# JMAP REGENTS BY COMMON CORE STATE STANDARD: TOPIC

NY Algebra II Regents Exam Questions from Spring 2015 to August 2016 Sorted by CCSS: Topic

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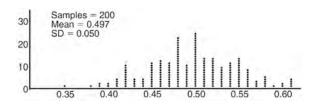
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## Algebra II Regents Exam Questions by Common Core State Standard: Topic

## GRAPHS AND STATISTICS S.IC.A.2, S.IC.B.4-6: ANALYSIS OF DATA

 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

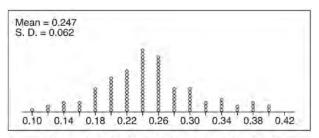


Given the results of her coin flips and of her computer simulation, which statement is most accurate?

- 1 73 of the computer's next 100 coin flips will be heads.
- 2 50 of her next 100 coin flips will be heads.
- 3 Her coin is not fair.
- 4 Her coin is fair.
- 2 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

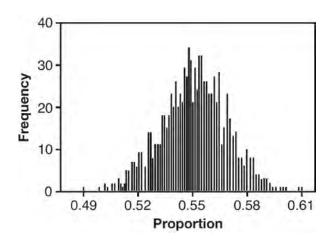
- 3 Which statement(s) about statistical studies is true?
  - I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
  - II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
  - III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
  - IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.
  - 1 I, only
  - 2 II, only
  - 3 I and III
  - 4 III and IV

4 Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products *A*, *B*, and the new product. Nine out of fifty participants preferred Stephen's new cola to products *A* and *B*. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen's new product, each of sample size 50, simulated 100 times.



**Proportion Preferring Stephen's Product** 

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision. 5 A candidate for political office commissioned a poll. His staff received responses from 900 likely voters and 55% of them said they would vote for the candidate. The staff then conducted a simulation of 1000 more polls of 900 voters, assuming that 55% of voters would vote for their candidate. The output of the simulation is shown in the diagram below.



Given this output, and assuming a 95% confidence level, the margin of error for the poll is closest to

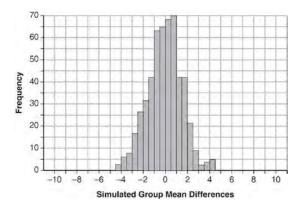
1	0.01
2	0.03
~	0.00

- 3 0.06
- 4 0.12

6 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

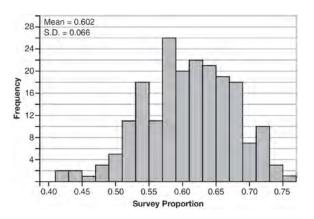
	Group 1	Group 2
x	80.16	83.8
Sx	6.9	5.2

Calculate the mean difference in the final grades (group 1 - group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

7 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

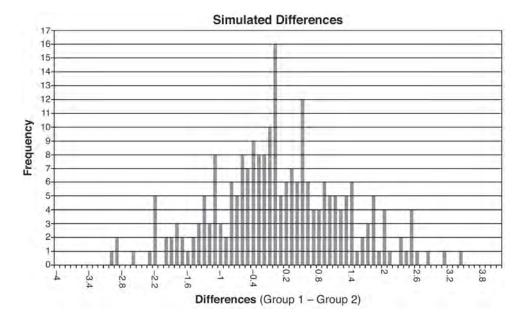


Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% - 50% split. Explain what statistical evidence supports this concern.

8 Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

Group 1 (seconds)	Group 2 (seconds)
17.4	23.3
18.1	18.8
18.2	22.1
19.6	12.7
18.6	16.9
16.2	24.4
16.1	21.2
15.3	21.2
17.8	16.3
19.7	14.5
Mean = 17.7	Mean = 19.1

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva's hypothesis may be *incorrect*. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.



Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.

4

9 Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.

#### S.IC.A.4: NORMAL DISTRIBUTIONS

- 10 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the *nearest ten thousandth*, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?
- 11 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is
  - 1 6
  - 2 48
  - 3 68
  - 4 95

- 12 The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?
  - 1 0.3803
  - 2 0.4612
  - 3 0.8415
  - 4 0.9612

## PROBABILITY S.CP.A.2, S.CP.B.7: THEORETICAL PROBABILITY

- 13 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on *both* issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.
- 14 Given events *A* and *B*, such that P(A) = 0.6, P(B) = 0.5, and  $P(A \cup B) = 0.8$ , determine whether *A* and *B* are independent or dependent.

15 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is  $\frac{974}{1376}$ , what is the probability that a student

participates in both sports and music?

#### S.CP.A.3-4: CONDITIONAL PROBABILITY

16 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

- 1 independent
- 2 dependent
- 3 mutually exclusive
- 4 complements
- 17 The results of a poll of 200 students are shown in the table below:

	Preferred Music Style			
	Techno	Rap	Country	
Female	54	25	27	
Male	36	40	18	

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer. 18 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

Ame Crown	Text Messages per Month			
Age Group	0-10	11-50	Over 50	
15-18	4	37	68	
19-22	6	25	87	
23-60	25	47	157	

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

157
229
157
312
157
384
157
456

1

2

3

4

## RATE F.IF.B.6: RATE OF CHANGE

19 The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

20 Which function shown below has a greater average rate of change on the interval [-2,4]? Justify your answer.

x	f(x)
-4	0.3125
-3	0.625
-2	1.25
-1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

### $g(x) = 4x^3 - 5x^2 + 3$

# A.REI.B.4: SOLVING OUADRATICS

- 21 The solutions to the equation  $-\frac{1}{2}x^2 = -6x + 20$  are
  - 1  $-6 \pm 2i$ 2  $-6 \pm 2\sqrt{19}$ 3  $6\pm 2i$ 4  $6+2\sqrt{19}$

 $4 \frac{1}{2}$ 

22 A solution of the equation  $2x^2 + 3x + 2 = 0$  is  $1 \quad -\frac{3}{4} + \frac{1}{4}i\sqrt{7}$ 2  $-\frac{3}{4} + \frac{1}{4}i$ 3  $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$ 

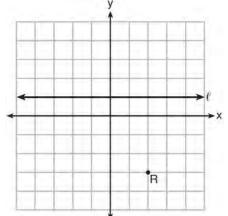
#### A.REI.B.4: COMPLEX CONJUGATE ROOT THEOREM

- 23 Which equation has 1 i as a solution?
  - $1 \quad x^2 + 2x 2 = 0$
  - $2 \qquad x^2 + 2x + 2 = 0$
  - $3 \qquad x^2 2x 2 = 0$
  - $4 \qquad x^2 2x + 2 = 0$

#### G.GPE.A.2: GRAPHING QUADRATIC FUNCTIONS

- 24 Which equation represents a parabola with a focus of (0,4) and a directrix of y = 2?
  - $1 y = x^{2} + 3$   $2 y = -x^{2} + 1$   $3 y = \frac{x^{2}}{2} + 3$   $4 y = \frac{x^{2}}{4} + 3$
- 25 The directrix of the parabola  $12(y+3) = (x-4)^2$ has the equation y = -6. Find the coordinates of the focus of the parabola.

26 Which equation represents the set of points equidistant from line  $\ell$  and point *R* shown on the graph below?



 $y = -\frac{1}{8}(x+2)^2 + 1$  $y = -\frac{1}{8}(x+2)^2 - 1$  $y = -\frac{1}{8}(x-2)^2 + 1$  $y = -\frac{1}{8}(x-2)^2 - 1$ 

## SYSTEMS A.REI.C.6: SOLVING LINEAR SYSTEMS

27 Solve the following system of equations algebraically for all values of *x*, *y*, and *z*: x + 3y + 5z = 45

$$6x - 3y + 2z = -10$$
$$-2x + 3y + 8z = 72$$

28 Which value is *not* contained in the solution of the system shown below?

$$a + 5b - c = -20$$

$$4a - 5b + 4c = 19$$

$$-a - 5b - 5c = 2$$

$$1 \quad -2$$

$$2 \quad 2$$

$$3 \quad 3$$

$$4 \quad -3$$

#### A.REI.C.7, A.REI.D.11: QUADRATIC-LINEAR SYSTEMS

29 Algebraically determine the values of x that satisfy the system of equations below.

$$y = -2x + 1$$
$$y = -2x^2 + 3x + 1$$

30 Solve the system of equations shown below algebraically.

$$(x-3)^{2} + (y+2)^{2} = 16$$
$$2x + 2y = 10$$

A.REI.D.11: OTHER SYSTEMS

32 Given: 
$$h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$$

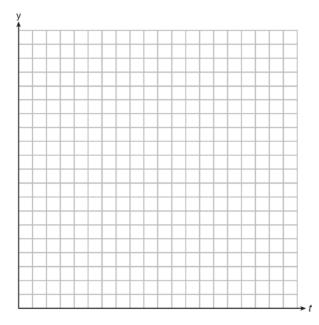
k(x) = -|0.7x| + 5

State the solutions to the equation h(x) = k(x), rounded to the *nearest hundredth*.

- 33 Which value, to the *nearest tenth*, is *not* a solution of p(x) = q(x) if  $p(x) = x^3 + 3x^2 - 3x - 1$  and q(x) = 3x + 8? 1 -3.9 2 -1.1 3 2.1
  - 4 4.7
- 34 To the *nearest tenth*, the value of x that satisfies  $2^x = -2x + 11$  is
  - 1 2.5
  - 2 2.6
  - 3 5.8
  - 4 5.9

- 31 Sally's high school is planning their spring musical. The revenue, *R*, generated can be determined by the function  $R(t) = -33t^2 + 360t$ , where *t* represents the price of a ticket. The production cost, *C*, of the musical is represented by the function C(t) = 700 + 5t. What is the highest ticket price, to the *nearest dollar*, they can charge in order to *not* lose money on the event?
  - 1 t = 3
  - 2 t = 5
  - 3 *t* = 8
  - 4 *t* = 11

35 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function  $N(t) = N_0(e)^{-rt}$ , where N(t) is the amount left in the body,  $N_0$  is the initial dosage, *r* is the decay rate, and *t* is time in hours. Patient *A*, *A*(*t*), is given 800 milligrams of a drug with a decay rate of 0.347. Patient *B*, *B*(*t*), is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, *A*(*t*) and *B*(*t*), to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.



To the *nearest hour*, *t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*? The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient *A* will have to wait to take another 800 milligram dose of the drug.

### POWERS A.SSE.B.3, F.IF.C.8, F.BF.A.1, F.LE.A.2, F.LE.B.5: MODELING EXPONENTIAL FUNCTIONS

- 36 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, B(t), can be represented by the function  $B(t) = 750(1.16)^t$ , where the *t* represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function
  - 1  $B(t) = 750(1.012)^{t}$
  - 2  $B(t) = 750(1.012)^{12t}$
  - $3 \quad B(t) = 750(1.16)^{12t}$

4 
$$B(t) = 750(1.16)^{\frac{1}{12}}$$

- 37 Which function represents exponential decay?
  - $1 y = 2^{0.3t}$   $2 y = 1.2^{3t}$   $3 y = \left(\frac{1}{2}\right)^{-t}$
  - $4 \qquad y = 5^{-t}$

- 38 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let *m* represent months.]
  - 1  $(1.0525)^m$
  - 2  $(1.0525)^{\overline{m}}$
  - $3 (1.00427)^m$
  - 4  $(1.00427)^{\frac{m}{12}}$
- 39 A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1 300(.30) 
$$\frac{14}{365}$$

2 
$$300(1.30)^{\frac{14}{365}}$$

4 
$$300(1.30)^{\frac{365}{14}}$$

- 40 An equation to represent the value of a car after t months of ownership is  $v = 32,000(0.81)^{\frac{t}{12}}$ . Which statement is *not* correct?
  - 1 The car lost approximately 19% of its value each month.
  - 2 The car maintained approximately 98% of its value each month.
  - 3 The value of the car when it was purchased was \$32,000.
  - 4 The value of the car 1 year after it was purchased was \$25,920.
- 41 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the *nearest tenth*. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.

## F.IF.C.7: GRAPHING EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- 42 Which statement about the graph of  $c(x) = \log_6 x$  is *false*?
  - 1 The asymptote has equation y = 0.
  - 2 The graph has no *y*-intercept.
  - 3 The domain is the set of positive reals.
  - 4 The range is the set of all real numbers.

#### A.CED.A.1, F.LE.A.4: EXPONENTIAL EQUATIONS

43 Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment P = amount borrowed r = annual interest rate n = number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the *fewest* number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than \$720.

44 Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after n years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the nearest cent. Algebraically determine, to the nearest tenth of a year, how long it would take for option B to double Seth's initial investment.

45 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

 $T_a$  = the temperature surrounding the object

 $T_0$  = the initial temperature of the object

t = the time in hours

- T = the temperature of the object after t hours
- k = decay constant

The turkey reaches the temperature of approximately  $100^{\circ}$  F after 2 hours. Find the value of *k*, to the *nearest thousandth*, and write an equation to determine the temperature of the turkey after *t* hours. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

- 46 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.
- 47 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

## POLYNOMIALS A.SSE.A.2: FACTORING POLYNOMIALS

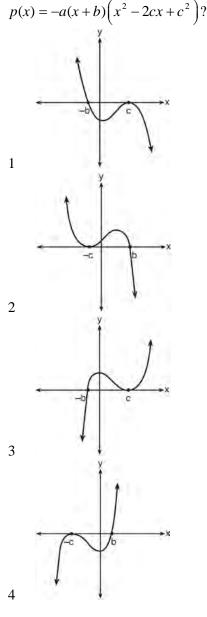
- 48 What is the completely factored form of
  - $k^4 4k^2 + 8k^3 32k + 12k^2 48?$
  - 1 (k-2)(k-2)(k+3)(k+4)
  - 2 (k-2)(k-2)(k+6)(k+2)
  - 3 (k+2)(k-2)(k+3)(k+4)
  - $4 \quad (k+2)(k-2)(k+6)(k+2)$
- 49 Rewrite the expression

 $(4x^2 + 5x)^2 - 5(4x^2 + 5x) - 6$  as a product of four linear factors.

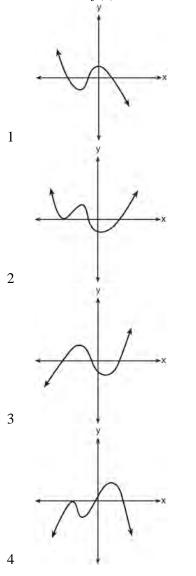
- 50 Which factorization is *incorrect*?
  - 1  $4k^2 49 = (2k + 7)(2k 7)$
  - 2  $a^3 8b^3 = (a 2b)(a^2 + 2ab + 4b^2)$
  - 3  $m^3 + 3m^2 4m + 12 = (m-2)^2(m+3)$
  - 4  $t^{3} + 5t^{2} + 6t + t^{2} + 5t + 6 = (t+1)(t+2)(t+3)$
- 51 The completely factored form of  $2d^4 + 6d^3 18d^2 54d$  is
  - 1  $2d(d^2 9)(d + 3)$
  - 2  $2d(d^2+9)(d+3)$
  - 3  $2d(d+3)^2(d-3)$
  - 4  $2d(d-3)^2(d+3)$

#### A.APR.B.3: ZEROS OF POLYNOMIALS

52 If *a*, *b*, and *c* are all positive real numbers, which graph could represent the sketch of the graph of  $a = (a^2 + a^2)^2$ 

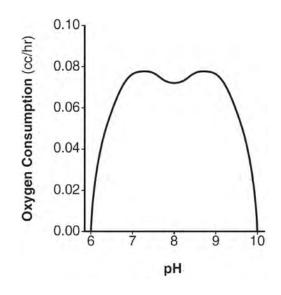


- 53 Which graph has the following characteristics?
  - three real zeros
  - as  $x \to -\infty$ ,  $f(x) \to -\infty$
  - as  $x \to \infty$ ,  $f(x) \to \infty$



#### F.IF.B.4, F.IF.C.7: GRAPHING POLYNOMIAL FUNCTIONS

55 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.

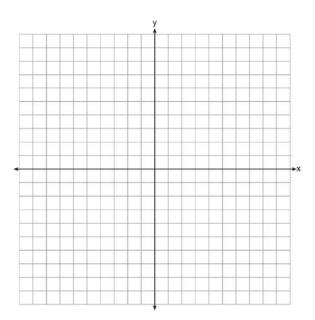


Which statement about this function is *incorrect*?

- 1 The degree of the polynomial is even.
- 2 There is a positive leading coefficient.
- 3 At two pH values, there is a relative maximum value.
- 4 There are two intervals where the function is decreasing.

- 54 The zeros for  $f(x) = x^4 4x^3 9x^2 + 36x$  are
  - 1  $\{0,\pm 3,4\}$
  - $2 \{0,3,4\}$
  - $3 \{0, \pm 3, -4\}$
  - 4 {0,3,-4}

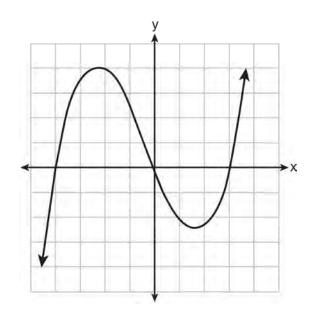
56 Find algebraically the zeros for  $p(x) = x^3 + x^2 - 4x - 4$ . On the set of axes below, graph y = p(x).



#### A.APR.B.2: REMAINDER THEOREM

- 57 Use an appropriate procedure to show that x 4 is a factor of the function  $f(x) = 2x^3 - 5x^2 - 11x - 4$ . Explain your answer.
- 58 Given  $z(x) = 6x^3 + bx^2 52x + 15$ , z(2) = 35, and z(-5) = 0, algebraically determine all the zeros of z(x).
- 59 Determine if x 5 is a factor of  $2x^3 4x^2 7x 10$ . Explain your answer.

60 The graph of p(x) is shown below.



What is the remainder when p(x) is divided by x + 4?

- $1 \quad x 4$
- 2 –4
- 3 0
- 4 4

#### A.APR.C.4: POLYNOMIAL IDENTITIES

- 61 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.
- 62 Algebraically prove that  $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$ , where  $x \neq -2$ .

- 63 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?
  - I  $(m+p)^2 = m^2 + 2mp + p^2$ II  $(x+y)^3 = x^3 + 3xy + y^3$ III  $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$ I, only
  - 2 I and II

1

- 3 II and III
- 4 I and III

#### N.RN.A.1-2: RADICALS AND RATIONAL EXPONENTS

67 Explain how 
$$\left(3^{\frac{1}{5}}\right)^2$$
 can be written as the equivalent radical expression  $\sqrt[5]{9}$ .

68 Use the properties of rational exponents to determine the value of *y* for the equation:

$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y, \ x > 1$$

69 When b > 0 and d is a positive integer, the expression  $(3b)^{\frac{2}{d}}$  is equivalent to  $1 \quad \frac{1}{\left(\frac{d}{\sqrt{3b}}\right)^2}$  $2 \quad \left(\sqrt{3b}\right)^d$  $3 \quad \frac{1}{\sqrt{3b^d}}$  $4 \quad \left(\frac{d}{\sqrt{3b}}\right)^2$ 

#### N.CN.A.2: OPERATIONS WITH COMPLEX NUMBERS

70 Write (5+2yi)(4-3i) - (5-2yi)(4-3i) in a+bi form, where y is a real number.

## RADICALS A.REI.A.2: SOLVING RADICALS

- 64 Solve algebraically for all values of *x*:  $\sqrt{x-5} + x = 7$
- 65 The solution set for the equation  $\sqrt{56-x} = x$  is 1  $\{-8,7\}$ 
  - $2 \{-7,8\}$
  - 2 (7) 3 {7}
  - 4 { }
  - <del>т</del> ( )
- 66 Solve the equation  $\sqrt{2x-7} + x = 5$  algebraically, and justify the solution set.

- 71 Given *i* is the imaginary unit,  $(2 yi)^2$  in simplest form is
  - $1 \qquad y^2 4yi + 4$

$$2 \quad -y^2 - 4yi + 4$$

- 3  $-y^2 + 4$
- $4 y^2 + 4$
- 72 Simplify  $xi(i-7i)^2$ , where *i* is the imaginary unit.

## RATIONALS A.APR.D.6: RATIONAL EXPRESSIONS

73 The expression 
$$\frac{6x^{3} + 17x^{2} + 10x + 2}{2x + 3}$$
 equals  
1  $3x^{2} + 4x - 1 + \frac{5}{2x + 3}$   
2  $6x^{2} + 8x - 2 + \frac{5}{2x + 3}$   
3  $6x^{2} - x + 13 - \frac{37}{2x + 3}$   
4  $3x^{2} + 13x + \frac{49}{2} + \frac{151}{2x + 3}$ 

74 The expression 
$$\frac{4x^3 + 5x + 10}{2x + 3}$$
 is equivalent to  
1  $2x^2 + 3x - 7 + \frac{31}{2x + 3}$   
2  $2x^2 - 3x + 7 - \frac{11}{2x + 3}$   
3  $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$   
4  $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$ 

75 The expression 
$$\frac{x^3 + 2x^2 + x + 6}{x + 2}$$
 is equivalent to  
1  $x^2 + 3$   
2  $x^2 + 1 + \frac{4}{x + 2}$   
3  $2x^2 + x + 6$   
4  $2x^2 + 1 + \frac{4}{x + 2}$ 

#### A.CED.A.1: MODELING RATIONALS

76 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, *T*, are left in the semester?

$$1 \quad \frac{255 + 93T}{3T} = 90$$

$$2 \quad \frac{255 + 90T}{3T} = 93$$

$$3 \quad \frac{255 + 93T}{T+3} = 90$$

$$4 \quad \frac{255 + 90T}{T+3} = 93$$

#### A.REI.A.2: SOLVING RATIONALS

77 What is the solution set of the equation  $\frac{3x+25}{x+7} - 5 = \frac{3}{x}?$ 1  $\left\{\frac{3}{2},7\right\}$ 2  $\left\{\frac{7}{2},-3\right\}$ 3  $\left\{-\frac{3}{2},7\right\}$ 4  $\left\{-\frac{7}{2},-3\right\}$ 

78 Solve for *x*: 
$$\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$$

79 The focal length, F, of a camera's lens is related to the distance of the object from the lens, J, and the distance to the image area in the camera, W, by the formula below.

$$\frac{1}{J} + \frac{1}{W} = \frac{1}{F}$$

When this equation is solved for J in terms of F and W, J equals

$$1 \quad F - W$$

 $2 \quad \frac{FW}{F-W}$ 

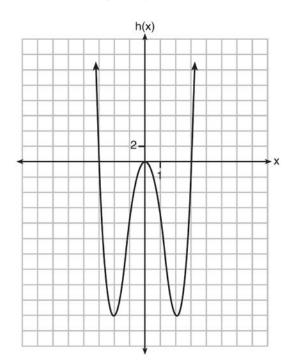
$$3 \quad \frac{FW}{W = F}$$

$$4 \quad \frac{1}{F} - \frac{1}{W}$$

## FUNCTIONS F.BF.A.2: EVEN AND ODD FUNCTIONS

80 Functions f, g, and h are given below.

$$f(x) = \sin(2x)$$
$$g(x) = f(x) + 1$$



Which statement is true about functions f, g, and h?

- 1 f(x) and g(x) are odd, h(x) is even.
- 2 f(x) and g(x) are even, h(x) is odd.
- 3 f(x) is odd, g(x) is neither, h(x) is even.
- 4 f(x) is even, g(x) is neither, h(x) is odd.
- 81 Which equation represents an odd function?

1 
$$y = \sin x$$

$$2 \quad y = \cos x$$

3 
$$y = (x+1)^3$$

$$\begin{array}{ll}
3 & y = (x + y) \\
4 & y = e^{5x}
\end{array}$$

F.BF.A.1: OPERATIONS WITH FUNCTIONS

- 82 If  $g(c) = 1 c^2$  and m(c) = c + 1, then which statement is *not* true?
  - $1 \qquad g(c) \cdot m(c) = 1 + c c^2 c^3$
  - 2  $g(c) + m(c) = 2 + c c^2$
  - $3 \quad m(c) g(c) = c + c^2$
  - $4 \qquad \frac{m(c)}{g(c)} = \frac{-1}{1-c}$

#### F.IF.C.9: COMPARING FUNCTIONS

83 The *x*-value of which function's *x*-intercept is larger, *f* or *h*? Justify your answer.

$$f(x) = \log(x - 4)$$

x	h(x)
-1	6
0	4
1	2
2	0
3	-2

#### F.BF.B.4: INVERSE OF FUNCTIONS

84 For the function  $f(x) = (x-3)^3 + 1$ , find  $f^{-1}(x)$ .

85 Given  $f^{-1}(x) = -\frac{3}{4}x + 2$ , which equation represents f(x)?

1 
$$f(x) = \frac{4}{3}x - \frac{6}{3}$$
  
2  $f(x) = -\frac{4}{3}x + \frac{8}{3}$   
3  $f(x) = \frac{3}{4}x - 2$   
4  $f(x) = -\frac{3}{4}x + 2$ 

## SEQUENCES AND SERIES F.LE.A.2, F.BF.A.2: SEQUENCES

86 Write an explicit formula for  $a_n$ , the *n*th term of the recursively defined sequence below.

$$a_1 = x + 1$$

$$a_n = x(a_{n-1})$$
  
For what values of *x* would  $a_n = 0$  when  $n > 1$ ?

- The population of Jamesburg for the years 2010-2013, respectively, was reported as follows: 250,000 250,937 251,878 252,822
  How can this sequence be recursively modeled?
  - 1  $j_n = 250,000(1.00375)^{n-1}$

2 
$$j_n = 250,000 + 937^{(n-1)}$$

3  $j_1 = 250,000$ 

$$j_n = 1.00375 j_{n-1}$$

4 
$$j_1 = 250,000$$

$$j_n = j_{n-1} + 937$$

88 A recursive formula for the sequence 18,9,4.5,...

is  
1 
$$g_1 = 18$$
  
 $g_n = \frac{1}{2}g_{n-1}$   
2  $g_n = 18\left(\frac{1}{2}\right)^{n-1}$   
3  $g_1 = 18$ 

$$g_n = 2g_{n-1}$$
  
4  $g_n = 18(2)^{n-1}$ 

- 89 The sequence  $a_1 = 6$ ,  $a_n = 3a_{n-1}$  can also be written as
  - 1  $a_n = 6 \cdot 3^n$
  - 2  $a_n = 6 \cdot 3^{n+1}$
  - 3  $a_n = 2 \cdot 3^n$
  - 4  $a_n = 2 \cdot 3^{n+1}$
- 90 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State *t* years after 2010?
  - 1  $P_t = 19,378,000(1.5)^t$
  - 2  $P_0 = 19,378,000$ 
    - $P_t = 19,378,000 + 1.015P_{t-1}$
  - 3  $P_t = 19,378,000(1.015)^{t-1}$

4 
$$P_0 = 19,378,000$$

$$P_t = 1.015 P_{t-1}$$

91 The formula below can be used to model which scenario?

$$a_1 = 3000$$
  
 $a_n = 0.80a_{n-1}$ 

- 1 The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
- 2 The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
- 3 A bank account starts with a deposit of \$3000, and each year it grows by 80%.
- 4 The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

#### A.SSE.B.4: SERIES

92 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula,  $S_n$ , for Alexa's total earnings over *n* years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

93 Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

$$1 \qquad \sum_{n=1}^{6} 8(1.10)^{n-1}$$

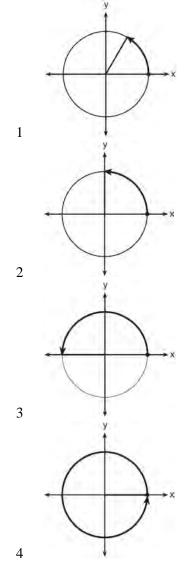
2 
$$\sum_{n=1}^{6} 8(1.10)^n$$

$$3 \frac{8-8(1.10)^6}{0.90}$$

$$4 \quad \frac{8 - 8(0.10)^n}{1.10}$$

## TRIGONOMETRY F.TF.A.1: UNIT CIRCLE

94 Which diagram shows an angle rotation of 1 radian on the unit circle?



#### F.TF.A.2, F.TF.C.8: DETERMINING TRIGONOMETRIC FUNCTIONS

- 95 If the terminal side of angle  $\theta$ , in standard position, passes through point (-4,3), what is the numerical value of  $\sin \theta$ ?
  - $\frac{3}{5}$  $\frac{4}{5}$  $\frac{-3}{5}$  $-\frac{4}{5}$ 1 2 3 4
- 96 A circle centered at the origin has a radius of 10 units. The terminal side of an angle,  $\theta$ , intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of  $\cos\theta$ ?
  - 2

$$3 \frac{3}{5}$$
  
 $4 \frac{4}{5}$ 

- 4
- 97 Using the identity  $\sin^2 \theta + \cos^2 \theta = 1$ , find the value of tan  $\theta$ , to the *nearest hundredth*, if  $\cos \theta$  is -0.7 and  $\theta$  is in Quadrant II.

#### F.TF.B.5: MODELING TRIGONOMETRIC **FUNCTIONS**

- 98 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Which equation best represents the value of the voltage as it flows through the electric wires, where *t* is time in seconds?
  - $V = 120 \sin(t)$ 1
  - $V = 120 \sin(60t)$ 2
  - 3  $V = 120 \sin(60\pi t)$
  - 4  $V = 120 \sin(120\pi t)$

#### F.IF.B.4, F.IF.C.7: GRAPHING TRIGONOMETRIC FUNCTIONS

99 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, *H*, in feet, above the ground of one of the six-person cars can be modeled by

 $H(t) = 70 \sin\left(\frac{2\pi}{7}(t-1.75)\right) + 80$ , where *t* is time,

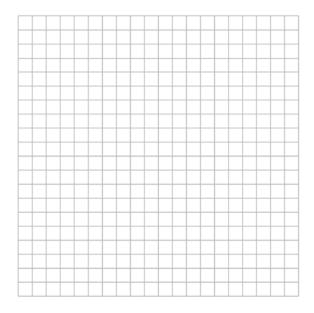
in minutes. Using H(t) for one full rotation, this car's minimum height, in feet, is

- 1 150 2 70
- 3 10
- 0
- 4
- 100 A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?
  - 1 (0, 200)
  - 2 (100, 300)
  - 3 (200, 400)
  - 4 (300, 400)

101 Which statement is *incorrect* for the graph of the

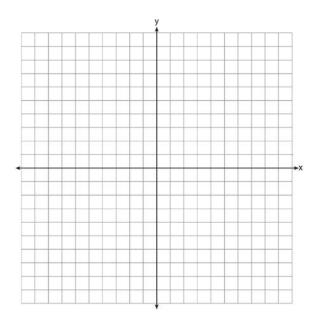
function 
$$y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 72$$

- 1 The period is 6.
- 2 The amplitude is 3.
- 3 The range is [4,10].
- 4 The midline is y = -4.
- 102 The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.
- 103 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form  $f(t) = A\cos(Bt)$ , where A and B are real numbers, that models the water level, f(t), in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m. On the grid below, graph one cycle of this function.



People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

104 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period  $\frac{\pi}{2}$ , midline y = -1, and passing through the point (0,2).



## CONICS A.SSE.A.2: EQUATIONS OF CONICS

- 105 The equation  $4x^2 24x + 4y^2 + 72y = 76$  is equivalent to
  - $1 \quad 4(x-3)^2 + 4(y+9)^2 = 76$
  - 2  $4(x-3)^2 + 4(y+9)^2 = 121$
  - $3 \quad 4(x-3)^2 + 4(y+9)^2 = 166$
  - 4  $4(x-3)^2 + 4(y+9)^2 = 436$

### Algebra II Regents Exam Questions by Common Core State Standard: Topic **Answer Section**

1 ANS: 3 PTS: 2 REF: 061607aii NAT: S.IC.A.2

TOP: Analysis of Data

2 ANS:

Randomly assign participants to two groups. One group uses the toothpaste with ingredient X and the other group uses the toothpaste without ingredient *X*.

PTS: 2 NAT: S.IC.B.3 REF: 061626aii TOP: Analysis of Data KEY: type

3 ANS: 1

II. Ninth graders drive to school less often; III.Students know little about adults; IV. Calculus students love math!

PTS: 2 REF: 081602aii NAT: S.IC.B.3 TOP: Analysis of Data KEY: bias

4 ANS:

Yes. The margin of error from this simulation indicates that 95% of the observations fall within  $\pm 0.12$  of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to

be 0.06 in the dotplot, by 2, or applying the estimated standard error formula,  $\left| \sqrt{\frac{p(1-p)}{n}} \right| or \left| \sqrt{\frac{p(1-p)}{n}} \right|$ 

and multiplying by 2. The interval  $0.25 \pm 0.12$  includes plausible values for the true proportion of people who prefer Stephen's new product. The company has evidence that the population proportion could be at least 25%. As seen in the dotplot, it can be expected to obtain a sample proportion of 0.18 (9 out of 50) or less several times, even when the population proportion is 0.25, due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

	PTS: 4	REF: spr15	12aii NAT:	S.IC.B.4 TOP:	Analysis of Data
-	ANS: 2				
		$p(1-p)$ = $\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$	).55)(0.45)	02	

900

$$ME =$$

PTS: 2 REF: 081612aii NAT: S.IC.B.4 TOP: Analysis of Data

6 ANS:

The mean difference between the students' final grades in group 1 and group 2 is -3.64. This value indicates that students who met with a tutor had a mean final grade of 3.64 points less than students who used an on-line subscription. One can infer whether this difference is due to the differences in intervention or due to which students were assigned to each group by using a simulation to rerandomize the students' final grades many (500) times. If the observed difference -3.64 is the result of the assignment of students to groups alone, then a difference of -3.64 or less should be observed fairly regularly in the simulation output. However, a difference of -3 or less occurs in only about 2% of the rerandomizations. Therefore, it is quite unlikely that the assignment to groups alone accounts for the difference; rather, it is likely that the difference between the interventions themselves accounts for the difference between the two groups' mean final grades.

PTS: 4 REF: fall1514aii NAT: S.IC.B.5 TOP: Analysis of Data

8 ANS:

 $0.602 \pm 2 \cdot 0.066 = 0.47 - 0.73$ . Since 0.50 falls within the 95% interval, this supports the concern there may be an even split.

- PTS: 4 REF: 061635aii NAT: S.IC.B.5 TOP: Analysis of Data
- Some of the students who did not drink energy drinks read faster than those who did drink energy drinks.

17.7 - 19.1 = -1.4 Differences of -1.4 and less occur  $\frac{25}{232}$  or about 10% of the time, so the difference is not unusual.

PTS: 4 REF: 081636aii NAT: S.IC.B.5 TOP: Analysis of Data

9 ANS:

Using a 95% level of confidence,  $x \pm 2$  standard deviations sets the usual wait time as 150-302 seconds. 360 seconds is unusual.

PTS: 2 REF: 081629aii NAT: S.IC.B.6 TOP: Analysis of Data 10 ANS:

normcdf(510, 540, 480, 24) = 0.0994  $z = \frac{510 - 480}{24} = 1.25$   $1.25 = \frac{x - 510}{20}$   $2.5 = \frac{x - 510}{20}$  535 - 560 $z = \frac{540 - 480}{24} = 2.5$  x = 535 x = 560

PTS: 4 REF: fall1516aii NAT: S.ID.A.4 TOP: Normal Distributions KEY: probability

11 ANS: 2



 $x + 2\sigma$  represents approximately 48% of the data.

PTS: 2 REF: 061609aii NAT: S.ID.A.4 TOP: Normal Distributions KEY: percent

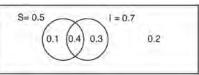
12 ANS: 3



PTS: 2 REF: 081604aii NAT: S.ID.A.4 TOP: Normal Distributions KEY: probability

Since

13 ANS:



This scenario can be modeled with a Venn Diagram:  $\bot$ 

 $P(S \cup I)_c = 0.2$ ,  $P(S \cup I) = 0.8$ . Then,  $P(S \cap I) = P(S) + P(I) - P(S \cup I)$  If S and I are independent, then the

= 0.5 + 0.7 - 0.8

= 0.4

Product Rule must be satisfied. However,  $(0.5)(0.7) \neq 0.4$ . Therefore, salary and insurance have not been treated independently.

 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  A and B are independent since  $P(A \cap B) = P(A) \cdot P(B)$ 

$$0.8 = 0.6 + 0.5 - P(A \cap B) \qquad \qquad 0.3 = 0.6 \cdot 0.5$$

$$P(A \cap B) = 0.3 \qquad \qquad 0.3 = 0.3$$

PTS: 2 REF: 081632aii NAT: S.CP.A.2 TOP: Theoretical Probability 15 ANS:

$$P(S \cap M) = P(S) + P(M) - P(S \cup M) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376} = \frac{108}{1376}$$

PTS: 2 REF: 061629aii NAT: S.CP.B.7 TOP: Theoretical Probability 16 ANS: 1

The probability of rain equals the probability of rain, given that Sean pitches.

PTS: 2 REF: 061611aii NAT: S.CP.A.3 TOP: Conditional Probability 17 ANS:

Based on these data, the two events do not appear to be independent.  $P(F) = \frac{106}{200} = 0.53$ , while

 $P(F|T) = \frac{54}{90} = 0.6$ ,  $P(F|R) = \frac{25}{65} = 0.39$ , and  $P(F|C) = \frac{27}{45} = 0.6$ . The probability of being female are not the same as the conditional probabilities. This suggests that the events are not independent.

PTS: 2 REF: fall1508aii NAT: S.CP.A.4 TOP: Conditional Probability  
18 ANS: 1  
$$\frac{157}{25+47+157}$$

PTS: 2 REF: 081607aii NAT: S.CP.A.4 TOP: Conditional Probability

 $\frac{156.25 - 56.25}{70 - 50} = \frac{150}{20} = 7.5$  Between 50-70 mph, each additional mph in speed requires 7.5 more feet to stop.

PTS: 2 REF: 081631aii NAT: F.IF.B.6 TOP: Rate of Change KEY: AII

20 ANS:

 $\frac{f(4) - f(-2)}{4 - -2} = \frac{80 - 1.25}{6} = 13.125 \ g(x) \text{ has a greater rate of change}$ 

 $\frac{g(4) - g(-2)}{4 - 2} = \frac{179 - 49}{6} = 38$ 

- PTS: 4 REF: 061636aii NAT: F.IF.B.6 TOP: Rate of Change KEY: AII
- 21 ANS: 3

$$-2\left(-\frac{1}{2}x^{2} = -6x + 20\right)$$

$$x^{2} - 12x = -40$$

$$x^{2} - 12x + 36 = -40 + 36$$

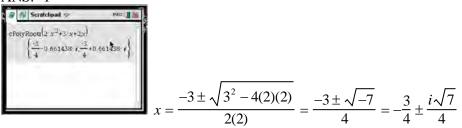
$$(x - 6)^{2} = -4$$

$$x - 6 = \pm 2i$$

$$x = 6 \pm 2i$$

PTS: 2 REF: fall1504aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | completing the square

22 ANS: 1



PTS: 2 REF: 061612aii NAT: A.REI.B.4 TOP: Solving Quadratics KEY: complex solutions | quadratic formula

If 1-i is one solution, the other is 1+i. (x-(1-i))(x-(1+i)) = 0

$$x^{2} - x - ix - x + ix + (1 - i^{2}) = 0$$
$$x^{2} - 2x + 2 = 0$$

PTS: 2 REF: 081601aii NAT: A.REI.B.4 TOP: Complex Conjugate Root Theorem 24 ANS: 4



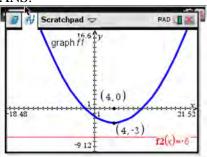
A parabola with a focus of (0,4) and a directrix of y = 2 is sketched as follows: By inspection, it is determined that the vertex of the parabola is (0,3). It is also evident that the distance, p, between the vertex and the focus is 1. It is possible to use the formula  $(x - h)^2 = 4p(y - k)$  to derive the equation of the parabola as follows:  $(x-0)^2 = 4(1)(y-3)$ 

$$x^{2} = 4y - 12$$
$$x^{2} + 12 = 4y$$
$$\frac{x^{2}}{4} + 3 = y$$

or A point (x, y) on the parabola must be the same distance from the focus as it is from the directrix. For any such point (x,y), the distance to the focus is  $\sqrt{(x-0)^2 + (y-4)^2}$  and the distance to the directrix is y-2. Setting this equal leads to:  $x^{2} + y^{2} - 8y + 16 = y^{2} - 4y + 4$ 

$$x^{2} + 16 = 4y + 4$$
$$\frac{x^{2}}{4} + 3 = y$$

PTS: 2 REF: spr1502aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions 25 ANS:



The vertex of the parabola is (4, -3). The x-coordinate of the focus and the vertex is the same. Since the distance from the vertex to the directrix is 3, the distance from the vertex to the focus is 3, so the y-coordinate of the focus is 0. The coordinates of the focus are (4,0).

PTS: 2 REF: 061630aii NAT: G.GPE.A.2 **TOP:** Graphing Quadratic Functions

The vertex is (2,-1) and p = 2.  $y = -\frac{1}{4(2)}(x-2)^2 - 1$ 

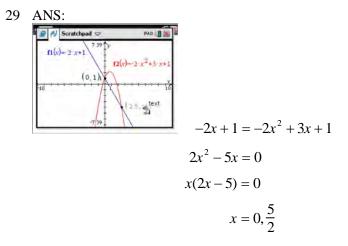
#### REF: 081619aii PTS: 2 NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions 27 ANS: Scratchpad 🤝 +3-1+5 2=45 -3 2+2 == 10 { x3 = } x+3 2+8 == 72 1-247} 6x - 3y + 2z = -10 x + 3y + 5z = 45 4x + 10z = 62 4x + 4(7) = 20-2x + 3y + 8z = 72 6x - 3y + 2z = -10 4x + 4z = 204x = -84x + 10z = 62 7x + 7z = 35 6z = 42x = -2 $4x + 4z = 20 \qquad \qquad z = 7$ 6(-2) - 3y + 2(7) = -10-3y = -12y = 4

PTS: 4 REF: spr1510aii NAT: A.REI.C.6 TOP: Solving Linear Systems KEY: three variables

28 ANS: 2

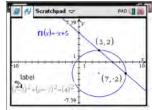
Combining (1) and (3): -6c = -18 Combining (1) and (2): 5a + 3c = -1 Using (3): -(-2) - 5b - 5(3) = 2 c = 3 5a + 3(3) = -1 2 - 5b - 15 = 2 5a = -10 b = -3a = -2

PTS: 2 REF: 081623aii NAT: A.REI.C.6 TOP: Solving Linear Systems KEY: three variables



PTS: 2 REF: fall1507aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems KEY: AII





$$y = -x + 5 \quad y = -7 + 5 = -2$$
$$(x - 3)^{2} + (-x + 5 + 2)^{2} = 16 \qquad y = -3 + 5 = 2$$
$$x^{2} - 6x + 9 + x^{2} - 14x + 49 = 16$$
$$2x^{2} - 20x + 42 = 0$$
$$x^{2} - 10x + 21 = 0$$
$$(x - 7)(x - 3) = 0$$
$$x = 7, 3$$

PTS: 4 REF: 061633aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems KEY: AII

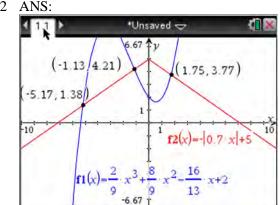
31 ANS: 3

$$-33t^2 + 360t = 700 + 5t$$

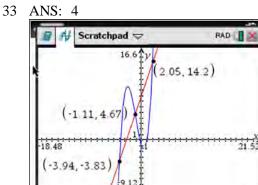
 $-33t^2 + 355t - 700 = 0$ 

$$t = \frac{-355 \pm \sqrt{355^2 - 4(-33)(-700)}}{2(-33)} \approx 3,8$$

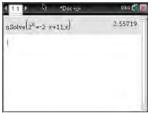
PTS: 2 REF: 081606aii NAT: A.REI.D.11 TOP: Quadratic-Linear Systems KEY: AII



PTS: 2 REF: fall1510aii NAT: A.REI.D.11 TOP: Other Systems KEY: AII

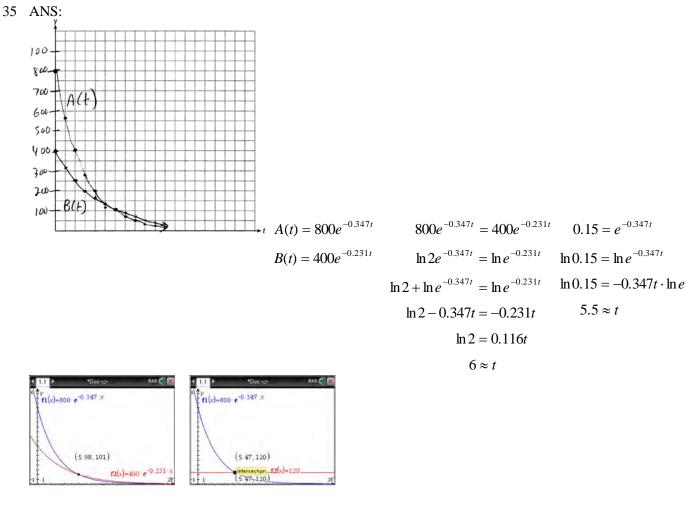


PTS: 2 NAT: A.REI.D.11 TOP: Other Systems REF: 061622aii KEY: AII



NAT: A.REI.D.11 TOP: Other Systems PTS: 2 REF: 081603aii KEY: AII

32 ANS:



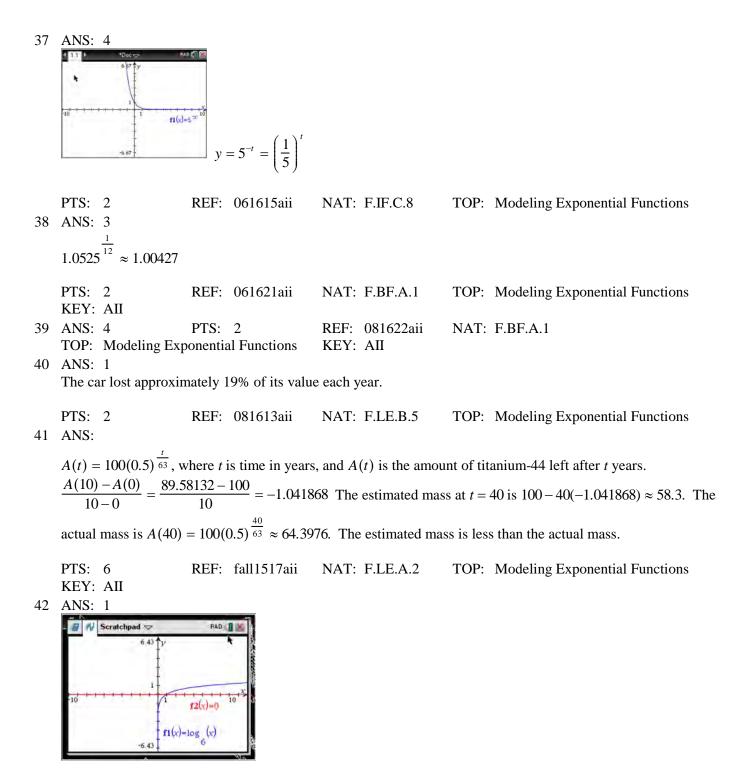
PTS: 6 REF: 061637aii NAT: A.REI.D.11 TOP: Other Systems KEY: AII

36 ANS: 2

$$B(t) = 750 \left( 1.16^{\frac{1}{12}} \right)^{12t} \approx 750(1.012)^{12t} \quad B(t) = 750 \left( 1 + \frac{0.16}{12} \right)^{12t}$$
 is wrong, because the growth is an annual rate

that is not compounded monthly.

PTS: 2 REF: spr1504aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions KEY: AII



PTS: 2 REF: 061618aii NAT: F.IF.C.7 TOP: Graphing Exponential and Logarithmic Functions

< 1.1 >	'Doc 🗢	RAE
1000 y	1	
f1(x)=72	0	
	(275-202,720)	
•	r2(x)_120000 0	0.004 (1.0
1.	12(x)-	004) <sup>x</sup> -1

$$720 = \frac{120000 \left(\frac{.048}{12}\right) \left(1 + \frac{.048}{12}\right)^n}{\left(1 + \frac{.048}{12}\right)^n - 1} \quad \frac{275.2}{12} \approx 23 \text{ years}$$

 $720(1.004)^{n} - 720 = 480(1.004)^{n}$  $240(1.004)^{n} = 720$  $1.004^{n} = 3$  $n \log 1.004 = \log 3$ 

 $n \approx 275.2$  months

PTS: 4 REF: spr1509aii NAT: A.CED.A.1 TOP: Exponential Growth 44 ANS:  $A = 5000(1.045)^n$   $5000\left(1 + \frac{.046}{4}\right)^{4(6)} - 5000(1.045)^6 \approx 6578.87 - 6511.30 \approx 67.57$   $10000 = 5000\left(1 + \frac{.046}{4}\right)^{4n}$   $B = 5000\left(1 + \frac{.046}{4}\right)^{4n}$   $2 = 1.0115^{4n}$   $\log 2 = 4n \cdot \log 1.0115$   $n = \frac{\log 2}{4\log 1.0115}$  $n \approx 15.2$ 

PTS: 6 REF: 081637aii NAT: A.CED.A.1 TOP: Exponential Growth

$$100 = 325 + (68 - 325)e^{-2k} \quad T = 325 - 257e^{-0.066t}$$

$$-225 = -257e^{-2k} \quad T = 325 - 257e^{-0.066(7)} \approx 163$$

$$k = \frac{\ln\left(\frac{-225}{-257}\right)}{-2}$$

$$k \approx 0.066$$
PTS: 4 REF: fall1513aii NAT: ELEA 4 TOP: Exponential Growth

REF: tall1513aii NAT: F.LE.A.4 TOP: Exponential Growth PTS: 446 ANS:  $A = Pe^{rt}$  $135000 = 100000e^{5r}$ 

 $1.35 = e^{5r}$  $\ln 1.35 = \ln e^{5r}$  $\ln 1.35 = 5r$ 

 $.06 \approx r \text{ or } 6\%$ 

PTS: 2 REF: 061632aii NAT: F.LE.A.4 TOP: Exponential Growth 47 ANS:

$$7 = 20(0.5)^{\frac{t}{8.02}}$$
$$\log 0.35 = \log 0.5^{\frac{t}{8.02}}$$
$$\log 0.35 = \frac{t \log 0.5}{8.02}$$
$$\frac{8.02 \log 0.35}{\log 0.5} = t$$
$$t \approx 12$$

PTS: 4

REF: 081634aii NAT: F.LE.A.4 TOP: Exponential Decay

48 ANS: 4  $k^{4} - 4k^{2} + 8k^{3} - 32k + 12k^{2} - 48$  $k^{2}(k^{2}-4) + 8k(k^{2}-4) + 12(k^{2}-4)$  $(k^2 - 4)(k^2 + 8k + 12)$ (k+2)(k-2)(k+6)(k+2)PTS: 2 REF: fall1505aii NAT: A.SSE.A.2 **TOP:** Factoring Polynomials KEY: factoring by grouping 49 ANS: The expression is of the form  $y^2 - 5y - 6$  or (y - 6)(y + 1). Let  $y = 4x^2 + 5x$ :  $(4x^2+5x-6)(4x^2+5x+1)$ (4x-3)(x+2)(4x+1)(x+1)PTS: 2 REF: fall1512aii NAT: A.SSE.A.2 **TOP:** Factoring Polynomials KEY: a>1 50 ANS: 3  $(m-2)^{2}(m+3) = (m^{2}-4m+4)(m+3) = m^{3}+3m^{2}-4m^{2}-12m+4m+12 = m^{3}-m^{2}-8m+12$ **PTS:** 2 REF: 081605aii NAT: A.SSE.A.2 **TOP:** Factoring Polynomials KEY: factoring by grouping 51 ANS: 3  $2d(d^3 + 3d^2 - 9d - 27)$  $2d(d^2(d+3) - 9(d+3))$  $2d(d^2 - 9)(d + 3)$ 2d(d+3)(d-3)(d+3) $2d(d+3)^2(d-3)$ PTS: 2 REF: 081615aii NAT: A.SSE.A.2 **TOP:** Factoring Polynomials KEY: factoring by grouping

52 ANS: 1

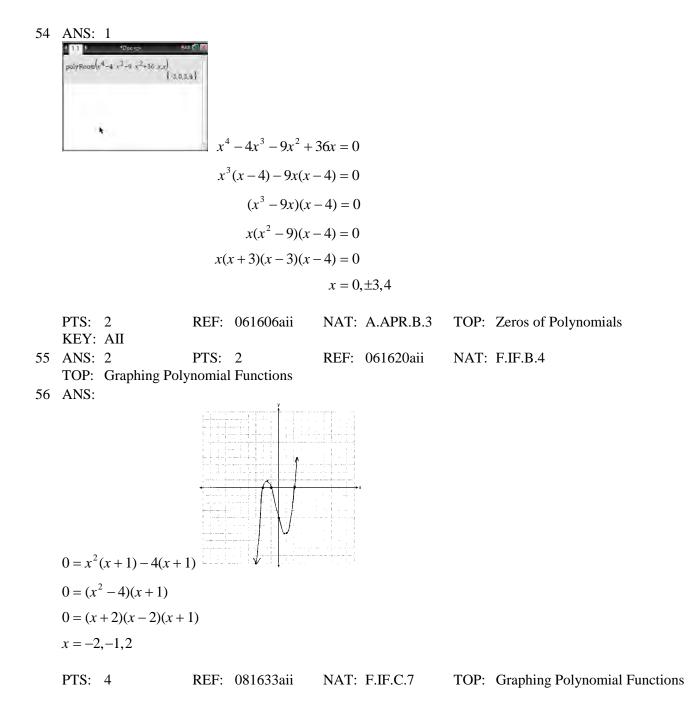
The zeros of the polynomial are at -b, and c. The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as x goes to negative infinity, f(x) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

PTS: 2 REF: spr1501aii NAT: A.APR.B.3 TOP: Zeros of Polynomials KEY: AII

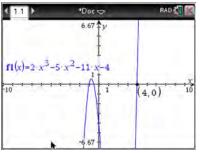
53 ANS: 3

The graph shows three real zeros, and has end behavior matching the given end behavior.

PTS: 2 REF: 061604aii NAT: A.APR.B.3 TOP: Zeros of Polynomials KEY: AII



 $f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0$  Any method that demonstrates 4 is a zero of f(x) confirms



that x - 4 is a factor, as suggested by the Remainder Theorem.

PTS: 2 REF: spr1507aii NAT: A.APR.B.2 TOP: Remainder Theorem 58 ANS:  $0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 - 5(x) = 6x^3 + 10x^2 - 52x + 15$ 

$$0 = 6(-5)^{2} + b(-5)^{2} - 52(-5) + 15 \quad z(x) = 6x^{2} + 19x^{2} - 52x + 15$$

$$0 = -750 + 25b + 260 + 15$$

$$475 = 25b$$

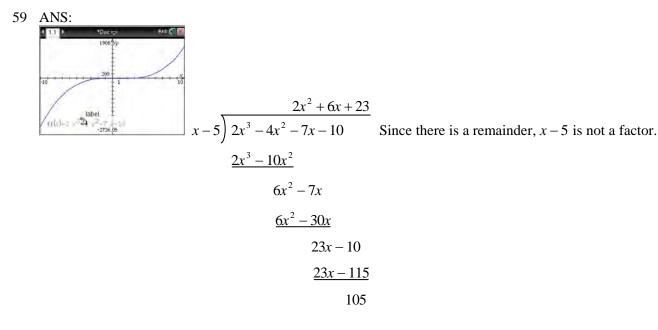
$$19 = b$$

$$-5 \boxed{\begin{array}{c} 6 & 19 & -52 & 15 \\ \hline -30 & 55 & 15 \\ \hline -6 & -11 & 3 & 0 \end{array}}$$

$$6x^{2} - 11x + 3 = 0$$

$$(2x - 3)(3x - 1) = 0$$

$$x = \frac{3}{2}, \frac{1}{3}, -5$$
PTS: 4 REF: fall1515aii NAT: A.APR.B.2 TOP: Remainder Theorem



PTS: 2 REF: 061627aii NAT: A.APR.B.2 TOP: Remainder Theorem 60 ANS: 3

Since x + 4 is a factor of p(x), there is no remainder.

PTS: 2 REF: 081621aii NAT: A.APR.B.2 TOP: Remainder Theorem 61 ANS:

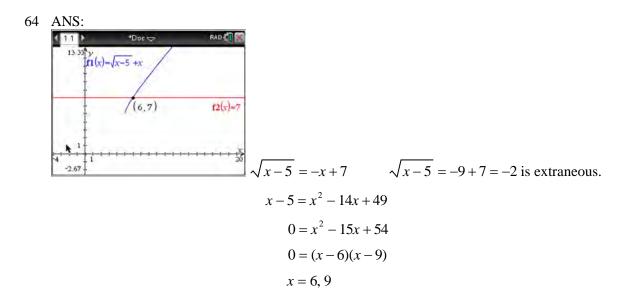
Let x equal the first integer and x + 1 equal the next.  $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$ . 2x + 1 is an odd integer.

PTS: 2 REF: fall1511aii NAT: A.APR.C.4 TOP: Polynomial Identities 62 ANS:

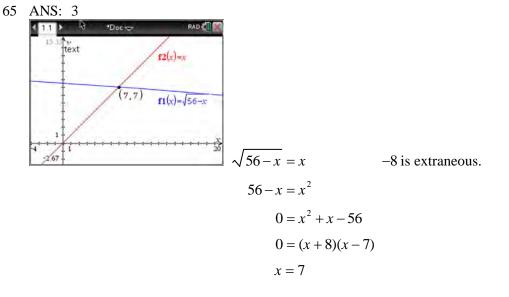
 $\frac{x^3+9}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$  $\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$ 

PTS: 2 REF: 061631aii NAT: A.APR.C.4 TOP: Polynomial Identities 63 ANS: 4  $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3$ 

PTS: 2 REF: 081620aii NAT: A.APR.C.4 TOP: Polynomial Identities



PTS: 2 REF: spr1508aii NAT: A.REI.A.2 TOP: Solving Radicals KEY: extraneous solutions



PTS: 2 REF: 061605aii NAT: A.REI.A.2 TOP: Solving Radicals KEY: extraneous solutions

$$\left(\sqrt{2x-7}\right)^2 = (5-x)^2 \qquad \sqrt{2(4)-7} + 4 = 5 \quad \sqrt{2(8)-7} + 8 = 5$$
$$2x - 7 = 25 - 10x + x^2 \qquad \sqrt{1} = 1 \qquad \sqrt{9} \neq -3$$
$$0 = x^2 - 12x + 32$$
$$0 = (x-8)(x-4)$$
$$x = 4, 8$$

PTS: 4 REF: 081635aii NAT: A.REI.A.2 TOP: Solving Radicals KEY: extraneous solutions

67 ANS:

Applying the commutative property,  $\left(3^{\frac{1}{5}}\right)^2$  can be rewritten as  $\left(3^2\right)^{\frac{1}{5}}$  or  $9^{\frac{1}{5}}$ . A fractional exponent can be

rewritten as a radical with the denominator as the index, or  $9^{\frac{1}{5}} = \sqrt[5]{9}$ .

PTS: 2 REF: 081626aii NAT: N.RN.A.1 TOP: Radicals and Rational Exponents 68 ANS:  $\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^{y}$ 

 $x^{\frac{4}{3}} = x^{y}$  $\frac{4}{3} = y$ 

REF: spr1505aii PTS: 2 NAT: N.RN.A.2 **TOP:** Radicals and Rational Exponents 69 ANS: 4 PTS: 2 REF: 061601aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents 70 ANS: (4-3i)(5+2yi-5+2yi)(4 - 3i)(4vi) $16yi - 12yi^2$ 12y + 16yiPTS: 2 REF: spr1506aii NAT: N.CN.A.2 TOP: Operations with Complex Numbers 71 ANS: 2  $(2-yi)(2-yi) = 4 - 4yi + y^{2}i^{2} = -y^{2} - 4yi + 4$ PTS: 2 REF: 061603aii NAT: N.CN.A.2 TOP: Operations with Complex Numbers

 $xi(-6i)^2 = xi(36i^2) = 36xi^3 = -36xi$ 

5

PTS: 2 73 ANS: 1  $2x+3\sqrt{6x^3+17x^2+10x+2}$   $6x^3+9x^2$   $8x^2+10x$  -2x+2 -2x-3REF: 081627aii NAT: N.CN.A.2 TOP: Operations with Complex Numbers

PTS: 2 REF: fall1503aii NAT: A.APR.D.6 TOP: Rational Expressions KEY: remainder

74 ANS: 2

$$2x^{2} - 3x + 7$$

$$2x + 3 \overline{\smash{\big)}} 4x^{3} + 0x^{2} + 5x + 10$$

$$4x^{3} + 6x^{2}$$

$$- 6x^{2} + 5x$$

$$-6x^{2} - 9x$$

$$14x + 10$$

$$14x + 21$$

$$- 11$$

PTS: 2 REF: 061614aii NAT: A.APR.D.6 TOP: Rational Expressions KEY: remainder

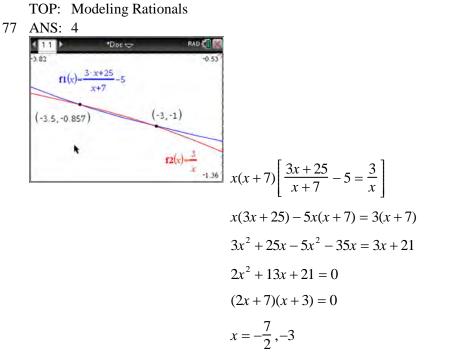
75 ANS: 2  

$$\begin{array}{rcl}
x^{2} + 0x + 1 \\
x + 2 \end{array} \\
x^{3} + 2x^{2} + x + 6 \\
\underline{x^{3} + 2x^{2}} \\
0x^{2} + x \\
\underline{0x^{2} + 0x} \\
x + 6 \\
\underline{x + 2} \\
4
\end{array}$$
PTS: 2  
REF: 081611aii  
KEY: remainder  
76 ANS: 3
PTS: 2

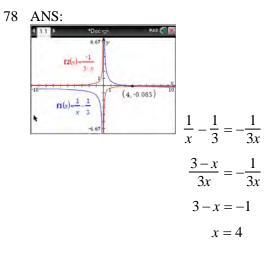
NAT: A.APR.D.6 **TOP:** Rational Expressions

NAT: A.CED.A.1

REF: 061602aii



PTS: 2 REF: fall1501aii NAT: A.REI.A.2 **TOP:** Solving Rationals KEY: rational solutions



PTS: 2 REF: 061625aii NAT: A.REI.A.2 TOP: Solving Rationals KEY: rational solutions

79 ANS: 3

$$\frac{1}{J} = \frac{1}{F} - \frac{1}{W}$$
$$\frac{1}{J} = \frac{W - F}{FW}$$
$$J = \frac{FW}{W - F}$$

PTS: 2 REF: 081617aii NAT: A.REI.A.2 TOP: Solving Rationals KEY: rational solutions

80 ANS: 3

f(x) = -f(x), so f(x) is odd.  $g(-x) \neq g(x)$ , so g(x) is not even.  $g(-x) \neq -g(x)$ , so g(x) is not odd. h(-x) = h(x), so h(x) is even.

PTS: 2 REF: fall1502aii NAT: F.BF.B.3 TOP: Even and Odd Functions 81 ANS: 1

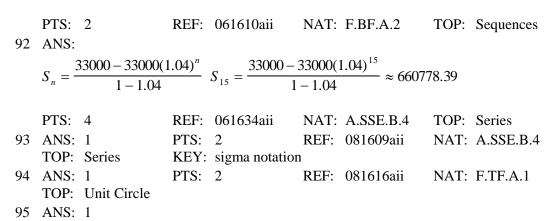
The graph of  $y = \sin x$  is unchanged when rotated 180° about the origin.

PTS: 2 REF: 081614aii NAT: F.BF.B.3 TOP: Even and Odd Functions 82 ANS: 4  $\frac{m(c)}{g(c)} = \frac{c+1}{1-c^2} = \frac{c+1}{(1+c)(1-c)} = \frac{1}{1-c}$ PTS: 2 REF: 061608aii NAT: F.BF.A.1 TOP: Operations with Functions

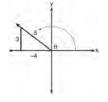
 $0 = \log_{10}(x-4)$  The *x*-intercept of *h* is (2,0). *f* has the larger value.

 $10^0 = x - 4$ 1 = x - 4*x* = 5 PTS: 2 REF: 081630aii NAT: F.IF.C.9 **TOP:** Comparing Functions KEY: AII 84 ANS:  $x = \left(y - 3\right)^3 + 1$  $x-1 = \left(y-3\right)^3$  $\sqrt[3]{x-1} = y - 3$  $\sqrt[3]{x-1} + 3 = y$  $f^{-1}(x) = \sqrt[3]{x-1} + 3$ NAT: F.BF.B.4 PTS: 2 REF: fall1509aii **TOP:** Inverse of Functions **KEY:** equations 85 ANS: 2  $x = -\frac{3}{4}y + 2$ -4x = 3y - 8-4x + 8 = 3y $-\frac{4}{3}x + \frac{8}{3} = y$ TOP: Inverse of Functions PTS: 2 REF: 061616aii NAT: F.BF.B.4 **KEY:** equations 86 ANS:  $a_n = x^{n-1}(x+1) \ x^{n-1} = 0 \ x+1 = 0$ x = 0 x = -1PTS: 4 REF: spr1511aii NAT: F.LE.A.2 **TOP:** Sequences 87 ANS: 3 PTS: 2 REF: 061623aii NAT: F.LE.A.2 **TOP:** Sequences 88 ANS: 1 (2) is not recursive PTS: 2 REF: 081608aii NAT: F.LE.A.2 **TOP:** Sequences 89 ANS: 3 PTS: 2 REF: 081618aii NAT: F.LE.A.2 **TOP:** Sequences

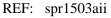
- 90 ANS: 4 PTS: 2 REF: 081624aii NAT: F.LE.A.2 TOP: Sequences 91 ANS: 4
  - The scenario represents a decreasing geometric sequence with a common ratio of 0.80.



A reference triangle can be sketched using the coordinates (-4,3) in the second quadrant to find the value of  $\sin \theta$ .



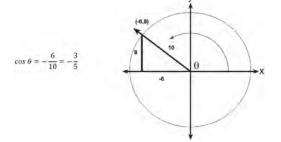
PTS: 2 96 ANS: 1



REF: 081628aii

NAT: F.TF.A.2 TOP: Determining Trigonometric Functions

TOP: Determining Trigonometric Functions



PTS: 2 REF: 061617aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions 97 ANS:

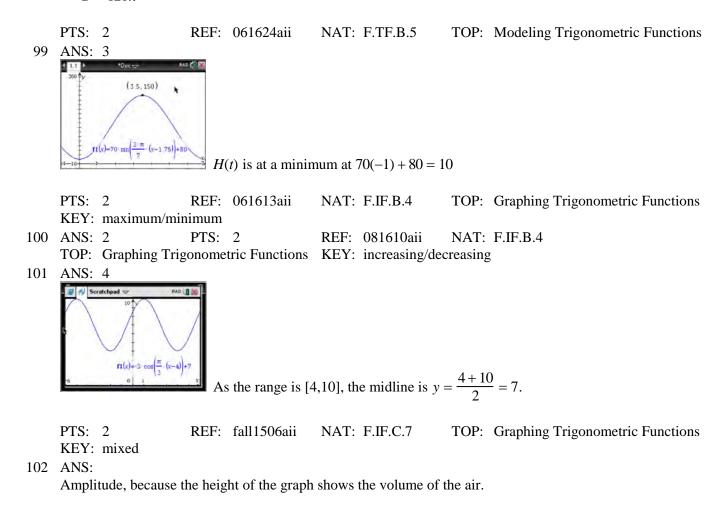
$$\sin^2 \theta + (-0.7)^2 = 1$$
 Since  $\theta$  is in Quadrant II,  $\sin \theta = \sqrt{.51}$  and  $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{.51}}{-0.7} \approx -1.02$   
$$\sin^2 \theta = .51$$
  
$$\sin \theta = \pm \sqrt{.51}$$

NAT: F.TF.C.8

PTS: 2

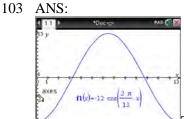
23

98 ANS: 4 period =  $\frac{2\pi}{B}$  $\frac{1}{60} = \frac{2\pi}{B}$  $B = 120\pi$ 



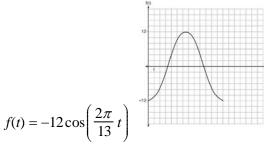
PTS: 2 REF: 081625aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: mixed

ID: A



The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of -12 and a maximum of 12. The value of A is -12 since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter B. By experimentation with

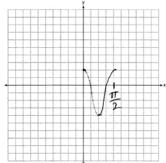
technology or using the relation  $P = \frac{2\pi}{B}$  (where P is the period), it is determined that  $B = \frac{2\pi}{13}$ .



In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from t = 13 to t = 19.5 (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

PTS: 6 REF: spr1514aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: graph

104 ANS:



PTS: 2 REF: 061628aii NAT: F.IF.C.7 TOP: Graphing Trigonometric Functions KEY: graph

105 ANS: 4

 $4(x^{2} - 6x + 9) + 4(y^{2} + 18y + 81) = 76 + 36 + 324$ 

 $4(x-3)^2 + 4(y+9)^2 = 436$ 

PTS: 2 REF: 061619aii NAT: A.SSE.A.2 TOP: Equations of Conics