

The University of the State of New York

299TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 29, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

- 21 Find, correct to the *nearest tenth*, the positive root of the equation $x^3 - 3x - 4 = 0$ [10]
- 22 Solve completely the equation $3x^4 - 16x^3 + 14x^2 + 24x - 9 = 0$ [10]
- 23 The present value V of an annual pension A which is to run for n years at rate r is given by the formula $V = \frac{A}{r} \left[1 - \frac{1}{(1+r)^n} \right]$
- a Using logarithms, find the value of $\frac{1}{(1+r)^n}$ to 4 decimal places if $r = 4\%$ and $n = 18$ years. [6]
- b Using the result obtained in answer to a, find the value of V if $A = \$1200$ [4]
- 24 a State and prove the remainder theorem. [6]
- b Find the remainder when $x^{50} - 2x^{24} + 1$ is divided by $x - 1$ [2]
- c Is $x - 1$ a factor of $x^{50} - 2x^{24} + 1$? [2]
- 25 Two trains are moving in the same direction on double tracks. During the interval from $t = 0$ to $t = 5$, the distance in miles of one train from a station is given by the equation $d = 20 + 10t - t^2$ and the distance of the other train from the same station is $d = 15t - 20$
- a On the same set of axes, draw the graphs of these equations from $t = 0$ to $t = 5$ inclusive. [7]
- b Using the graphs made in answer to a, find the distance between the two trains when $t = 0$ [1]
- c Using the graphs made in answer to a, find, correct to the *nearest integer*, the value of t at the time one train passes the other. [2]

[1]

[OVER]

26 A man receives a salary of \$2000 for the first year and is to receive an increase of \$150 each year. In how many years will he have earned a total of \$26,750? [10]

27 How many gallons of water must be added to n gallons of a solution of salt and water which is $r\%$ salt to reduce it to a solution which is $s\%$ salt? [10]

*28 A ball is thrown vertically upward with an initial velocity of 128 feet per second. Its height s reached in t seconds is given by the equation $s = 128t - 16t^2$

a Find its velocity when $t = 2$ [3]

b How high will it rise? [4]

c How long will it be in the air? [1]

d Find its velocity at the moment it reaches the ground. [1]

e Find its acceleration. [1]

*29 a Write $(2,135^\circ)$ in the form $a + bi$. [2]

b Find the modulus and the amplitude (angle) of the product of $3 + 4i$ and $3 - 4i$. [4]

c Write in polar form the real root and one of the imaginary roots of the equation $x^3 - 1 = 0$ [4]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Is 3 a root of the equation $x = 6 + \sqrt{3x}$? [Answer *yes* or *no*.] 1.....
- 2 How many imaginary roots has the equation $x^5 + 12 = 0$? 2.....
- 3 Find the sum of the roots of the equation $2x^3 - 4x + 1 = 0$ 3.....
- 4 Write the equation of the line which passes through the point (6, -1) and is parallel to the line $2y = 3x - 1$ 4.....
- 5 What value of m in the equation $x^2 - 18x + m = 0$ will make one root double the other? 5.....
- 6 Express $\frac{i}{3 + 2i}$ in the form of $a + bi$ 6.....
- 7 Write the *third* term of the expansion of $(x^3 + \frac{1}{x^2})^8$ 7.....
- 8 When y varies inversely as x , the equation which expresses the relationship is (a) linear (b) quadratic (c) neither linear nor quadratic. Which is correct (a), (b) or (c)? 8.....
- 9 Find the value of the expression $x^{-\frac{1}{2}} + x^0$ when $x = 9$ 9.....
- 10 Find the value of $\sqrt[3]{74.1}$ correct to the *nearest tenth*. 10.....
- 11 If $\log x^2 = 0.7604$, find $\log 10x$. 11.....
- 12 Given $x = \frac{y}{y + 2}$; express y in terms of x . 12.....
- 13 Write the equation of third degree with rational coefficients two of whose roots are -1 and $\sqrt{2}$. 13.....
- 14 Transform $2x^5 + 5x^2 - 4x + 6 = 0$ into an equation which has integral coefficients and unity for the coefficient of x^5 . 14.....
- 15 Transform $2x^3 - 7x^2 - 3x + 1 = 0$ into an equation whose roots are those of the original equation each diminished by 4. 15.....
- 16 The sum of an infinite geometric progression is 6 and the first term is 4. Find the ratio. 16.....
- 17 The graph of $9x^2 - 6x + 5 = y$ (a) intersects the x axis (b) is tangent to the x axis (c) neither intersects nor is tangent to the x axis. Which is correct (a), (b) or (c)? 17.....
- 18 How many different straight lines are determined by 10 points if no three of the points lie in the same straight line? 18.....
- 19 How many different numbers between 4000 and 6000 can be made with the digits 4, 5, 7, 8 if no digit is repeated in any number? 19.....
- 20 If 3 balls are drawn at random from a bag containing 4 red balls and 4 white balls, what is the probability that all three balls will be red? 20.....