

ADVANCED ALGEBRA

Monday, January 21, 1924—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 The first term of an arithmetic progression is 2, and the first, third and eleventh terms are also the first three terms of a geometric progression. Find the sum of the first eleven terms of the arithmetic progression. $[12\frac{1}{2}]$

2 Verify by substitution that $1 + \sqrt{-2}$ is a root of the equation $2x^3 - 7x^2 + 12x - 9 = 0$. Find the other roots. Add graphically the three roots of the equation. $[6, 3\frac{1}{2}, 3]$

3 Transform the equation $2x^3 - 3x^2 - 4x + 6 = 0$ into an equation in which the coefficients are integers, that of the highest degree term being unity. Find the roots of the resulting equation and of the given equation. $[7\frac{1}{2}, 3, 2]$

4 Find two values of y for which the equation $x^3 - 6x^2 + 9x - y = 0$ has two equal roots. $[12\frac{1}{2}]$

5 a In how many years will a sum of money double itself at 6% if the interest is compounded annually? $[4]$

b Given $p = \left(\frac{2}{x+1}\right)^{\frac{x}{x+1}}$; find the value of p if $x = 1.41$. $[8\frac{1}{2}]$

6 a How many different sums of money can be made from a cent, a nickel, a dime, a quarter, a half dollar and a dollar? $[6\frac{1}{2}]$

b How many different signals can be made with six flags of different colors if three are used at a time in a vertical line? $[6]$

7 By the use of Horner's method, find to the nearest tenth the positive root of

$$x^3 + 2x^2 - 5x - 9 = 0 \quad [12\frac{1}{2}]$$

8 Two motor boats race 25 miles down a river and the same distance back; at the end of the first half hour one boat is three quarters of a mile ahead, and later it meets the other boat one mile from the turning point. Find the rates of the boats in still water, if the rate of the river is 3 miles an hour and if both boats can make headway upstream. $[12\frac{1}{2}]$

9 Find the value of k that will make the two values of x equal in the solution of the simultaneous equations

$$y = 2x + k$$

$$y = x^2 - 5x + 6 \quad [12\frac{1}{2}]$$

[This is the condition that will make the straight line become tangent to the parabola.]

10 a Plot between the values $x = -3$ and $x = +5$ the curve represented by the following equation:

$$y = x^3 - 3x^2 - 9x + 15 \quad [9\frac{1}{2}]$$

b From the curve estimate to the nearest tenth the x intercepts. $[3]$

11 Assuming that every rational equation has a root, prove that an equation of the n th degree has exactly n roots. $[12\frac{1}{2}]$