

ADVANCED ALGEBRA

Monday, January 21, 1918—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

1 Draw the graph of the expression $2x^3 - 3x^2 - 7x + 5$. If the expression is set equal to zero and the roots of the resulting equation are required, explain why it is unnecessary to extend the table of values beyond $x = -3$ and $x = +4$.

2 Find to two decimal places the real root of the equation $x^3 + 7x - 11 = 0$.

3 a Solve the equation $2x^4 - 7x^3 + 11x^2 - 8x + 2 = 0$

b Check by forming the equation from the roots, clearly indicating the method.

4 a State and prove the remainder theorem.

b If $f(x) = 4x^3 - 12x^2 - 8x + 20$, find $f(1)$, $f(\frac{1}{2})$, $f(-2)$, $f(8)$, $f(\sqrt{2})$.

5 Given the equation $x^4 - 2x^2 - 6 = 0$

Without solving the equation, fill out the following statements by placing the proper number in each parenthesis; justify each statement by means of a theorem or principle:

a The equation has () roots.

b It has () positive roots and () negative roots.

c It has () fractional roots.

d It has at least () imaginary roots.

e All the integral roots, if any, must be factors of ().

f The sum of the roots is ().

6 A druggist has a set of 8 different weights and always weighs with the same scale pan; how many different combinations of these weights are possible, if he uses any number of them at a time?

7 Solve the equation $x^3 - 1 = 0$. Show graphically that the sum of the roots is 0.

8 A sum of money, P dollars, is put out at simple interest, at the annual rate r .

a Find a formula for the amount A at the end of n years.

b From this formula find P .

9 Given two geometric progressions. Prove that if their corresponding terms are multiplied together, the products will also form a geometric progression.

10 A certain point of a pendulum describes a path 8 inches long during the first swing; if each consecutive swing is $\frac{1}{2}$ less than the preceding one, what is the total length of the path described by the point before the pendulum comes to rest?

11 Examine the following "proof" and point out the mistake in the argument:

Given $x = a$. Then $x^2 = ax$. Hence $x^2 - a^2 = ax - a^2$, and $(x + a)(x - a) = a(x - a)$. Therefore $x + a = a$. But $x = a$. Hence $2a = a$, and $2 = 1$.

12 The capacity of a tank $3 \times 4 \times 5$ feet is to be doubled by increasing each dimension by the same amount; find the new dimensions algebraically.

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DIRECTIONS FOR RATING

The direction, "Less than 60% of the credit should be granted when an error in computation occurs," should be followed in rating all incorrect answers to questions which fall under the topics mentioned in "Suggestions on the Rating of Regents Examination Papers in Mathematics" under "General 3."

In all problems solved with two unknowns no credit should be given for one equation correctly formed if the other is not given or is inaccurate.

Except in schools where the "committee system" is used, teachers are urged to mark papers cumulatively, that is, to add the credits earned by each answer to the total credits earned by preceding answers so that the mark given to the last answer is the per cent to which the paper is entitled, e. g. consecutive answers earning 5, 7, 4 etc. respectively should be marked 5, 12, 16 etc. respectively.

- 1 $12\frac{1}{2}$ credits
Allow $8\frac{1}{2}$ credits for correct graph.
Allow 4 credits for explanation.
- 2 $12\frac{1}{2}$ credits
Allow 3 credits for finding first figure of root.
Allow 4 credits for finding second figure of root.
Allow $5\frac{1}{2}$ credits for finding third figure of root.
- 3 $12\frac{1}{2}$ credits
a $6\frac{1}{2}$ credits
b 6 credits
- 4 $12\frac{1}{2}$ credits
a $7\frac{1}{2}$ credits
b 5 credits
- 5 $12\frac{1}{2}$ credits
Allow $2\frac{1}{2}$ credits for b , 2 credits for each of the others.
- 6 $12\frac{1}{2}$ credits
See "General Suggestion 3."
- 7 $12\frac{1}{2}$ credits
Allow 4 credits for correct solution.
Allow $8\frac{1}{2}$ credits for correct graph.

DIRECTIONS FOR RATING—concluded

- 8 $12\frac{1}{2}$ credits
a 6 credits
b $6\frac{1}{2}$ credits
- 9 $12\frac{1}{2}$ credits
See "General Suggestion 4."
- 10 $12\frac{1}{2}$ credits
Allow 1 credit for correct formula.
Allow 8 credits for correct substitution.
Allow $3\frac{1}{2}$ credits for correct solution.
- 11 $12\frac{1}{2}$ credits
See "General Suggestion 4."
- 12 $12\frac{1}{2}$ credits
Allow 6 credits for correct equation.
Allow $6\frac{1}{2}$ credits for correct solution.