

The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

ADVANCED ALGEBRA

Monday, January 17, 1916—9.15 a. m. to 12.15 p. m., only

Answer eight questions. Each answer should be reduced to its simplest form. Papers entitled to less than 75 credits will not be accepted.

1 Find the linear factors of $3x^2 + 4x + 7$. Verify your result by multiplying the factors together. Perform the operation in full on your paper.

2 a Five flags, no two of the same color, can either be displayed singly or displayed any number at a time one above another; how many different signals can be made with them?

b From 14 books, in how many ways can a selection of 5 be made (1) when one specified book is always included, (2) when one specified book is always excluded?

3 A man is in a boat 3 miles from the nearest point of a straight beach; if he rows at the rate of 4 miles an hour and walks at the rate of 5 miles an hour, where ought he to land in order to reach in 1 hour and 27 minutes a point on the beach 5 miles from the point directly opposite him? Would it be possible for him to reach his destination in an hour and a quarter?

4 Reduce *each* of the following expressions to the standard form $a + ib$:

$$(2 + \sqrt{-3})(7 - \sqrt{-4}); \left(\frac{-1 + \sqrt{-3}}{2}\right)^3; \frac{2}{-1 + \sqrt{3}i}; \left(\frac{1+i}{1-i}\right)^2; \frac{1}{1}$$

5 Use Descartes' rule of signs to obtain all the information you can about the number of positive and of negative roots of *each* of the following equations:

$$x^4 + 3x^2 + 5x - 1 = 0$$

$$x^4 + 3x^2 - 5x + 1 = 0$$

$$x^5 - 6 = 0$$

$$6x^4 - x^3 - x^2 - 75x - 25 = 0 \quad [\text{This equation has an imaginary root.}]$$

6 Show that $1+i$ is a root of the following equation: $2x^3 - x^2 - 2x + 6 = 0$. Find the other roots, represent all of the roots graphically and show graphically that their sum is related to the first two coefficients in the given equation.

ADVANCED ALGEBRA—concluded

7 Prove that when $f(x)$ is divided by $x-r$ the remainder is $f(r)$. [Given $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$]

8 The volume of a box $10 \times 12 \times 15$ inches is to be increased 50 cubic inches by adding the same amount to each dimension; what should this amount be? [Solve by Horner's method to *three* decimal places.]

9 Plot the graphs of the equations $2x^2 - 3y = 20$ and $y^2 + 5x = 36$. Estimate from the graphs the values of the roots of this pair of simultaneous equations.

10 a Reduce $x - \frac{\left(1 + \frac{4p^2}{y^2}\right) \frac{2p}{y}}{\frac{-4p^2}{y^3}}$ to $3x + 2p$ when $y^2 = 4px$

b For what values of k will the roots of $9x^2 + 6kx + k^2 = 10x$ be equal?

DIRECTIONS FOR RATING

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The direction, "Less than 60% of the credit should be granted when an error in computation occurs," should be followed in rating all incorrect answers to questions which fall under the topics mentioned in "Suggestions on the Rating of Regents Examination Papers in Mathematics" under "General 3."

Except in schools where the "committee system" is used, teachers are urged to mark papers cumulatively, that is, to add the credits earned by each answer to the total credits earned by preceding answers so that the mark given to the last answer is the per cent to which the paper is entitled, e. g. consecutive answers earning 5, 7, 4 etc. respectively should be marked 5, 12, 16 etc. respectively.

12½ credits each

- 1 Allow $6\frac{1}{2}$ credits for finding correct factors.
Allow 6 credits for correct multiplication.
- 2 a Allow $6\frac{1}{2}$ credits.
b1 Allow 3 credits.
b2 Allow 3 credits.
Allow no partial credits on (b1) or on (b2).
- 3 Allow $7\frac{1}{2}$ credits for correct equation.
Allow 3 credits for correct solution.
Allow 2 credits for correct answer to second question.
- 4 Allow $2\frac{1}{2}$ credits for each correct result.
Allow no partial credit for any one expression.
- 5 Allow 3 credits each for the first three.
Allow $3\frac{1}{2}$ credits for the fourth.
- 6 Allow 3 credits for showing that $1+i$ is a root.
Allow 3 credits for finding other roots correctly.
Allow $3\frac{1}{2}$ credits for correct graph.
Allow 3 credits for showing that the sum is related to the coefficients.
- 7 See "Suggestions" under "General 7."
- 8 Allow 3 credits for correct formation of equation.
Allow 2 credits for finding the first figure of the root correctly.
Allow 3 credits for finding the second figure of the root correctly.
Allow $4\frac{1}{2}$ credits for finding the third figure of the root correctly.

DIRECTIONS FOR RATING—concluded

- 9 Allow 2 credits for each correct graph.
Allow 2 credits each for the values of the roots correct within $\frac{2}{10}$ of the value.
Allow $\frac{1}{2}$ credit for neatness and accuracy.
- 10 a $6\frac{1}{2}$ credits
Allow no partial credit.
- b 6 credits
Allow 4 credits for equation $(6k - 10)^2 = 36k^2$
Allow 2 credits for correct values of k .