The University of the State of New York

259TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Thursday, January 25, 1934 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and reduced to its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

ADVANCED ALGEBRA

Thursday, January 25, 1934

Fill in the following lines:

Name of school..Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has $2\frac{1}{2}$ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 Find the result of multiplying $x + i\sqrt{3}$ by its conjugate. *Ans*............

2 If the numbers 3, $-1 + i\sqrt{3}$ and $-1 - i\sqrt{3}$ are represented graphically by the points P, Q and R, of what kind of triangle are these points the vertices? *Ans*............

3 Write in the form $y = mx + b$ the equation of the straight line passing through the points (0, 3) and (3, 5). *Ans*............

4 In the equation $K^2x^2 + Kx + 1 = 0$, is it possible to assign any value to K that will produce real roots? [Answer *Yes* or *No*.] *Ans*............

5 Is the following statement *true* or *false?*
A straight line may intersect a hyperbola in more than two points. *Ans*............

6 What values must b have in order that the graph of the function $y = x^2 + bx + 9$ will be tangent to the x-axis? *Ans*............

7 Transform the equation $x^3 - 6x^2 + 12x + 2 = 0$ into an equation whose roots are less by 2 than the roots of the given equation. *Ans*............

8 Transform the equation $x^3 - x - 1 = 0$ into an equation whose roots are the negative of the roots of the given equation. *Ans*............

9 Can the equation $x^{2n} + 1 = 0$ have any real positive or negative roots if n is a positive integer? [Answer *Yes* or *No*.] *Ans*............

10 Find the real rational root of the equation $2x^3 + x^2 - 7x + 3 = 0$ *Ans*............

11 Given $\sqrt[n]{b} = c$; express n as a function of the logarithms of b and c. *Ans*............

12 How many lines can be drawn joining n points, no three of which lie in the same straight line? *Ans*............

13 From the first five letters of the alphabet how many "code" words of three letters each are possible if no letter is repeated in any word? *Ans*............

14 If from a group of six secret-service men, of which A is a member, two are chosen, what is the probability that A will be among those chosen? *Ans*............

15 Write the first *three* terms of the expansion $(x^2 - \frac{1}{x})^{10}$ *Ans*............

16 The electrical resistance R of a wire varies inversely as the square of its diameter d; that is, $R = \frac{k}{d^2}$; express R as a function of d, given that $R = 20$ when $d = 2$ *Ans*............

17 The length of one wave of sodium light, expressed in scientific notation, is 5.89×10^{-5} cm; write this number as a decimal. *Ans*............

18 In finding $\log_e 3$, by what logarithm must $\log_{10} 3$ be divided? *Ans*............

19 In an arithmetic progression the first term is 2 and the difference is 2. Express in simplest form the sum S as a function of the number of terms n. *Ans*............

20 If the volume of gas in an inclosed vessel is represented by A and each stroke of the pump removes one half of the gas remaining in the vessel, find the amount remaining in the vessel after the nth stroke. [Express your answer in terms of A and n.] *Ans*............

ADVANCED ALGEBRA

Thursday, January 25, 1934

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.
The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 The roots of the equation $2x^3 - 11x^2 + 10x + 8 = 0$ are represented by r, $2r$ and s.

a Using the coefficient of the second term of this equation, write a linear equation connecting r and s. [2]

b Using the coefficient of the third term of this equation, write a quadratic equation connecting r and s. [2]

c Using the two equations found in answer to *a* and *b*, find the roots of the given equation. [6]

22 Find to the *nearest tenth* the positive root of the equation $x^3 + 3x^2 - 2x - 5 = 0$ [10]

23 *a* Using the same set of axes, plot the graph of $(x - 3)^2 + y^2 = 9$ and of $x + y^2 = 6$ from $x = 0$ to $x = 6$ inclusive. [8]

b Indicate on the graphs made in answer to *a* the points that represent the real solutions of the following set of equations and estimate these solutions correct to the *nearest tenth*:

$$(x - 3)^2 + y^2 = 9$$
$$x + y^2 = 6$$
[2]

24 The sum of the digits of a certain two-digit number is a. If the order of the digits is reversed, the number is decreased by b. Find the two digits in terms of a and b and from them show that in order for the solution to be admissible, $9a$ must either equal b or be greater than b and that $9a \pm b$ must be divisible by 18. [10]

25 The parabola $y = ax^2 + bx + c$ passes through the points $(1, -2)$, $(2, 1)$ and $(0, -1)$; determine a, b and c. [10]

26 The amount A that must be invested at annual rate of interest r, interest compounded semi-annually, in order that a sum S may be withdrawn every 6 months for a period of n years, is given by the formula

$$A = S\left[\frac{1 - \left(1 + \frac{r}{2}\right)^{-2n}}{\frac{r}{2}}\right]$$

Find A, correct to the *nearest dollar*, if $S = \$500$, $r = .04$ and $n = 4$ years [10]

27 A motorist traveling at the rate of 35 miles an hour leaves Albany for New York, a distance of 155 miles, half an hour before a second motorist, traveling at the rate of 40 miles an hour, leaves New York for Albany.

a Using one pair of axes, and representing on the horizontal axis time in hours and on the vertical axis distance in miles, show graphically the distance traveled by each motorist at any time. [4]

b From the graph made in answer to *a*, determine when and where the two motorists pass each other. [3]

c Determine the equation of *one* of the two lines in the graph made in answer to *a*. [3]

176

*28 Find the equation of the straight line tangent to the parabola $y = -x^2 + 3$ at the point whose abscissa is -1. [10]

*29 A business man estimates that if he charges 8¢ a pound for a certain commodity he will sell 600 pounds a week and that for each decrease of ½¢ a pound on the selling price he will increase his sales by 150 pounds a week.

a Representing the number of decreases of ½¢ a pound by x, express the gross returns y as a function of x. [6]

b Using differentiation, determine the selling price a pound that will produce the greatest gross returns. [4]

---

* This question is based on one of the optional topics in the syllabus.