Strategy 2. Solve using the quadratic equation.

$$(x+3)^2 = 7$$
$$x^2 + 6x + 9 = 7$$
$$x^2 + 6x + 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$a = 1,\ b = 6,\ c = 2$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{36-8}}{2}$$
$$x = \frac{-6 \pm \sqrt{28}}{2}$$
$$x = \frac{-6 \pm 2\sqrt{7}}{2}$$
$$x = -3 \pm \sqrt{7}$$

PTS: 2 REF: 081523ai NAT: A.REI.4 TOP: Solving Quadratics
KEY: completing the square

6. ANS:
No Real Solutions

Strategy 1. Evaluate the discriminant $b^2 - 4ac$ for $a = 1$, $b = -2$, and $c = 5$.

$b^2 - 4ac$

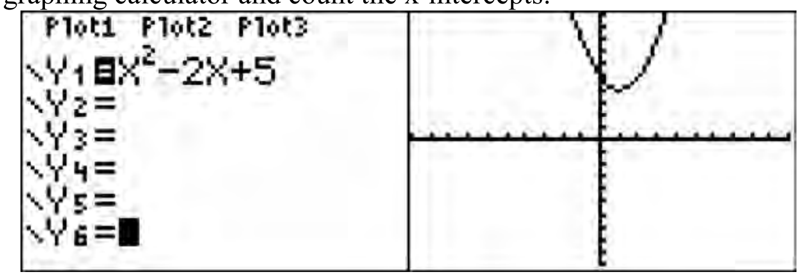
$(-2)^2 - 4(1)(5)$

$4 - 20$

-16

Because the value of the discriminant is negative, there are no real solutions.

Strategy 2.
Input the equation in a graphing calculator and count the x-intercepts.



The graph does not intercept the x-axis, so there are no real solutions.

PTS: 2 REF: 081529ai NAT: A.REI.4 TOP: Using the Discriminant

7. ANS: C
Strategy: Solve using root operations.

$$4x^2 - 100 = 0$$
$$4x^2 = 100$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$

Answer choice *c* is correct.

PTS: 2 REF: 081403a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: taking square roots

8. ANS: A

Strategy: Use the quadratic equation to solve $x^2 - 6x - 19 = 0$, where $a = 1$, $b = -6$, and $c = -19$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-19)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{112}}{2}$$

$$x = \frac{6 \pm \sqrt{16} \cdot \sqrt{7}}{2}$$

$$x = \frac{6 \pm 4\sqrt{7}}{2}$$

$$x = 3 \pm 2\sqrt{7}$$

Answer choice *a* is correct.

PTS: 2 REF: fall1302a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: quadratic formula

9. ANS: B

Strategy 1: Use the quadratic equation to solve $x^2 + 4x - 16 = 0$, where $a = 1$, $b = 4$, and $c = -16$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{80}}{2}$$

$$x = \frac{-4 \pm \sqrt{16}\sqrt{5}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{5}}{2}$$

$$x = -2 \pm 2\sqrt{5}$$

Answer choice *b* is correct.

Strategy 2: Solve by completing the square:

$$x^2 + 4x - 16 = 0$$

$$x^2 + 4x = 16$$

$$(x+2)^2 = 16 + 2^2$$

$$(x+2)^2 = 20$$

$$\sqrt{(x+2)^2} = \sqrt{20}$$

$$x + 2 = \pm 2\sqrt{5}$$

$$x = -2 \pm 2\sqrt{5}$$

Answer choice *b* is correct.

PTS: 2 REF: 061410a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: quadratic formula

10. ANS: A
Strategy 1: Use the quadratic equation to solve $x^2 - 8x = 24$, where $a = 1$, $b = -8$, and $c = -24$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-24)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{160}}{2}$$

$$x = \frac{8 \pm \sqrt{16}\sqrt{10}}{2}$$

$$x = \frac{8 \pm 4\sqrt{10}}{2}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.
Strategy 2. Solve by completing the square.

$$x^2 - 8x = 24$$

$$(x-4)^2 = 24 + (-4)^2$$

$$(x-4)^2 = 24 + 16$$

$$(x-4)^2 = 40$$

$$\sqrt{(x-4)^2} = \sqrt{40}$$

$$x - 4 = \pm 2\sqrt{10}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

PTS: 2 REF: 061523AI NAT: A.REI.4b TOP: Solving Quadratics
KEY: completing the square

11. ANS:
$x = \pm 2\sqrt{2}$

Strategy: Use root operations to solve for x in the equeation $y = \frac{1}{2}x^2 - 4$.

$$\frac{1}{2}x^2 - 4 = 0$$
$$x^2 - 8 = 0$$
$$x^2 = 8$$
$$\sqrt{x^2} = \sqrt{8}$$
$$x = \pm\sqrt{8}$$
$$x = \pm\sqrt{4}\sqrt{2}$$
$$x = \pm 2\sqrt{2}$$

PTS: 2 REF: fall1306a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: taking square roots

12. ANS:

Strategy 1: Solve using factoring by grouping.

$$4x^2 - 12x = 7$$

$$4x^2 - 12x - 7 = 0$$

$$|ac| = 28$$

The factors of 28 are

1 and 28

2 and 14 (use these)

$$4x^2 - 14x + 2x - 7 = 0$$

$$\left(4x^2 - 14x\right) + (2x - 7) = 0$$

$$2x(2x - 7) + 1(2x - 7) = 0$$

$$(2x + 1)(2x - 7) = 0$$

$$x = -\frac{1}{2}$$

$$x = \frac{7}{2}$$

Strategy 2: Solve by completing the square.

$$4x^2 - 12x = 7$$

$$\frac{4x^2}{4} - \frac{12x}{4} = \frac{7}{4}$$

$$x^2 - 3x = \frac{7}{4}$$

$$x^2 - 3x + \left(\frac{-3}{2}\right)^2 = \frac{7}{4} + \left(\frac{-3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{16}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{4}$$

$$x - \frac{3}{2} = \pm 2$$

$$x = \frac{3}{2} \pm 2$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

Strategy 3. Solve using the quadratic formula, where $a = 4$, $b = -12$, and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{8}$$

$$x = \frac{12 \pm \sqrt{256}}{8}$$

$$x = \frac{12 \pm 16}{8}$$

$$x = \frac{3 \pm 4}{2}$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

PTS: 2 REF: 011529a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: factoring

13. ANS:

The value of c that creates a perfect square trinomial is $\left(\frac{6}{2}\right)^2$, which is equal to 9.

The value of c is determined by taking half the value of b, when $a = 1$, and squaring it. In this problem, $b = 6$, so $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$.

PTS: 2 REF: 081432a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: completing the square

14. ANS:
$x = -8$ and $x = -2$

Strategy: Transform the expression $(3x-1)(3-x)+4x^2+19$ to a trinomial, then set the expression equal to 0 and solve it.
STEP 1. Transform $(3x-1)(3-x)+4x^2+19$ into a trinomial.

$$(3x-1)(3-x)+4x^2+19$$
$$9x-3x^2-3+x+4x^2+19$$
$$x^2+10x+16$$

STEP 2. Set the trinomial expression equal to 0 and solve.

$$x^2+10x+16=0$$
$$(x+8)(x+2)=0$$
$$x=-8 \text{ and } -2$$

PTS: 4 REF: 061433a1 NAT: A.REI.4b TOP: Solving Quadratics
KEY: factoring

15. ANS:
The solutions are $y = 3$ and $y = 7$.

$$\left(y-3\right)^2 = 4y-12$$
$$y^2-6y+9 = 4y-12$$
$$y^2-10y+21 = 0$$
$$(y-7)(y-3) = 0$$
$$y-7 = 0$$
$$y = 7$$
$$y-3 = 0$$
$$y = 3$$

PTS: 2 REF: 011627ai NAT: A.REI.4 TOP: Solving Quadratics
KEY: factoring

16. ANS: B

$$x^2-8x-7 = 0$$
$$x^2-8x = 7$$
$$x^2-8x+(-4)^2 = 7+(-4)^2$$
$$x^2-8x+16 = 7+16$$
$$(x-4)^2 = 23$$

PTS: 2 REF: 011614ai NAT: A.REI.4 TOP: Solving Quadratics
KEY: completing the square

17. ANS: D
Strategy: Simplify the equation $y-34 = x(x-12)$.

$$y-34 = x(x-12)$$
$$y-34 = x^2-12x$$
$$y = x^2-12x+34$$
$$y = x^2-12x+36-2$$
$$y = (x-6)^2-2$$

PTS: 2 REF: 011607ai NAT: A.REI.4 TOP: Solving Quadratics
KEY: completing the square

F.IF.8a: Identify Characteristics of Parabolas by Completing the Square

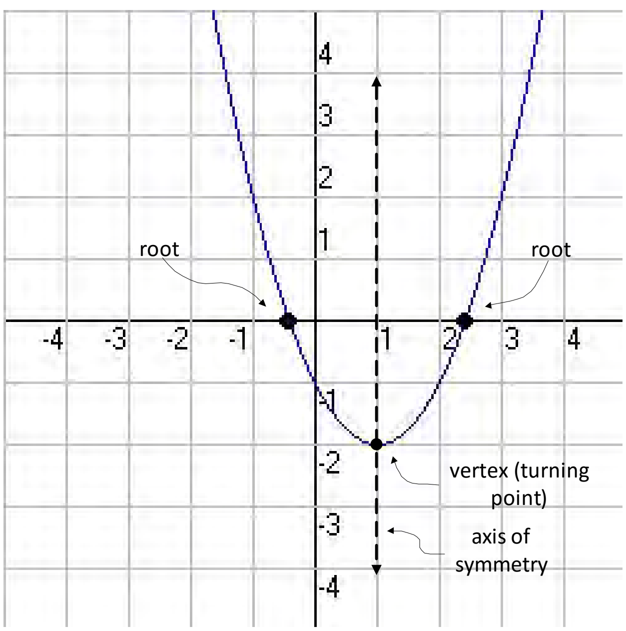
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Big Ideas:

Completing the Square is an efficient method to find the zeros, vertex, and axis of symmetry of a parabola.

The graph of a quadratic equation is called a **parabola**.



- The vertex form of a quadratic function is given by $a(x-h)^2+k=0$, where (h,k) is the vertex of the parabola and $x=h$ is the axis of symmetry.
- The x-value of the point where a parabola touches the x-axis is called:
  - Root
  - Zero
  - Solution
  - X-axis intercept
- Completing the square can be used to find the zeros of a quadratic.

COMPLETE THE SQUARE TO FIND CHARACTERISTICS OF A PARABOLA	
Start with any quadratic equation of the general form $ax^2 + bx + c = 0$	
STEP 1 **Isolate all terms with** x^2 **and** x **on one side of the equation. If** $a \neq 1$**, divide every term in the equation by** a **to get one expression in the form of** $x^2 + bx$	
STEP 2 **Complete the Square by adding** $\left(\frac{b}{2}\right)^2$ **to both sides of the equation.**	
STEP 3 **Factor the side containing** $x^2 + bx + \left(\frac{b}{2}\right)^2$ **into a binomial expression of the form** $\left(x + \frac{b}{2}\right)^2$	
STEP 4a **(solving for roots and zeros only)** **Take the square root of both sides of the equation and simplify,**	**STEP 4b** **(solving for maxima and minima only)** **Multiply both sides of the equation by** a**.** **Move all terms to left side of equation.** **Solve the factor in parenthesis for axis of symmety and x-value of the vertex..** **The number not in parentheses is the y-value of the vertes.**

Being fluent in completing the square makes solving quadratics easy and fast.

EXEMPLAR OF FINDING THE CHARACTERISTICS OF A PARABOLA	
Find the axis of symmetry, vertex, and zeros of $4x^2 - 12x = 7$	
STEP 1 $4x^2 - 12x = 7$ $x^2 - 3x = \frac{7}{4}$	
STEP 2 $x^2 - 3x = \frac{7}{4}$ $x^2 - 3x + \left(-\frac{3}{2}\right)^2 = \frac{7}{4} + \frac{9}{4}$ $x^2 - 3x + \left(-\frac{3}{2}\right)^2 = \frac{16}{4}$	
STEP 3 $\left(x - \frac{3}{2}\right)^2 = \frac{16}{4}$	
STEP 4a **(solving for zeros)** $\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{16}{4}}$ $x - \frac{3}{2} = \pm 2$ $x = \frac{3}{2} \pm 2$ $x = \left\{-\frac{1}{2} \ and \ 3\frac{1}{2}\right\}$ **Solutions are** $-\frac{1}{2}$ **and** $3\frac{1}{2}$	**STEP 4b** **(solving for axis of symmetry & extreme)** $4\left(x - \frac{3}{2}\right)^2 = 4\left(\frac{16}{4}\right)$ $4\left(x - \frac{3}{2}\right)^2 = 16$ $4\left(x - \frac{3}{2}\right)^2 - 16 = 0$ **axis of symmetry is** $+\frac{3}{2}$ **vertex is** $\left(\frac{3}{2}, -16\right)$

REGENTS PROBLEMS

1. The zeros of the function $f(x) = (x+2)^2 - 25$ are
   a. -2 and 5
   b. -3 and 7
   c. -5 and 2
   d. -7 and 3

2. a) Given the function $f(x) = -x^2 + 8x + 9$, state whether the vertex represents a maximum or minimum point for the function. Explain your answer.
   b) Rewrite *f(x)* in vertex form by completing the square.

3. In the function $f(x) = (x-2)^2 + 4$, the minimum value occurs when x is
   a. -2
   b. 2
   c. -4
   d. 4

F.IF.8a: Identify Characteristics of Quadratics
Answer Section

1. ANS: D
   Strategy: Use root operations to solve $f(x) = (x+2)^2 - 25$ for $f(x) = 0$.

$$\begin{aligned} f(x) &= (x+2)^2 - 25 \\ 0 &= (x+2)^2 - 25 \\ 25 &= (x+2)^2 \\ \sqrt{25} &= \sqrt{(x+2)^2} \\ \pm 5 &= x+2 \\ -2 \pm 5 &= x \\ -7 \text{ and } 3 &= x \end{aligned}$$

   PTS: 2 REF: 081418a1 NAT: F.IF.8a TOP: Zeros of Polynomials

2. ANS:
a) The vertex represents a maximum since $a < 0$.
b) $f(x) = -(x-4)^2 + 25$

$$\left.\begin{array}{r} f(x) = -x^2 + 8x + 9 \\ -x^2 + 8x + 9 = 0 \end{array}\right\}\text{(set } f(x) \text{ to 0)}$$

$$\left.\begin{array}{c} -x^2 + 8x = -9 \\ \dfrac{-x^2}{-1} + \dfrac{8x}{-1} = \dfrac{-9}{-1} \\ x^2 - 8x = 9 \end{array}\right\}\text{(isolate both variables with 1 as coefficient of leading variable)}$$

$$\left.\begin{array}{r} \mathrm{x}^2 - 8x + (-4)^2 = 9 + (-4)^2 \\ (x-4)^2 = 9 + 16 \\ (x-4)^2 = 25 \end{array}\right\}\text{(complete the square)}$$

$$\left.\begin{array}{r} -1(x-4)^2 = -1(25) \\ -1(x-4)^2 + 25 = 0 \end{array}\right\}\text{(multiply by a)}$$

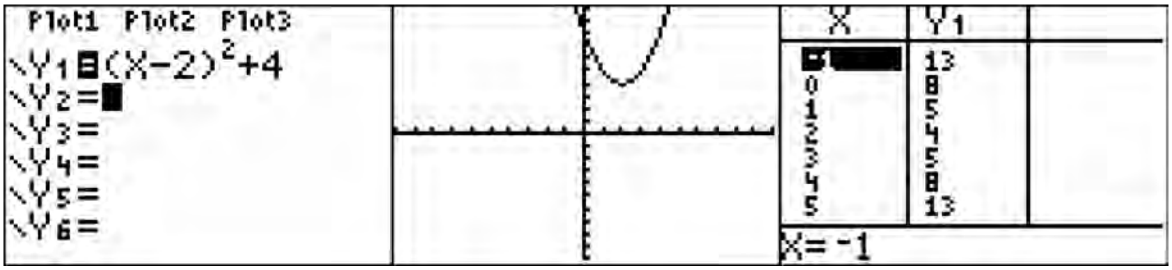
The vertex is at (4,25), but this information is not required by the problem.

PTS: 4 REF: 011536a1 NAT: F.IF.8a TOP: Graphing Quadratic Functions

3. ANS: B

Strategy #1. Recognize that the function $f(x) = (x-2)^2 + 4$ is expressed in vertex form, and that the vertex is located at $(2,4)$. Accordingly, the minimum value of $f(x)$ occurs when $x = 2$.

Strategy #2: Input the function rule in a graphing calculator, then examine the graph and tabler views to determine the vertex. The problem wants to know the x value of the when $f(x)$ is at its minimum.



The minimum value of $f(x) = 4$ when x is equal to 2.

Strategy #3: Substitute each value of x into the equation and determine the minimum value of $f(x)$.

$f(x) = (x-2)^2 + 4$

$f(-2) = (-2-2)^2 + 4$

$f(-2) = (-4)^2 + 4$

$f(-2) = 16 + 4$

$f(-2) = 20$

$f(2) = (2-2)^2 + 4$

$f(2) = (0)^2 + 4$

$f(2) = 4$

$f(-4) = (-4-2)^2 + 4$

$f(-4) = (-6)^2 + 4$

$f(-4) = 36 + 4$

$f(-4) = 40$

$f(4) = (4-2)^2 + 4$

$f(4) = (2)^2 + 4$

$f(4) = 4 + 4$

$f(4) = 8$

PTS: 2 REF: 011601ai NAT: A.SSE.3, jmap F.IF.8A
TOP: Vertex Form of a Quadratic

SYSTEMS

A-REI.5: Solve Systems by Elimination

Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Vocabulary:

A **term** is a number {1,2,3,…}, a variable {x,y,z,a,b,c…}, or the product of a number and a variable {2x, 3y, ½ a, etc.). Terms are separated by + or – signs in an expression, and the + or – signs are part of each term. (Everything inside parenthesis is treated as one term until the parentheses are removed.)

A **variable** is a letter that represents an unknown value(s). When we are asked to solve an equation, it usually means that we must isolate the variable and find its value.

A **coefficient** is a number that comes in front of a variable. A coefficient can be an integer, a decimal, or a fraction. A coefficient multiplies the variable. Every variable has a coefficient. If a variable appears to have no coefficient, it's coefficient is an "invisible 1"

An **expression** is a mathematical statement consisting of one or more terms.

An **equation** is two expressions that have an equal (=) sign between them.

A **solution to a system of equations** is the set of values for each variable that solve all equations in the system simultaneously (at the same time). A system of equations may have one, two, or more solutions.

Big Ideas:

An equation describes a relationship between its terms and expressions. If every term is multiplied or divided by the same factor, the relationship is unchanged.

Cookie Analogy: A cookie recipe describes a relationship between the different ingredients in the cookies. If the amount of every ingredient is multiplied by the same amount, the relationship between the ingredients will be unchanged and the cookies will taste the same. If you want to make twice the number of cookies, you double the recipe by multiplying everything by two. If you want to make three times the number of cookies, you multiply all the ingredients by three. You can make half the number of cookies by dividing all the ingredients by two. The secret is to multiply or divide *everything* by the same number. Your cookies will not be very good if you multiply only some of the ingredients and don't multiply all of the ingredients. The same is true with equations. You can multiply or divide any equation by any number, so long as you multiply or divide every term and expression by the same number, and the relationships between the terms and expressions will be unchanged.

Solving Systems by Elimination

Strategy: Multiply or divide one or both equations so that the coefficients of one variable are the same or opposites. Then, eliminate that variable by adding or subtracting both equations. The result is a new equation with one variable instead of two variables.

EXAMPLE #1:

Solve the following system of equations by elimination.

$$4M + 3C = 12.00$$
$$5C + 6M = 19.00$$

STEP #1 Line up the like terms in columns.

$$3C + 4M = 12.00$$
$$5C + 6M = 19.00$$

STEP #2. Multiply or divide one or both equations to ensure that one of the variables has the same or opposite coefficients. In this example, the C variable has a coefficient of 3 in the first equation and a coefficient of 5 in the second equation, so we can make the coefficient of C be 15 in both equations by multiplying the first equation by 5 and the second equation by 3.

$$5(3C + 4M = 12.50) \rightarrow 15C + 20M = 60.00$$
$$3(5C + 6M = 19.00) \rightarrow 15C + 18M = 57.00$$

STEP #3. After ensuring that one of the variables has the same or opposite coefficients, add or subtract the like terms in the two equations to form a third equation, in which the coefficient of one of the variables is zero. In this example, we will subtract the second equation from the first, as follows:

$$\begin{array}{r} 15C + 20M = 60.00 \\ \underline{-(15C + 18M = 57.00)} \\ 2M = \ \ 3.00 \end{array}$$

Note that, after this step, the new equation has only one variable.

STEP #4. Solve the new equation with one variable.

$$2M = \ \ 3.00$$
$$M = 1.50$$

STEP #5. Substitute the value of the known variable into either equation and solve for the second variable.

$$\begin{aligned} 3C + 4M &= 12.00 \\ 3C + 4(1.50) &= 12.00 \\ 3C + 6.00 &= 12.00 \\ 3C &= 6.00 \\ C &= 2.00 \end{aligned}$$

Step #6. Check your answers in both equations:

$$4M + 3C = 12.00$$
$$4(1.50) + 3(2.00) = 12.00$$
$$6.00 + 6.00 = 12.00$$
$$12.00 = 12.00$$

$$5C + 6M = 19.00$$
$$5(2.00) + 6(1.50) = 19$$
$$10.00 + 9.00 = 19.00$$
$$19.00 = 19.00$$

REGENTS PROBLEMS

1. Which system of equations has the same solution as the system below?

$$2x + 2y = 16$$
$$3x - y = 4$$

a. $2x + 2y = 16$
$6x - 2y = 4$

b. $2x + 2y = 16$
$6x - 2y = 8$

c. $x + y = 16$
$3x - y = 4$

d. $6x + 6y = 48$
$6x + 2y = 8$

2. Which pair of equations could *not* be used to solve the following equations for x and y?

$$4x + 2y = 22$$
$$-2x + 2y = -8$$

a. $4x + 2y = 22$
$2x - 2y = 8$

b. $4x + 2y = 22$
$-4x + 4y = -16$

c. $12x + 6y = 66$
$6x - 6y = 24$

d. $8x + 4y = 44$
$-8x + 8y = -8$

A-REI.5: Prove Equivalent Forms of Systems
Answer Section

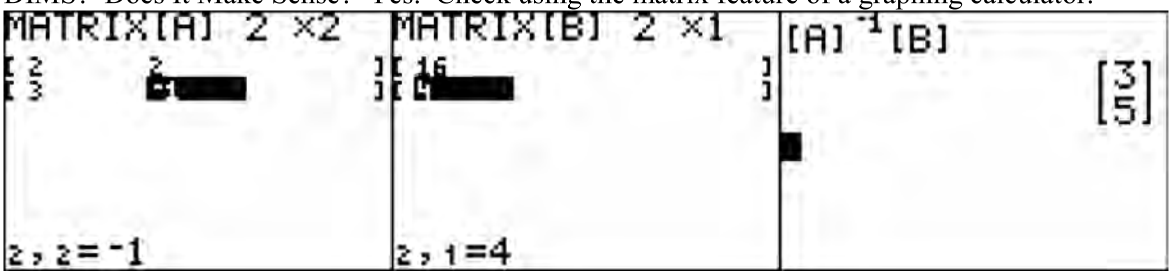
1. ANS: B
Strategy: Find equivalent forms of the system and eliminate wrong answers.

STEP 1. Eliminate answer choices *c* and *d* because the first equation in each system is not a multiple of any equation in the original system.

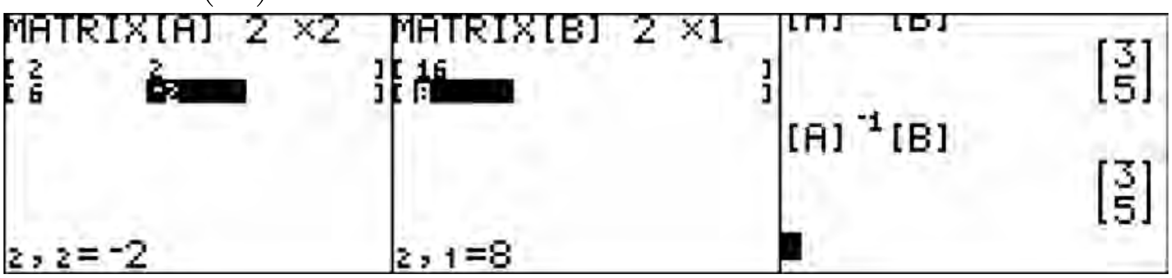
STEP 2. Eliminate answer choice *a* because $6x - 2y = 4$ is not a multiple of $3x - y = 4$.

Choose answer choice *b* as the only remaining choice.

DIMS? Does It Make Sense? Yes. Check using the matrix feature of a graphing calculator.



The solution set $(3,5)$ also works for the system in answer choice *b*.



PTS: 2 REF: 061414a1 NAT: A.REI.5 TOP: Solving Linear Systems

2. ANS: D
Strategy: Eliminate wrong answers by deciding which systems of equations are made of multiples of the original system of equations and which system is made of equations that are not multiples of the orginal system of equations.
Choice (a) is a multiple of the original system of equations.
$$\begin{pmatrix} 4x+2y=22 \\ 2x-2y=8 \end{pmatrix} = \begin{pmatrix} 1(4x+2y=22) \\ -1(-2x+2y=-8) \end{pmatrix}$$
Choice (b) is a multiple of the original system of equations.
$$\begin{bmatrix} 4x+2y=22 \\ -4x+4y=-16 \end{bmatrix} = \begin{bmatrix} 1(4x+2y=22) \\ 2(-2x+2y=-8) \end{bmatrix}$$
Choice (c) is a multiple of the original system of equations.
$$\begin{bmatrix} 12x+6y=66 \\ 6x-6y=24 \end{bmatrix} = \begin{bmatrix} 3(4x+2y=22) \\ -3(-2x+2y=-8) \end{bmatrix}$$
Choice (d) is ***<u>not</u>*** a multiple of the original system of equations.
$$\begin{bmatrix} 8x+4y=44 \\ -8x+8y=-8 \end{bmatrix} \neq \begin{bmatrix} 8x+4y=44 \\ -8x+8y=-8 \end{bmatrix}$$

PTS: 2 REF: 011621ai NAT: A.REI.5 TOP: Solving Linear Systems

A-REI.6: Solve Linear Systems

Solve systems of equations.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

<u>Solutions to systems of equations</u>

Solutions to systems of equations are those values of variables which solve all equations in the system simultaneously (at the same time). A system of equations may have one, two, or more solutions.

EXAMPLE: The system $\begin{aligned} 2x-y&=3 \\ x+y&=3 \end{aligned}$ has a common solution of $(2,1)$.

When $x=2$ and $y=1$, both equations balance, which means both equations are true.

You can verify this by substituting the values $(2,1)$ into both equations.

$$2(\cancel{x}2)-(\cancel{y}1)=3$$
$$4-1=3$$
$$3=3 \ \textit{check}$$
$$(\cancel{x}2)+(\cancel{y}1)=3$$
$$2+1=3$$
$$3=3 \ \textit{check}$$

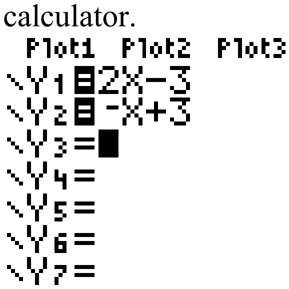
You can also verify this by looking at the graphs of both equations.

STEP #1. Put both equations into slope intercept form.

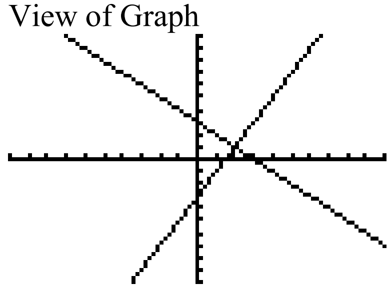
2x - y = 3	x + y = 3
-y=-2x+3	y=-x+3
y=2x-3	

STEP #2. Graph both equations on the same coordinate plane.

Input for transformed equations in a graphing calculator.

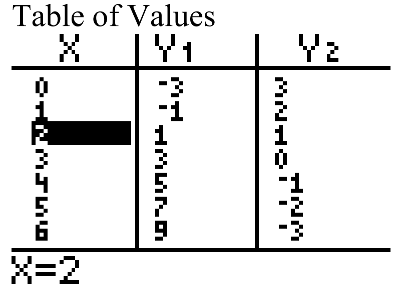


View of Graph



You can see that the graphs of the two equations intersect at $(2,1)$ This is the solution for this system of equations.

Table of Values



You can also see in the table of values that when $x=2$, the value of the dependent variable is the same in both equations.

Elimination Method (See A.REI.5)

Substitution Method
Strategy: Find the easiest variable to isolate in either equation, and substitute its equivalent expression into the other equation. This results in a new equation with only one variable.

EXAMPLE:

Solve the system of equations $\begin{matrix} 3C+4M=12.50 \\ 3C+2M=8.50 \end{matrix}$ by isolating one variable in one equation and substituting its equivalent expression into the other equation.

STEP #1 Isolate one variable in one equation. Normally, you should pick the equation and the variable that seems easiest to isolate.

$$\begin{aligned} &Eq.\#1 \\ 3C+4M&=12.50 \\ 3C&=12.50-4M \\ C&=\frac{12.50-4M}{3} \end{aligned}$$

STEP #2. Substitute the equivalent expression for the variable in the other equation.

$$\begin{aligned} &Eq.\#2 \\ &3\left(\cancel{C}\,\frac{12.50-4M}{3}\right)+2M=8.50 \end{aligned}$$

Note that , after the substitution, equation #2 has only one variable.

STEP #3. Solve the other equation with one variable, which in this case is M.

$$\begin{aligned} &Eq.\#2 \\ \cancel{3}\left(\frac{12.50-4M}{\cancel{3}}\right)+2M&=8.50 \\ 1\left(\frac{12.50-4M}{1}\right)+2M&=8.50 \\ 12.50-4M+2M&=8.50 \\ 12.50-8.50&=4M-2M \\ 4.00&=2M \\ &\boxed{\frac{4.00}{2}=M=2.00} \end{aligned}$$

STEP #4. Substitute the value of the variable you found in the first equation and solve for the second variable.

$$\begin{aligned} &Eq.\#1 \\ 3C + 4\left(\cancel{M}\ 2.00\right) &= 12.50 \\ 3C + 8 &= 12.50 \\ 3C &= 4.50 \\ &\boxed{C = \frac{4.50}{3} = 1.50} \end{aligned}$$

One again, these are the same values you found using the tables method, so you do not have to check them. Normally, you would do a check.

<u>Graphing Method</u>

STEP #1.

Put the equations into slope-intercept form (Y=mx+b) and identify slope (m) and the y-intercept (b).

STEP #2.

Graph both equations on the same coordinate plane. Pick either equation to start.

STEP #3.

Identify the location of the point or points where the two lines intersect. This is the point(s) that makes both equations balance. This is the solution to the system of equations. Write its address on the coordinate plane as an ordered pair, as in (x,y).

STEP #4.

Check your solution by substituting it into the original equations. If both equations balance, you have the correct solution and you are done. If not, find your mistake.

NOTE: Graphing solutions are best performed with the aid of a graphing calculator. Input both equations in the $\boxed{\text{Y=}}$ feature of the TI-83+ and identify the solution in either the graph or table of values views. In the graph view, input

$$\boxed{\text{2nd}}\boxed{\text{calculate}}\boxed{\text{5.intersection}}\boxed{\text{enter}}\boxed{\text{enter}}\boxed{\text{enter}}$$

and the intersection of the two linear equations will appear on the screen.

REGENTS PROBLEMS

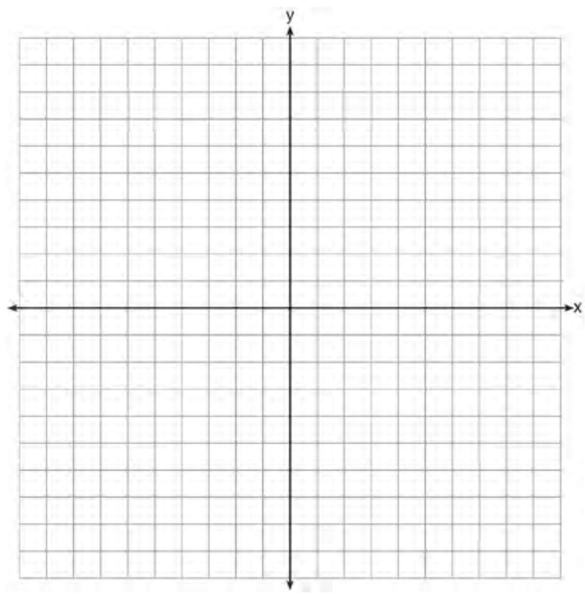
1. Last week, a candle store received \$355.60 for selling 20 candles. Small candles sell for \$10.98 and large candles sell for \$27.98. How many large candles did the store sell?

a. 6 c. 10
b. 8 d. 12

2. Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for \$1.75 per pound and peaches for \$2.50 per pound. If she made \$337.50, how many pounds of peaches did she sell?

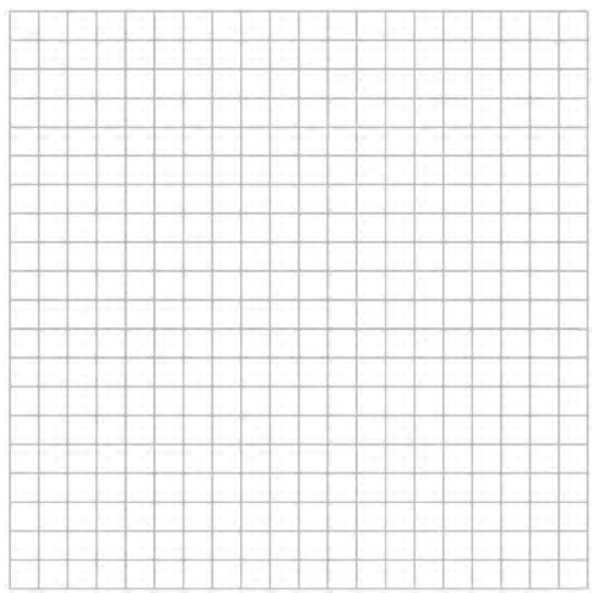
a. 11	c. 65
b. 18	d. 100

3. Next weekend Marnie wants to attend either carnival *A* or carnival *B*. Carnival *A* charges \$6 for admission and an additional \$1.50 per ride. Carnival *B* charges \$2.50 for admission and an additional \$2 per ride.
a) In function notation, write $A(x)$ to represent the total cost of attending carnival *A* and going on *x* rides. In function notation, write $B(x)$ to represent the total cost of attending carnival *B* and going on *x* rides.
b) Determine the number of rides Marnie can go on such that the total cost of attending each carnival is the same. [Use of the set of axes below is optional.]
c) Marnie wants to go on five rides. Determine which carnival would have the lower total cost. Justify your answer.



4. Guy and Jim work at a furniture store. Guy is paid \$185 per week plus 3% of his total sales in dollars, x, which can be represented by $g(x) = 185 + 0.03x$. Jim is paid \$275 per week plus 2.5% of his total sales in dollars, x, which can be represented by $f(x) = 275 + 0.025x$. Determine the value of x, in dollars, that will make their weekly pay the same.

5. A local business was looking to hire a landscaper to work on their property. They narrowed their choices to two companies. Flourish Landscaping Company charges a flat rate of \$120 per hour. Green Thumb Landscapers charges \$70 per hour plus a \$1600 equipment fee. Write a system of equations representing how much each company charges. Determine and state the number of hours that must be worked for the cost of each company to be the same. [The use of the grid below is optional.] If it is estimated to take at least 35 hours to complete the job, which company will be less expensive? Justify your answer.



6. Albert says that the two systems of equations shown below have the same solutions.

First System	Second System
$8x + 9y = 48$ $12x + 5y = 21$	$8x + 9y = 48$ $-8.5y = -51$

Determine and state whether you agree with Albert. Justify your answer.

7. A gardener is planting two types of trees:
   Type *A* is three feet tall and grows at a rate of 15 inches per year.
   Type *B* is four feet tall and grows at a rate of 10 inches per year.
   Algebraically determine exactly how many years it will take for these trees to be the same height.

8. Jacob and Zachary go to the movie theater and purchase refreshments for their friends. Jacob spends a total of \$18.25 on two bags of popcorn and three drinks. Zachary spends a total of \$27.50 for four bags of popcorn and two drinks. Write a system of equations that can be used to find the price of one bag of popcorn and the price of one drink. Using these equations, determine and state the price of a bag of popcorn and the price of a drink, to the *nearest cent.*

9. During the 2010 season, football player McGee's earnings, m, were 0.005 million dollars more than those of his teammate Fitzpatrick's earnings, f. The two players earned a total of 3.95 million dollars. Which system of equations could be used to determine the amount each player earned, in millions of dollars?

a. $m + f = 3.95$
   $m + 0.005 = f$

b. $m - 3.95 = f$
   $f + 0.005 = m$

c. $f - 3.95 = m$
   $m + 0.005 = f$

d. $m + f = 3.95$
   $f + 0.005 = m$

A-REI.6: Solve Linear Systems Algebraically and by Graphing
Answer Section

1. ANS: B
Strategy: Write and solve a system of equations to represent the problem.

Let L represent the number of large candles sold.
Let S represent the number of small candles sold.

STEP 1. Write a system of equations.
Eq. 1 $L + S = 20$
Eq. 2 $\$27.98 \times L + \$10.98 \times S = \$355.60$

STEP 2. Solve the system.

$$L + S = 20$$
$$S = 20 - L$$
$$27.98L + 10.98S = 355.60$$
Substitute
$$27.98L + 10.98(20 - L) = 355.60$$
$$27.98L + 219.6 - 10.98L = 355.60$$
$$17L = 355.60 - 219.6$$
$$17L = 136$$
$$L = \frac{136}{17}$$
$$L = 8$$

DIMS? Does It Make Sense? Yes. If $L = 8$, then $S = 12$, and these values make both equations balance.

Eq. 1 $L + S = 20$	Eq. 2 $\$27.98 \times L + \$10.98 \times S = \$355.60$
$8 + 12 = 20$	$\$27.98 \times 8 + \$10.98 \times 12 = \$355.60$
$20 = 20$	$\$223.84 + \$131.76 = \$355.60$
	$\$355.60 = \$355.60$

PTS: 2 REF: 081510ai NAT: A.REI.6 TOP: Modeling Linear Systems

2. ANS: C
Strategy: Write and solve a system of equations to represent the problem.

Let a represent the number pounds of apples sold.
Let p represent the number of pounds of peaches sold.

STEP 1. Write a system of equations.
Eq. 1 $a + p = 165$
Eq. 2 $\$1.75a + \$2.50p = \$337.50$

STEP 2. Solve the system.

$$Eq.1 \qquad a + p = 165$$

$$Eq.2 \qquad 1.75a + 2.5p = 337.50$$

Multiply Eq. 1 by 1.75

$$Eq.1a \qquad 1.75\,a + 1.75p = 1.75(165)$$

Subtract Eq.1a from Eq.2

$$.75p = 337.5 - 1.75(165)$$

$$.75p = 48.75$$

$$p = \frac{48.75}{.75}$$

$$p = 65$$

DIMS? Does It Make Sense? Yes. If $p = 65$, then $a = 100$, and these values make both equations balance.

Eq. 1		Eq. 2	
	$a + p = 165$		$\$1.75a + \$2.50p = \$337.50$
	$100 + 65 = 165$		$\$1.75(100) + \$2.50(65) = \$337.50$
	$165 = 165$		$\$175.00 + \$162.50 = \$337.50$
			$\$337.50 = \$337.50$

PTS: 2 REF: 061506AI NAT: A.REI.6 TOP: Solving Linear Systems

3. ANS:
a) $A(x) = 1.50x + 6$

$B(x) = 2x + 2.50$

b) The total costs are the same if Marnie goes on 7 rides.
c) Carnival *B* has the lower cost for admission and 5 rides. Carnival B costs $12.50 for admission and 5 rides and Carnival A costs $13.50 for admission and 5 rides.

Strategy: Write a system of equations, then input it into a graphing calculator and use it to answer parts *b* and *c* of the problem.

STEP 1. Write a system of equations.

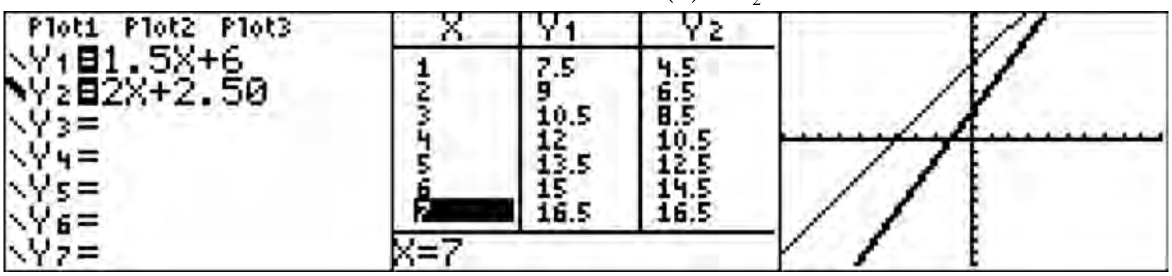
$$A(x) = 1.50x + 6$$

$$B(x) = 2x + 2.50$$

STEP 2. Input the system into a graphing calculator.

Let $A(x) = Y_1$

Let $B(x) = Y_2$



STEP 3. Use the different views of the function to answer parts b and c of the problem.
Part a) The total costs are the same at 7 rides.
Part b) Carnival B costs $12.50 for admission and 5 rides and Carnival A costs $13.50 for admission and 5 rides, so Carnival B has the lower total cost.

PTS: 6 REF: spr1308a1 NAT: A.REI.6 TOP: Modeling Linear Systems

4. ANS:
$18,000

Strategy: Set both function equal to one another and solve for *x*.

STEP 1. Set both functions equal to one another.

$$g(x) = 185 + 0.03x$$

$$f(x) = 275 + 0.025x$$

$$185 + 0.03x = 275 + 0.025x$$

$$0.03x - 0.025x = 275 - 185$$

$$0.005x = 90$$

$$x = 18{,}000$$

PTS: 2 REF: 081427a1 NAT: A.REI.6 TOP: Solving Linear Systems

5. ANS:
a) $F(x) = 120x$

$G(x) = 70x + 1600$

b) The costs will be the same when 32 hours are worked.
c) If the job takes at least 35 hours, Green Thumb Landscapers will be less expensive.

Strategy: Write a system of equations, then set both equations equal to one another and solve for x, then answer the questions

STEP 1. Write a system of equations.
Let x represent the number of hours worked.
Let $F(x)$ represent the total costs of Flourish Landscape Company.
Let $G(x)$ represent the total costs of Green Thumb Landscapers.
Write: $F(x) = 120x$

$G(x) = 70x + 1600$

STEP 2. Set both functions equal to one another to find the break even hours..

$$F(x) = 120x$$

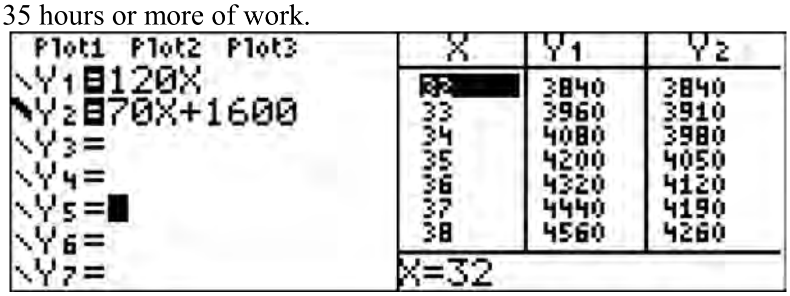
$$G(x) = 70x + 1600$$

$$120x = 70x + 1600$$

$$50x = 1600$$

$$x = 32$$

STEP 3. Input the equations into a graphing calculator to verify the break even amount and determine which company is cheaper for 35 hours or more of work.



Green Thumb is less expensive.

PTS: 6 REF: fall1315a1 NAT: A.REI.6 TOP: Modeling Linear Systems

6. ANS:

Albert is correct. Both systems have the same solution $\left(\frac{-3}{4},6\right)$.

Strategy: Solve one system of equations, then test the solution in the second system of equations.

STEP 1. Solve the first system of equations.

$$Eq.\ 1\ \ 8x+9y=48$$

$$Eq.\ 2\ \ 12x+5y=21$$

Multiply Eq.1 by 3 and Multiply Eq. 2 by 2.

Then solve for the first variable

$$24x+27y=144$$

$$\underline{24x+10y=42}$$

$$17y=102$$

$$y=6$$

Solve for the second variable.

$$8x+9(6)=48$$

$$8x=-6$$

$$x=-\frac{3}{4}$$

The solution is $\left(\frac{-3}{4},6\right)$

STEP 2: Test the second system of equations using the same solution set.

$8x+9y=48$	$-8.5y=-51$
$8\left(\frac{-3}{4}\right)+9(6)=48$	$-8.5(6)=-51$
$-6+54=48$	$-51=-51$
$48=48$	

DIMS? Does It Make Sense? Yes. The solution $\left(\frac{-3}{4},6\right)$ makes both equations balance.

PTS: 4 REF: 061533AI NAT: A.REI.6 TOP: Solving Linear Systems

7. ANS:
2.4 years

Strategy: Convert all measurements to inches per year, then write two equations, then write and so;lve a new equation from the right expressions of the two equations.

STEP 1: Convert all measurements to inches per year.
Type A is 36 inches tall and grows at a rate of 15 inches per year.
Type B is 48 inches tall and grows at a rate of 10 inches per year.

STEP 2: Write 2 equations

$$G(A) = 36 + 15t$$
$$G(B) = 48 + 10t$$

STEP 3: Write and solve a break-even equation from the right expressions.

$$36 + 15t = 48 + 10t$$
$$15t - 10t = 48 - 36$$
$$5t = 12$$
$$t = \frac{12}{5}$$
$$t = 2.4 \text{ years}$$

DIMS? Does It Make Sense? Yes. After 2.4 years, the type A trees and the type B trees will both be 72 inches tall.

$$G(A) = 36 + 15(2.4) = 36 + 36 = 72$$
$$G(B) = 48 + 10(2.4) = 48 + 24 = 72$$

PTS: 2 REF: 011531a1 NAT: A.REI.6 TOP: Modeling Linear Equations
NOT: NYSED classifies this problem as A.CED.1: Create Inequations and Inequalities

8. ANS:

a) $18.25 = 2p + 3d$

$27.50 = 4p + 2d$

b) Drinks cost \$2.25 and popcorn costs \$5.75

Strategy: Write one equation for Jascob and one equation for Zachary, then solve them as a system of equations.

STEP 1: Write 2 equations.

$$\underbrace{18.25 = \qquad +2p \qquad +3d}_{\text{Jacob spends a total of \$18.25 on two bags of popcorn and three drinks}}$$

$$18.25 = 2p + 3d$$

$$\underbrace{27.50 = \qquad +4p \qquad +2d}_{\text{Zachary spends a total of \$27.50 for four bags of popcorn and two drinks.}}$$

$$27.50 = 4p + 2d$$

STEP 2. Solve both equations as a system of equations.

$$\begin{array}{ll}
\text{Eq.1} & 18.25 = 2p + 3d \\
\text{Eq.2} & 27.50 = 4p + 2d \\
 & \text{Rewrite both equations} \\
\text{Eq.1} & 2p + 3d = 18.25 \\
\text{Eq.2} & 4p + 2d = 27.50 \\
 & \text{Multiply Eq.1 by 2} \\
\text{Eq.1}a & 4p + 6d = 36.50 \\
\text{Eq.2} & 4p + 2d = 27.50 \\
 & \text{Subtract Eq.2 from Eq1a} \\
\text{Eq.3} & 4d = 9.00 \\
 & d = \$2.25 \\
 & \text{Substitute 2.25 for } d \text{ in Eq.1} \\
\text{Eq.1} & 18.25 = 2p + 3d \\
\text{Eq.1} & 18.25 = 2p + 3(2.25) \\
\text{Eq.1} & 18.25 = 2p + 6.75 \\
\text{Eq.1} & 18.25 - 6.75 = 2p \\
\text{Eq.1} & 11.50 = 2p \\
\text{Eq.1} & \$5.75 = p
\end{array}$$

Drinks cost \$2.25 and popcorn costs \$5.75

DIMS? Does It Make Sense? Yes. Both equations balance if drinks cost \$2.25 and popcorn costs \$5.75, as shown below:

Eq.1 $\quad 18.25 = 2p + 3d$

Eq.2 $\quad 27.50 = 4p + 2d$

Substitute and Solve

Eq.1
$$\begin{aligned} 2(5.75) + 3(2.25) &= 18.25 \\ 11.50 + 6.75 &= 18.25 \\ 18.25 &= 18.25 \end{aligned}$$

Eq.2
$$\begin{aligned} 4(5.75) + 2(2.25) &= 27.50 \\ 23.00 + 4.50 &= 27.50 \\ 27.50 &= 27.50 \end{aligned}$$

PTS: 2 REF: 011533a1 NAT: A.REI.6 TOP: Modeling Linear Systems
NOT: NYSED classifies this problem as A.CED.2

9. ANS: D
Strategy: Eliminate wrong answers and choose between the remaining choices..

The problem states that McGee (*m*) and Fitzpatrick's (*f*) combined earning were 3.95 million dollars. This can be represented mathematically as $m+f=3.95$. Eliminate answer choices b and c because they state that $m-f=3.95$, which is the difference of their salaries, not the sum.

Choose between answer choices *a* and *d*. Choice a says that Fitzpatrick (*f*) makes more. Choice *d* says that McGee (*m*) makes more. The problem states that McGee (*m*) makes more, so choice *d* is the correct answer.

DIMS? Does It Make Sense? Yes. Solve the system in answer choice D using the substitution method, as follows:

$$\text{Eq.1} \quad m+f=3.95$$

$$\text{Eq.2} \quad f+0.005=m$$

$$\text{Substitute } \left(f+0.005\right) \text{ for } m \text{ in Eq. 1}$$

$$\left(f+0.005\right)+f=3.95$$

$$2f+0.005=3.95$$

$$2f=3.95-0.005$$

$$2f=3.945$$

$$f=\frac{3.945}{2}$$

$$f=1.9725 \text{ million dollars}$$

Fitzpatrick earns \$1,972,500 and McGee earns $\$3{,}950{,}000-\$1{,}972{,}500=\$1{,}977{,}500$, which is $\$1{,}977{,}500-\$1{,}972{,}500=\$5{,}000$ more than Fitzpatrick. \$5,000 is 0.005 million dollars, so everything agrees with the information contained in the problem.

PTS: 2 REF: 081419a1 NAT: A.REI.6 TOP: Modeling Linear Systems
NOT: A.CED.2

A.REI.7: Solve Quadratic-Linear Systems

Solve systems of equations.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Big Idea

Two or more equations together form a **system of equations**. Any solution common to all equations is a **solution of the system**. When one or more of the equations is a quadratic, the system is called a non-linear system of equations. Non-linear systems of equations are solved in the same way as linear systems, except there can be more than one solution.

Sample Regents Problem

Solve the following system of equations algebraically.

$$y = x^2 + 4x - 2$$
$$y = 2x + 1$$

One Solution:

Since both equations have y isolated, the two polynomial expressions can be set equal to one another.

$$x^2 + 4x - 2 = 2x + 1$$
$$x^2 + 4x - 2x - 2 - 1 = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x + 3)(x - 1) = 0$$
$$x = -3 \; and \; x = 1$$

Substitute the values of x into the simpler question to find the corresponding values of y.

When $x = -3$, $y = -5$

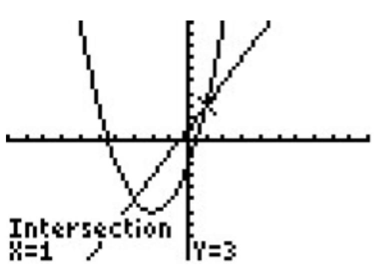
$$y = 2x + 1$$
$$y = 2(-3) + 1$$
$$y = -6 + 1$$
$$y = -5$$

When $x = 1$, $y = 3$

$$y = 2x + 1$$
$$y = 2(1) + 1$$
$$y = 2 + 1$$
$$y = 3$$

The solutions are (-3,-5) and (1,3)

Check by inputting the two original equations into a graphing calculator and visually inspect the points of intersection.

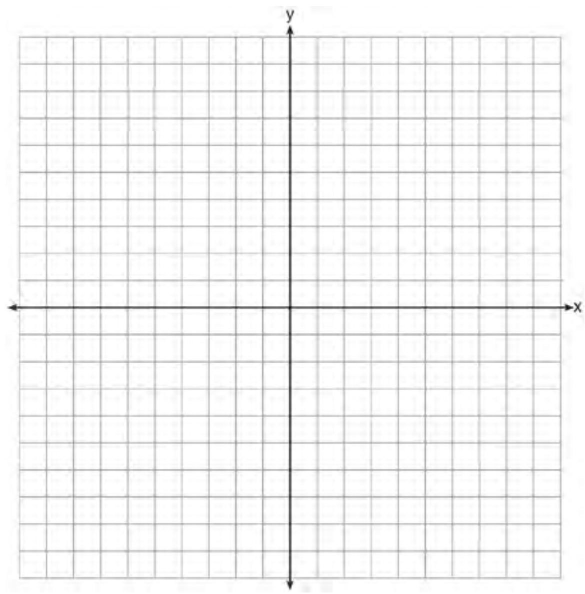


X	Y1	Y2
-3	-5	-5
-2	-6	-3
-1	-5	-1
0	-2	1
1	3	3
2	10	5
3	19	7

X= -3

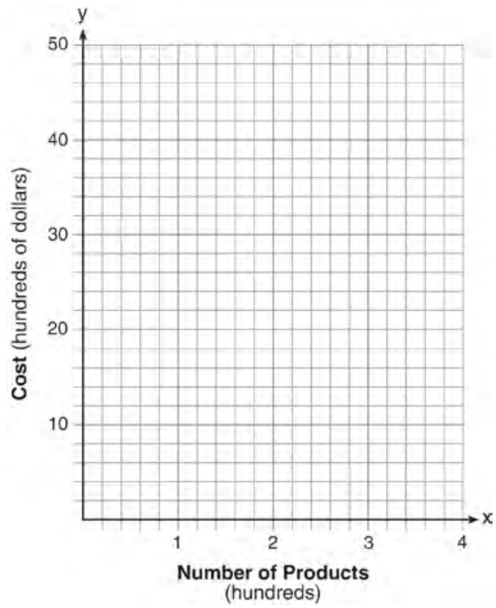
REGENTS PROBLEMS

1. Let $f(x) = -2x^2$ and $g(x) = 2x - 4$. On the set of axes below, draw the graphs of $y = f(x)$ and $y = g(x)$.



Using this graph, determine and state *all* values of x for which $f(x) = g(x)$.

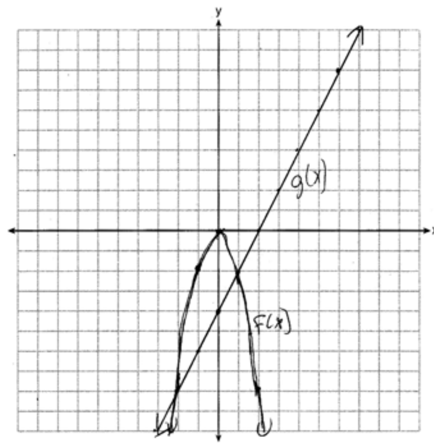
2. A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x)=3x^2$ while the production cost at site B is $B(x)=8x+3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*. Graph the production cost functions on the set of axes below and label them site A and site B.



State the positive value(s) of x for which the production costs at the two sites are equal. Explain how you determined your answer. If the company plans on manufacturing 200 products per week, which site should they use? Justify your answer.

A.REI.7: Solve Quadratic-Linear Systems Algebraically and by Graphing
Answer Section

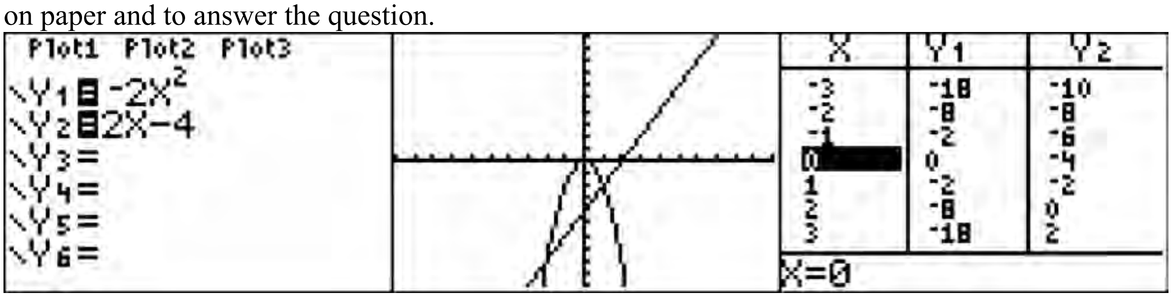
1. ANS:



a)

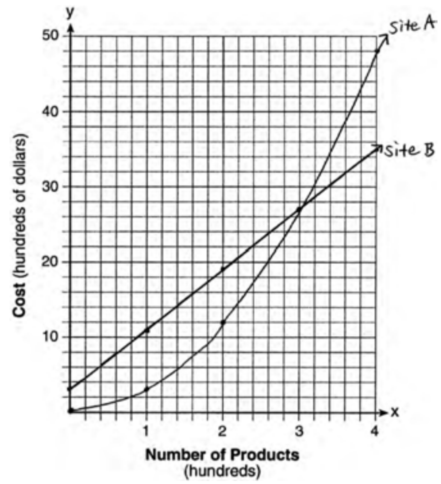
b) $f(x) = g(x)$ when $x = -2$ and $x = 1$.

Strategy: Input both functions into a graphing calculator and use the table and graph views to construct the graph on paper and to answer the question.



PTS: 4 REF: 081435a1 NAT: A.REI.7 TOP: Quadratic-Linear Systems

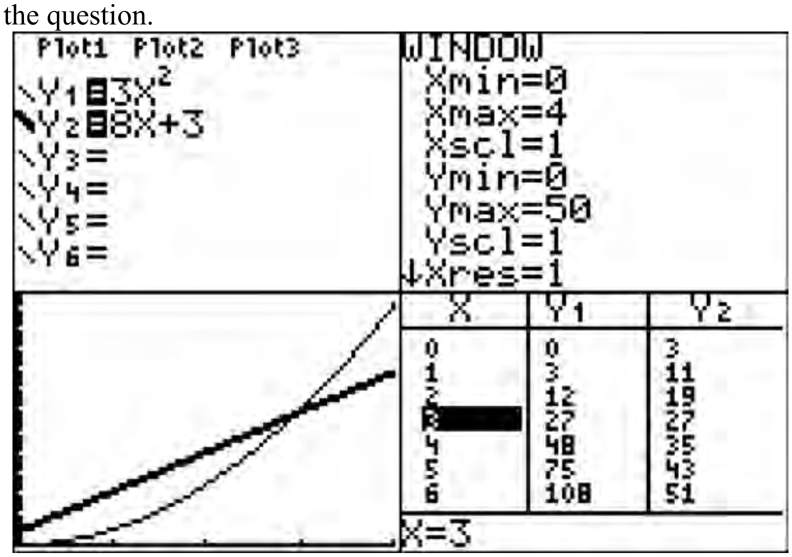
2. ANS:



a)

b) The graphs of the production costs are equal when $x = 3$.
c) The company should use Site A, because the costs of Site A are lower when $x = 2$.

Strategy: Input both functions into a graphing calculator and use the table and graph views to construct the graph on paper and to answer the question.



PTS: 6 REF: 061437a1 NAT: A.REI.7 TOP: Quadratic-Linear Systems

A.REI.11: Find and Explain Solutions to Systems

Represent and solve equations and inequalities graphically.

11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

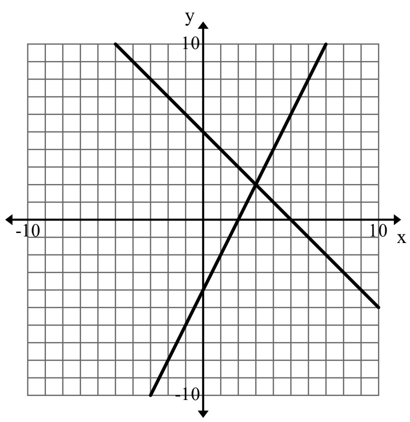
Big Ideas:

A **solution of a system** of equations makes each equation in the system true. Solutions can be found using three different views of a function.

Example: If $f(x) = -x+5$ and $g(x) = 2x-4$, then $f(3) = g(3)$

Graph View

of a Solution to a System of Equations



A solution occurs when the graphs of equations intersect.

Table View

of a Solution to System of Equations

X	Y1	Y2
0	-4	5
1	-2	4
2	0	3
3	2	2
4	4	1
5	6	0
6	8	-1

X=3

A solution occurs when one value of x creates the same value of y in all equations.

Note:

$Y_1 = f(x) = -x+5$

$Y_2 = g(x) = 2x-4$

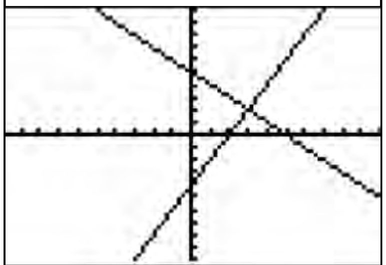
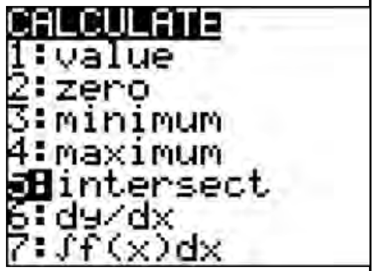
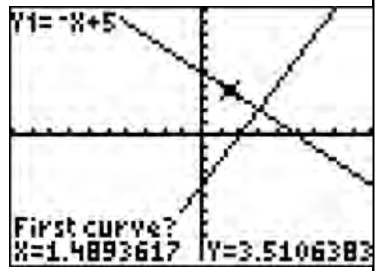
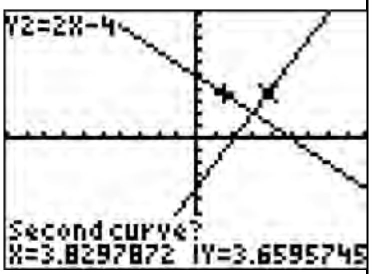
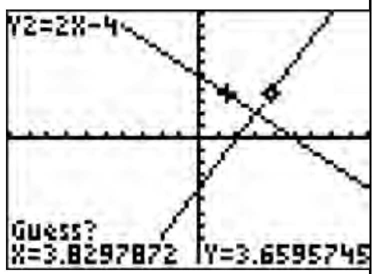
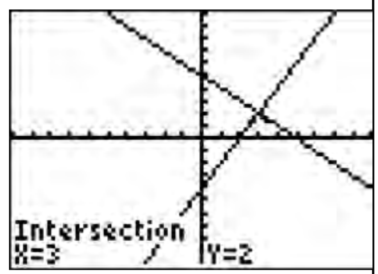
Function Rule View

of a Solution to a System of Equations

$$f(x) = -x+5$$
$$g(x) = 2x-4$$
$$f(x) = g(x)$$
$$-x+5 = 2x-4$$
$$9 = 3x$$
$$3 = x$$
$$f(3) = g(3)$$

A solution occurs when $f(x) = g(x)$ for a specific value(s) of x.

Using the TI-84 family of graphing calculators to calculate the intersection of a graph.

1	2	3
		
Start in the graph view of the system.	Select the 2nd - Calc - Intersect function. Press enter.	Move the cursor to the left of the intersection. Press enter.
4	5	6
		
Move the cursor to the right of the intersection. Press enter.	Press enter in response to Guess?	The x and y coordinates of the intersection will appear.

REGENTS PROBLEMS

1. Two functions, $y = |x-3|$ and $3x+3y=27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?
   a. $(3,0)$ is the solution to the system because it satisfies the equation $y = |x-3|$.
   b. $(9,0)$ is the solution to the system because it satisfies the equation $3x+3y=27$.
   c. $(6,3)$ is the solution to the system because it satisfies both equations.
   d. $(3,0)$, $(9,0)$, and $(6,3)$ are the solutions to the system of equations because they all satisfy at least one of the equations.

2. If $f(x) = x^2 - 2x - 8$ and $g(x) = \frac{1}{4}x - 1$, for which value of x is $f(x) = g(x)$?

a. -1.75 and -1.438
b. -1.75 and 4
c. -1.438 and 0
d. 4 and 0

3. John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, x, will they have the same amount of money saved? Explain how you arrived at your answer.

4. Given the functions $h(x) = \frac{1}{2}x + 3$ and $j(x) = |x|$, which value of x makes $h(x) = j(x)$?

a. -2
b. 2
c. 3
d. -6

A.REI.11: Find and Explain Solutions of Systems
Answer Section

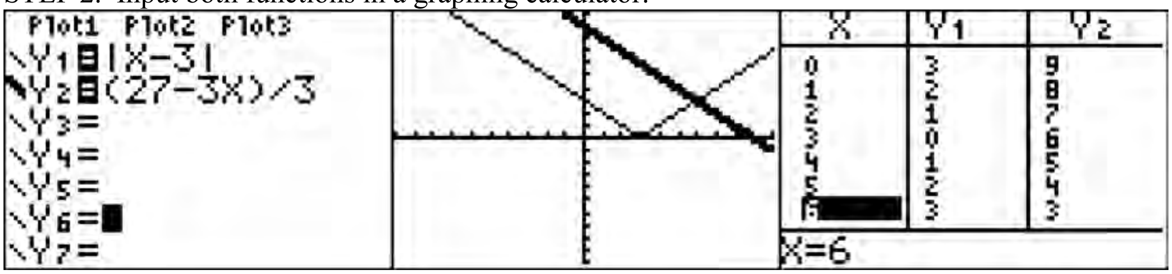
1. ANS: C
Strategy: Input both functions in a graphing calculator, then use the table and graph views of the function to select the correct answer.
STEP 1. Transpose the second function for input into a graphing calculator.

$$3x + 3y = 27$$

$$3y = 27 - 3x$$

$$y = \frac{27 - 3x}{3}$$

STEP 2. Input both functions in a graphing calculator.



When $x = 6$, the value of y in both equations is 3. (6, 3) is the solution to this system.

PTS: 2 REF: 011518a1 NAT: A.REI.11 TOP: Nonlinear Systems

2. ANS: B
Strategy: Set both expressions equal to one another and solve for x.

$$f(x) = x^2 - 2x - 8 \;\; and \;\; g(x) = \frac{1}{4}x - 1$$

$$Let\ f(x) = g(x)$$

$$x^2 - 2x - 8 = \frac{1}{4}x - 1$$

$$4x^2 - 8x - 32 = x - 4$$

$$4x^2 - 9x - 28 = 0$$

$$(4x + 7)(x - 4) = 0$$

$$x = -\frac{7}{4} \;\; and \;\; x = 4$$

$$f(-1.75) = g(-1.75)$$

$$and$$

$$f(4) = g(4)$$

PTS: 2 REF: 081517ai NAT: A.REI.11 TOP: Quadratic-Linear Systems

3. ANS:
John and Sarah will have the same amount of money saved at 7 weeks. I set the expressions representing their savings equal to each other and solved for the positive value of x by factoring.

Strategy: Set the expressions representing their savings equal to one another and solve for x.

$$f(x) = 60 + 5x \;\; and \;\; g(x) = x^2 + 46$$

$$Let \; f(x) = g(x)$$

$$x^2 + 46 = 60 + 5x$$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7$$

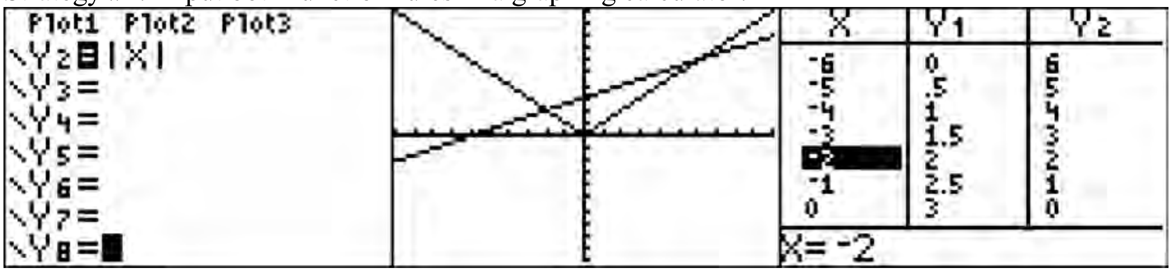
DIMS? Does It Make Sense? Yes. After 7 weeks, John and Sarah will each have $95.00.

John's Savings	Sarah's Savings
$f(x) = 60 + 5x$ $f(7) = 60 + 5(7)$ $f(7) = 60 + 35$ $f(7) = 95$	$g(x) = x^2 + 46$ $g(7) = (7)^2 + 46$ $g(7) = 49 + 46$ $g(7) = 95$

PTS: 2 REF: 061527AI NAT: A.REI.11 TOP: Quadratic-Linear Systems

4. ANS: A

Strategy #1: Input both function rules in a graphing calculator.



Strategy #2: Set the right expressions of both functions equal to one another. Then solve for the positive and negative values of $|x|$.

$$\frac{1}{2}x+3=|x|$$

$\frac{1}{2}x+3=x$ $x+6=2x$ $6=x$	$-\left(\frac{1}{2}x=3\right)=x$ $-\frac{1}{2}x-3=x$ $-x-6=2x$ $-6=3x$ $-2=x$

Check:

$h(x)=\frac{1}{2}x+3$ $h(-2)=\frac{1}{2}(-2)+3$ $h(-2)=-1+3$ $h(-2)=2$	$j(x)=\|x\|$ $j(-2)=\|-2\|$ $j(x)=2$

PTS: 2 REF: 011617ai NAT: A.REI.11 TOP: Other Systems

A.REI.12: Graph Systems of Inequalities

Represent and solve equations and inequalities graphically.

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Big Idea:

A linear inequality describes a region of the coordinate plane that has a **boundary line**. Every point in the region is a **solution of the inequality**.

Two or more linear inequalities together form a **system of linear inequalities**. Note that there are two or more boundary lines in a system of linear inequalities.
A **solution of a system of linear inequalities** makes each inequality in the system true. The graph of a system shows all of its solutions.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

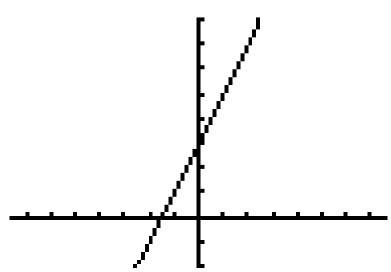
- When the inequality sign **contains** an equality bar beneath it, use a solid line for the boundary.
- When the inequality sign **does not contain** an equality bar beneath it, use a dashed or dotted line for the boundary

Step Two. Restore the inequality sign and test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. However, if the boundary line passes through the point (0,0), another point not on the boundary line must be selected for testing.

- If the test point makes the inequality true, shade the side of the boundary line that includes the test point.
- If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.

Example Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$. This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.



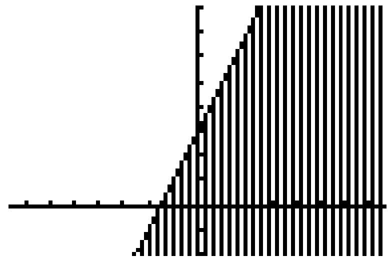
Next, **test a point** to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ True, so the solution of the inequality is the region that contains the point $(0,0)$.

Therefore, we shade the side of the boundary line that contains the point (0,0).



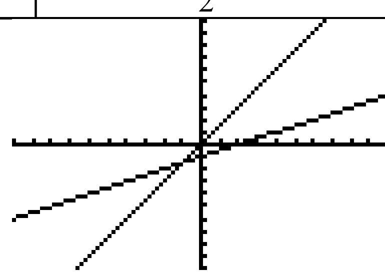
Note: The TI-83+ graphing calculator does not have the ability to distinguish between solid and dashed lines on a graph of an inequality. The less than and greater than symbols are input using the far-left column of symbols that can be accessed through the $\boxed{\text{Y=}}$ feature.

Graphing a System of Linear Inequalities. Systems of linear inequalities are graphed in the same manner as systems of equations are graphed. The solution of the system of inequalities is the region of the coordinate plane that is shaded by both inequalities.

Example: Graph the system: $\begin{array}{l} 4y \geq 6x \\ -3x + 6y \leq -6 \end{array}$

First, convert both inequalities to slope-intercept form and graph.

$4y \geq 6x$	$-3x + 6y \leq -6$
$\frac{4y}{4} \geq \frac{6x}{4}$	$6y \leq -6 + 3x$
	$6y \leq 3x - 6$
$y \geq \frac{3}{2}x$	$D_2 \quad \frac{6y}{6} \leq \frac{3x}{6} - \frac{6}{6}$
m=$\frac{3}{2}$, b=0	$y \leq \frac{1}{2}x - 1$
	m=$\frac{1}{2}$, b=-1



Next, test a point in each inequality and shade appropriately.

- Since point (0,0) is on the boundary line of $y \geq \frac{3}{2}x$, select another point, such as (0,1).

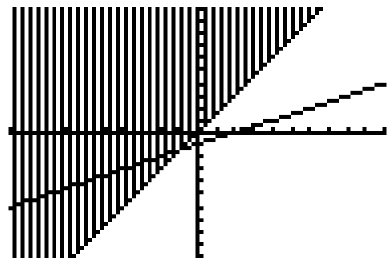
  $y \geq \frac{3}{2}x$

  Test (0,1)

  $1 \geq \frac{3}{2}(0)$

  $1 \geq 0$ This is true, so the point (0,1) is in the solution set of this inequality.

  Therefore, we shade the side of the boundary line that includes point (0,1).



- Since (0,0) is not on the boundary line of $y \leq \frac{1}{2}x - 1$, we can use (0,0) as our test point, as follows:

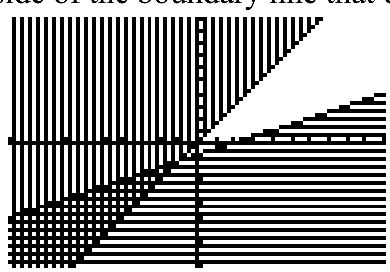
  $y \leq \frac{1}{2}x - 1$

  Test (0,0)

  $0 \leq \frac{1}{2}(0) - 1$

  $0 \leq -1$ This is not true, so the point (0,0) is not in the solution set of this inequality.

  We therefore must shade the side of the boundary line that does not incude the point (0,0).



Note that the system of inequalities divides the coordinate plane into four sections. The solution set for the system of inequalities is the area where the two shaded regions overlap.

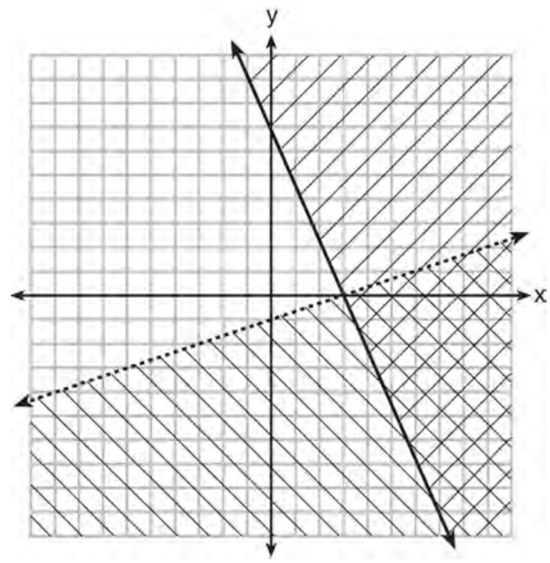
Remember The Big Rule for Solving Inequalities:

All the rules for solving equations apply to inequalties – plus one:

When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.

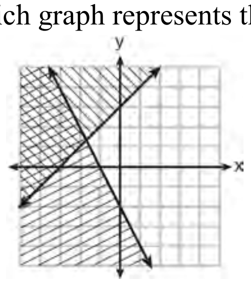
REGENTS PROBLEMS

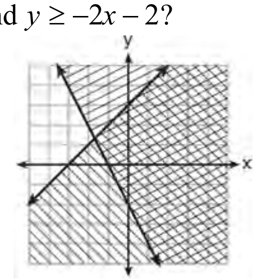
1. What is one point that lies in the solution set of the system of inequalities graphed below?

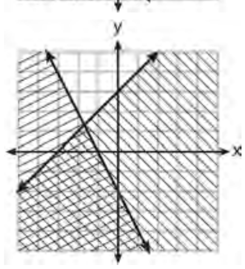


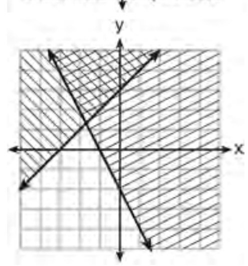
a. $(7,0)$ c. $(0,7)$
b. $(3,0)$ d. $(-3,5)$

2. Which graph represents the solution of $y \le x + 3$ and $y \ge -2x - 2$?

a. 

c. 

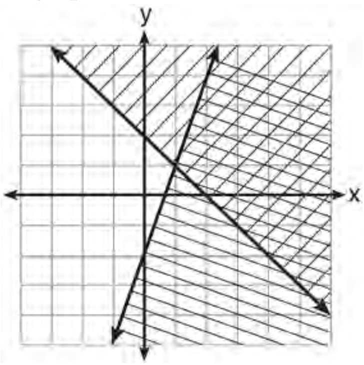
b. 

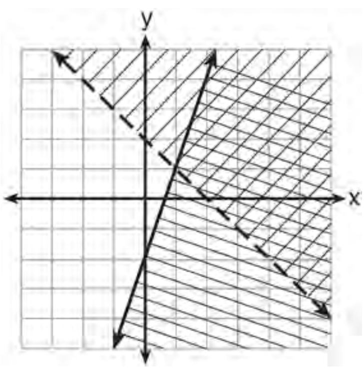
d. 

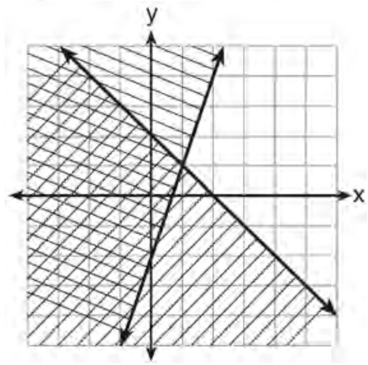
3. Given: $y + x > 2$

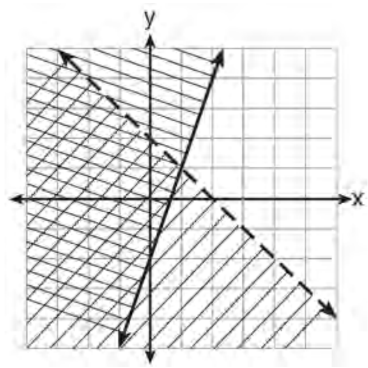
$y \le 3x - 2$

Which graph shows the solution of the given set of inequalities?

a. 

b. 

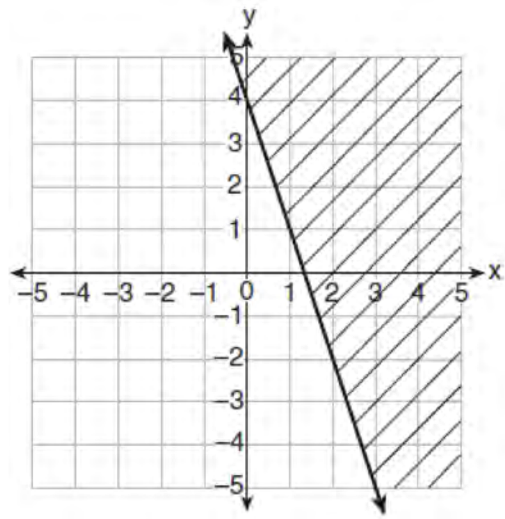
c. 

d. 

4. Which ordered pair is *not* in the solution set of $y > -\frac{1}{2}x + 5$ and $y \le 3x - 2$?

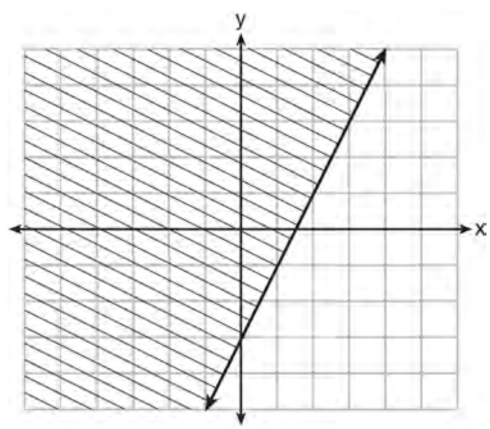
a. (5,3)
b. (4,3)
c. (3,4)
d. (4,4)

5. Which inequality is represented in the graph below?



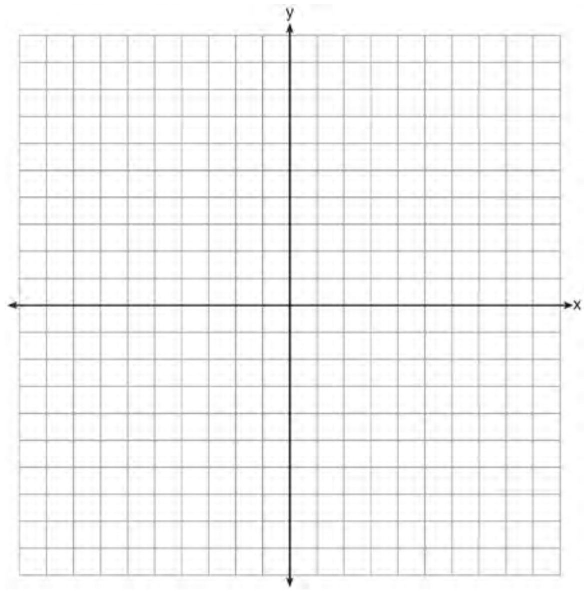
a. $y \geq -3x + 4$
b. $y \leq -3x + 4$
c. $y \geq -4x - 3$
d. $y \leq -4x - 3$

6. The graph of an inequality is shown below.

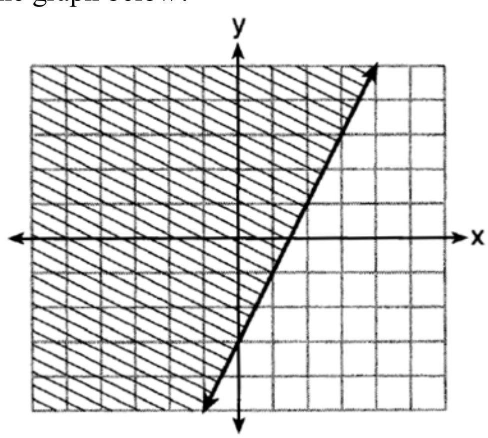


a) Write the inequality represented by the graph.
b) On the same set of axes, graph the inequality $x + 2y < 4$.
c) The two inequalities graphed on the set of axes form a system. Oscar thinks that the point $(2, 1)$ is in the solution set for this system of inequalities. Determine and state whether you agree with Oscar. Explain your reasoning.

7. On the set of axes below, graph the inequality $2x + y > 1$.



8. Which inequality is represented by the graph below?



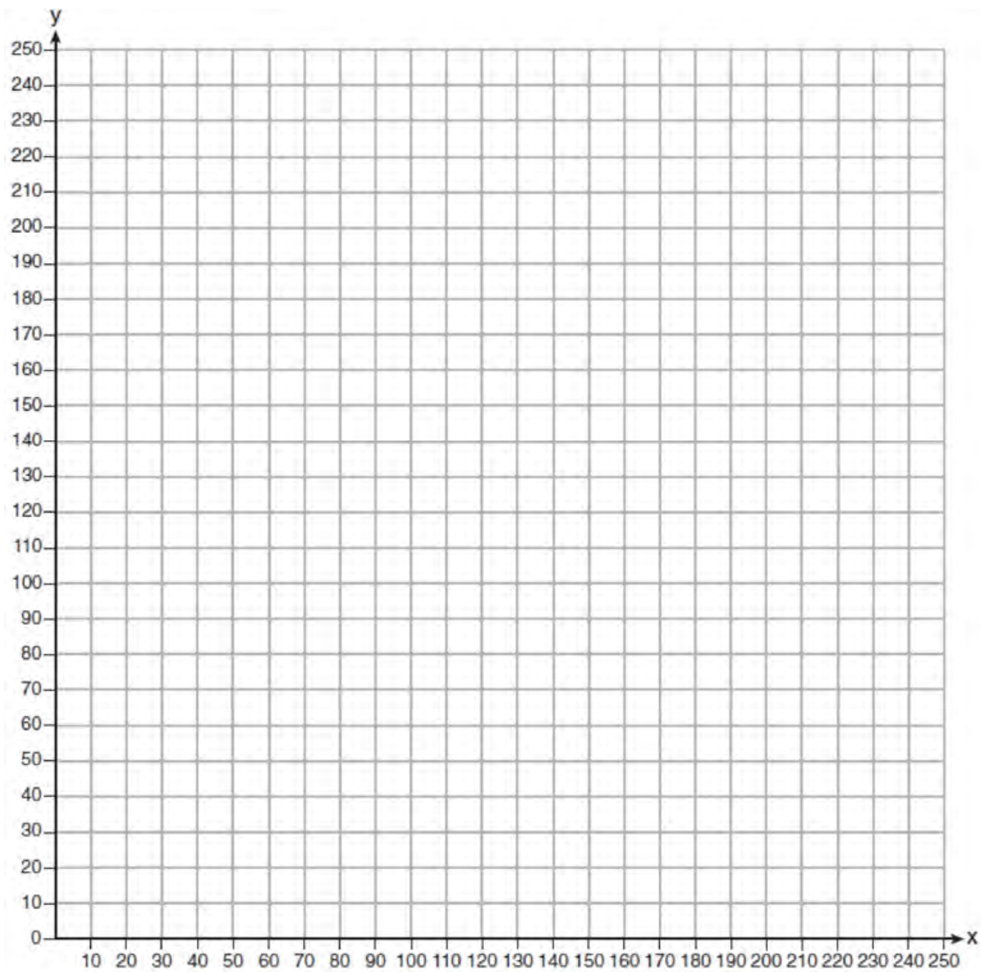
a. $y \le 2x - 3$
b. $y \ge 2x - 3$
c. $y \le -3x + 2$
d. $y \ge -3x + 2$

9. The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater.

Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x, and child tickets, y, that would satisfy the cinema's goal.

Graph the solution to this system of inequalities on the set of axes below. Label the solution with an S.

Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.

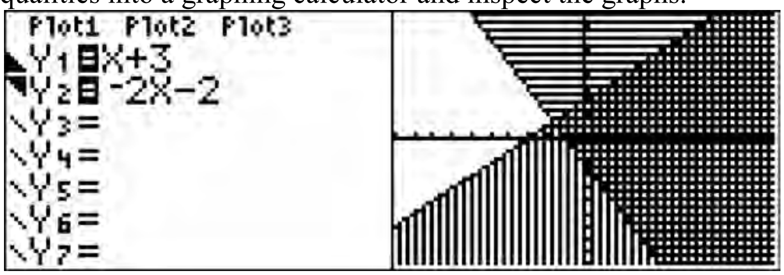


A.REI.12: Graph Systems of Inequalities
Answer Section

1. ANS: A
   Strategy: Visually estimate whether a point falls in the solution area and eliminate wrong answers.

   a. (7,0) clearly falls in the solutions area for both the solid line and the dotted line.
   b. (3,0) appears to be in the solution area for the solid line, but not for the dotted line.
   c. (0,7) is clearly not in the solution area for the dotted line.
   d. (-3,5) is clearly not in the solution area for either the solid line or the dotted line.

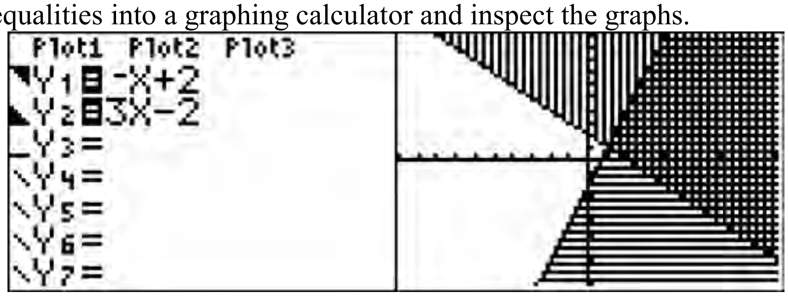
   PTS: 2 REF: 081407a1 NAT: A.REI.12 TOP: Graphing Systems of Linear Inequalities
2. ANS: C
   Strategy: Input both inequalities into a graphing calculator and inspect the graphs.



   Answer choice *c* is the correct answer.

   PTS: 2 REF: 081506ai NAT: A.REI.12 TOP: Graphing Systems of Linear Inequalities
3. ANS: B
   Strategy: Transpose the first inequality to slope intercept form ($y = mx + b$), then input both inequalities into a graphing calculator and eliminate wrong answers.

   STEP 1. Transpose the first inequality to slope intercept form ($y = mx + b$).
   $$y + x > 2$$
   $$y > -x + 2$$
   STEP 2. Input both inequalities into a graphing calculator and inspect the graphs.



   Eliminate answer choices *c* and *d*.

   STEP 3. Decide between answer choices *a* and *b*.
   Eliminate answer choice *a* because it shows two solid lines. The graph for $y > -x + 2$ must have a dotted line.
   Answer choice *b* is the correct answer.

   PTS: 2 REF: 061404a1 NAT: A.REI.12 TOP: Graphing Systems of Linear Inequalities
   KEY: bimodalgraph

4. ANS: B
Strategy: Test the answer choices in the system of equations and eliminate wrong answers.

a. (5,3)	c. (3,4)
$y > -\frac{1}{2}x + 5$ $y \le 3x - 2$ $3 > -\frac{1}{2}(5) + 5$ $3 \le 3(5) - 2$ $3 > 2\frac{1}{2}$ $3 \le 13$ True	$y > -\frac{1}{2}x + 5$ $y \le 3x - 2$ $4 > -\frac{1}{2}(3) + 5$ $4 \le 3(3) - 2$ $4 > 3\frac{1}{2}$ $4 \le 7$ True
b. (4,3)	**d. (4,4)**
$y > -\frac{1}{2}x + 5$ $y \le 3x - 2$ $3 > -\frac{1}{2}(4) + 5$ $3 \le 3(4) - 2$ $3 > 3$ $3 \le 10$ Not true	$y > -\frac{1}{2}x + 5$ $y \le 3x - 2$ $4 > -\frac{1}{2}(4) + 5$ $4 \le 3(4) - 2$ $4 > 3$ $4 \le 10$ True

PTS: 2 REF: fall1301a1 NAT: A.REI.12 TOP: Graphing Systems of Linear Inequalities

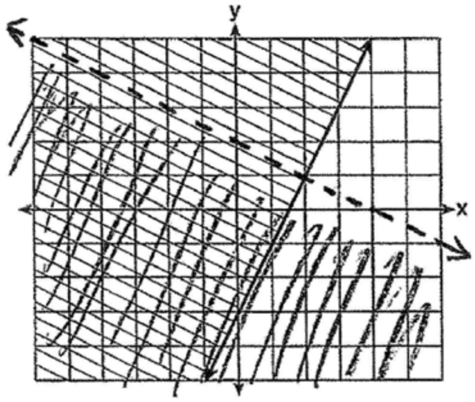
5. ANS: A
Strategy: Use the slope intercept form of a line, $y = mx + b$, to construct the inequality from the graph.

The line passes though points (0,4) and (1,1), so the slope is $m - \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{1-0} = \frac{-3}{1} = -3$. The y-intercept is 4.

The equation of the boundary line is $y = -3x + 4$, so eliminate choices *c* and *d*.
The shading is above the line, so eliminate choice *b*.
The inequality is $y \ge -3x + 4$, so answer choice *a* is correct.

PTS: 2 REF: 061505AI NAT: A.REI.12 TOP: Linear Inequalities

6. ANS:
a) $y \geq 2x - 3$.



b)
c Oscar is wrong. The point (2,1) is not in the solution set of both inequalities.

Strategy: Use information from the graph together with the slope intercept form of a line ($y = mx + b$) to write the inequality $y \geq 2x - 3$, where 2 is the slope (m) and -3 is the y-intercept b. Then, transform the new equation and put both equations in a graphing calculator. Use the graph and the table of values to finish the system of inequaulities on paper. Finally, determine if Oscar is right or wrong.

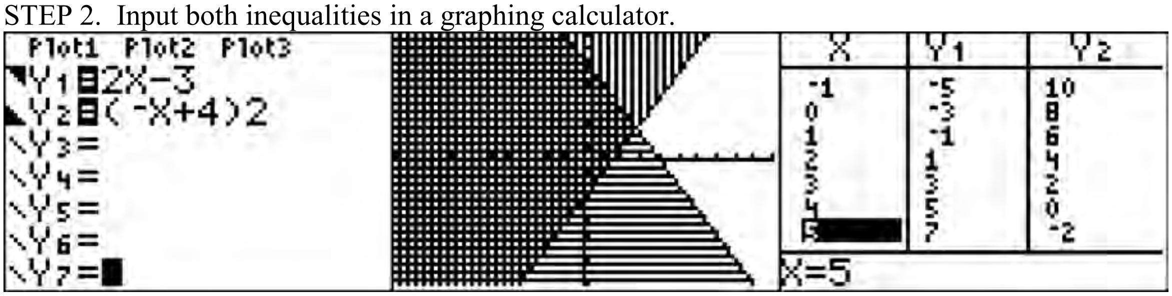
STEP 1. Transform $x + 2y < 4$ for input into a graphing calculator.

$$x + 2y < 4$$

$$2y < -x + 4$$

$$y < \frac{-x + 4}{2}$$

STEP 2. Input both inequalities in a graphing calculator.



STEP 3. Use the graph view and the table view to transfer the graph to paper. Be sure to make the line dotted for $y < \frac{-x + 4}{2}$. The line for $y \geq 2x - 3$ should be solid.

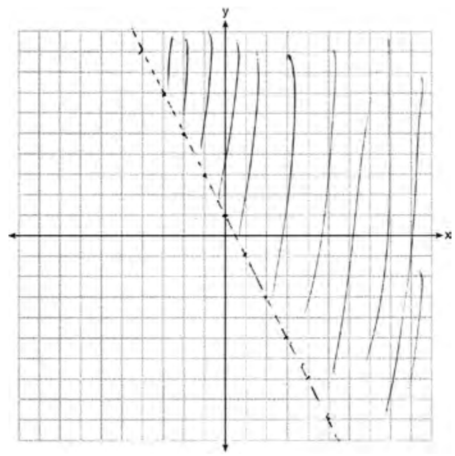
STEP 4. Test the point (2,1) in both equations.

$y < \frac{-x + 4}{2}$	$y \geq 2x - 3$
$1 < \frac{-2 + 4}{2}$	$1 \geq 2(2) - 3$
$1 < \frac{2}{2}$	$1 \geq 1$
$1 < 1$	True
Not True	

Oscar is wrong.

PTS: 4 REF: 011534a1 NAT: A.REI.12 TOP: Graphing Systems of Linear Inequalities

7. ANS:



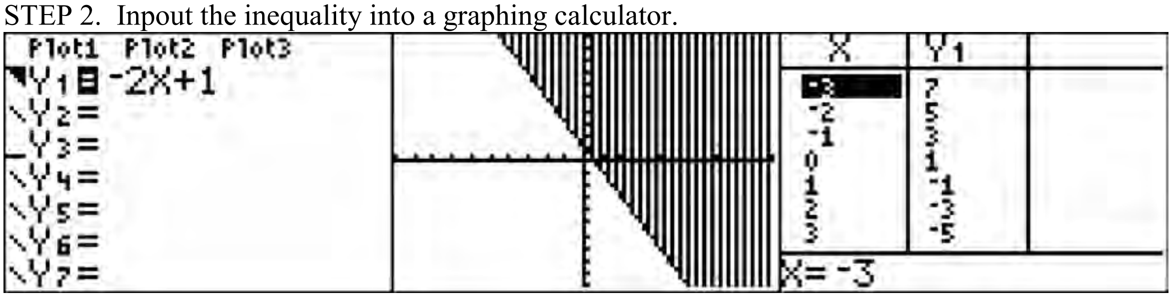
Strategy: Transpose the inequality, put it in a graphing calculator, then use the table and graph views to create the graph on paper.

STEP 1. Transpose the inequality for input into a graphing calculator.

$$2x + y > 1$$

$$y > -2x + 1$$

STEP 2. Inpout the inequality into a graphing calculator.



STEP 3. Use information from the graph and table views to create the graph on paper. Be sure to make the line dotted.

PTS: 2 REF: 081526ai NAT: A.REI.13 TOP: Graphing Linear Inequalities

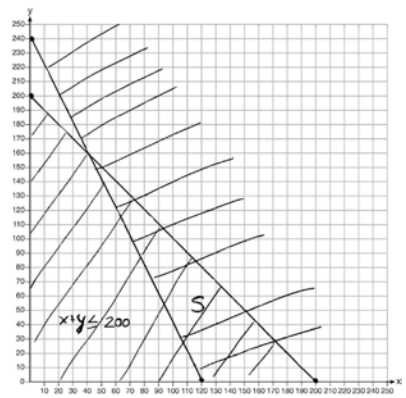
8. ANS: B PTS: 2 REF: 011605ai NAT: A.REI.12
TOP: Graphing Linear Inequalities

9. ANS:
System of Inequalities
Let x represent the number of adult tickets and let y represent the number of child tickets.

$$x + y \le 200$$

$$12.5x + 6.25y \ge 1500$$

Graph of the System



Marta is incorrect because the coordinates (30, 80) are not in the solution area.

Check: Marta is incorrect because $12.50(30) + $6.25(80) = $875.00.. This is less than the cinema’s goal of selling at least $1500 worth of tickets.

PTS: 6 REF: 011637ia NAT: A.REI.12 TOP: Graphing Systems of Linear Inequalities
KEY: graph

Revised "Writing the Math" Assignment

START	Write your name, date, topic of lesson, and class on your paper.
	NAME: Mohammed Chen
	DATE: December 18, 2015
	LESSON: Missing Number in the Average
	CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking.

Rationale for Assignment

Each New York Regents Algebra I (Common Core) examination contains 13 open response problems. An analysis of the first three Algebra I examinations revealed that approximately 51% (20 out of 39) of these open response problems instructed students to: 1) ***describe;*** 2) ***state***; 2) ***explain;*** 3) ***justify*** or otherwise write about their answers. It is theorized that students can benefit from explicit instruction and writing routines that are applicable to solving these problems.

Revised "Writing the Math" Assignment

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

Check: $88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$

POLYA'S APPROACH FOR SOLVING ANY MATH PROBLEM

STEP 1: UNDERSTAND THE PROBLEM

Read the entire problem.	Underline key words.	What is the problem asking you to do?	What information do you need?	What do you need to find out?	What should the answer look like?

STEP 2: DEVELOP A PROBLEM SOLVING STRATEGY

Have you seen a problem like this before?

If yes, how did you solve the previous problem?

If no, use a proven strategy from your problem solving toolkit.
Use a formula
Simplify the problem
Draw a picture
Look for a pattern
Eliminate wrong answers
Guess and check

STEP 3: DO THE STRATEGY

Show all work! Do not erase anything.

STEP 4: CHECK YOUR ANSWER TO SEE IF IT IS REASONABLE AND ACCURATE

after George Polya
Hungarian Mathematician
1887-1985