

The University of the State of New York

308TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 25, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

- 21 Find, to the *nearest tenth*, the real root of $x^3 - 6x - 13 = 0$. [10]
- 22 Find all the roots of $x^4 + 2x^3 - 3x^2 - 4x + 4 = 0$. [10]
- 23 The normal or probability curve used in statistics has for one form of its equation $y = 0.3989 e^{-\frac{x^2}{2}}$ where $e = 2.718$. If $x = 0.4$, find y to the *nearest hundredth*. [10]
- 24 For what positive value of k will the line whose equation is $x - 2y = k$ be tangent to the curve whose equation is $x^2 + y^2 = 9$, when the graphs are drawn on the same set of axes? [10]
- 25 a Using the same set of axes, draw the graphs of $(x - 3)^2 + y^2 = 4$ and $y = -x^2 + 4$. [4, 4]
b From these graphs estimate to *tenths* the values of x and y that satisfy both equations. [2]
- 26 A man wants to obtain 15 gallons of a 24% alcohol solution by combining a quantity of 20% alcohol solution, a quantity of 30% alcohol solution and 1 gallon of pure water. How many gallons of each of the alcohol solutions must he use? [10]
- 27 If $\frac{1}{b-a}$, $\frac{1}{2b}$, $\frac{1}{b-c}$ are in arithmetical progression, prove that a , b , c are in geometrical progression. [10]
- *28 a Express $-2i$ in the form $p(\cos \theta + i \sin \theta)$. [3]
b Express $4(\cos 210^\circ + i \sin 210^\circ)$ in the form $a + bi$. [3]
c Express in polar form one of the imaginary roots of the equation $x^5 - 1 = 0$. [4]
- *29 Given $y = \frac{1}{3}x^3 - 4x^2 + 15x - 6$.
a Find the coordinates of the maximum point. [5]
b Find the equation of the line tangent to the curve at the point where the curve crosses the y axis. [5]

* This question is based on one of the optional topics in the syllabus.

[1]

[OVER]

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

- 1 Express $\frac{6+i}{2-i}$ as a fraction with a real denominator. 1.....
- 2 If y varies directly as x and inversely as z , and if $y = 8$ when $x = 2$ and $z = 3$, find x when $y = 2$ and $z = 24$. 2.....
- 3 Write an equation of the line through the point $(-1, 3)$ parallel to the line whose equation is $3x - 2y = 6$. 3.....
- 4 When the graphs of the equations $xy = 8$ and $y = x^2 + 2$ are plotted on the same set of axes, how many points do the graphs have in common? 4.....
- 5 Write in simplest form the third term in the expansion of $\left(\frac{x}{2} + 2y\right)^8$. 5.....
- 6 For what value of k is $x - 3$ a factor of $kx^2 - 11x + 6$? 6.....
- 7 Find the remainder when $3x^7 + 6$ is divided by $x - 1$. 7.....
- 8 Find the rational root of $2x^3 - 3x^2 + 3x - 1 = 0$. 8.....
- 9 Find the sum of the roots of $x^3 + px^2 + qx + r = 0$. 9.....
- 10 An equation with real coefficients has $2 + 3i$ and $1 - 4i$ among its roots. What is the lowest possible degree of the equation? 10.....
- 11 Write an equation whose roots are the roots of $2x^3 + 3x^2 + x + 4 = 0$, each increased by 2. 11.....
- 12 Write an equation whose roots are the roots of $x^3 + 2x^2 - 3x + 1 = 0$, each multiplied by 2. 12.....
- 13 If $\log x = a$, $\log y = b$, $\log z = c$, express $\log \frac{x^2y}{\sqrt{z}}$ in terms of a , b and c . 13.....
- 14 Find $\log_3 2$ to the nearest tenth. 14.....
- 15 Find $\sqrt[3]{24.1}$ to the nearest hundredth. 15.....
- 16 In how many different ways can five boys line up for a race if the smallest boy is always to run in the left lane? 16.....
- 17 A man has 5 Jefferson nickels and 4 buffalo nickels. If he selects one coin at random, what is the probability that it is a buffalo nickel? 17.....
- 18 If the number of combinations of n things taken 2 at a time is 15, find n . 18.....
- 19 If $f(x) = x^2 - x$, find $f(2 - y)$. 19.....
- 20 If $x^2 - xy - y^2 = 0$, express x in terms of y . 20.....