The University of the State of New York

235TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Thursday, January 21, 1926 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Each answer should be reduced to its simplest form.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

1 *a* Represent graphically the irrational numbers $1+\sqrt{2}$, $1-\sqrt{2}$; and the complex numbers $-1+\sqrt{-3}$, $-1-\sqrt{-3}$. [6]

b Form the equation of which these numbers are the roots. [$6\frac{1}{2}$]

2 *a* For what values of k will the roots of the equation $x^2+kx+(2k+\frac{9}{4})=0$ be real and equal? real and different? imaginary? [4, 2, 2]

b Show that the equation $x^3-7x+3=0$ has either one or three real roots. [$4\frac{1}{2}$]

3 Find the four rational roots of the equation

$$8x^4-14x^3-9x^2+11x-2=0 \qquad [12\tfrac{1}{2}]$$

4 Find to the *nearest tenth* the positive root of the equation

$$x^3-3x-7=0 \qquad [12\tfrac{1}{2}]$$

5 *a* In how many years will a sum of money treble itself if the interest at 6% is compounded annually? [$7\frac{1}{2}$]

b Show that $\frac{1}{2}[\log(a+b)-\log a]=\log\sqrt{1+\frac{b}{a}}$ [5]

6 *a* How many different numbers greater than 2000 can be formed with the digits 1, 2, 3, 4, if repetitions are not allowed? [6]

b Determine how many words (counting any set a word) consisting of 2 vowels and 3 consonants can be formed from the letters of the word *stenographic*. [$6\frac{1}{2}$]

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7 Transform the equation $x^3-\frac{1}{2}x^2+\frac{1}{3}x+\frac{1}{12}=0$ into an equation of the form $x^3+px+q=0$, where p and q are integers. [$12\frac{1}{2}$]

8 The sum of an infinite number of terms of a decreasing geometric progression is 3 and the sum of their squares is also 3; write the first three terms of the progression. [$12\frac{1}{2}$]

9 In the following set of equations determine the values of k so that two values of x in the solution shall be equal, that is, the graph of the second equation shall be tangent to the graph of the first equation: [The drawing of the two graphs is not required.]

$$y=x^3-3x^2-8x+1$$
$$y=x+k \qquad [12\tfrac{1}{2}]$$

10 The edges of two hollow cubes differed by 10 inches. When a certain quantity of water was poured into the larger cube there remained 1578 cubic inches of space not filled with water. When the second cube was filled from this same quantity of water there were 142 cubic inches of water left. Find the dimensions of each cube. [7, $5\frac{1}{2}$]

11 Solve the following set of equations:

$$x^3+y^3=468$$
$$x^2y+xy^2=420 \qquad [12\tfrac{1}{2}]$$

12 A rectangular tin box with an open top and square base is to have a total outside area of 48 square feet.

a Express the volume y in terms of a base edge x. [5]

b Plot the graph of the relation expressed in answer to *a* for values of x from 0 to 7 inclusive. [5]

c From the graph determine the length of a base edge that will make the greatest volume. [$2\frac{1}{2}$]