

The University of the State of New York

314TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, January 23, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

- 21 Find to the *nearest tenth* the positive root of the equation  $x^3 - 3x^2 - x - 5 = 0$ . [10]
- 22 Solve the equation  $2x^4 + 3x^3 - x^2 - 8x - 6 = 0$ . [10]
- 23 Using the formula  $D = 0.632 \left( \frac{fq^2}{h} \right)^{\frac{1}{5}}$ , find  $D$  to the *nearest thousandth* when  $f = .012$ ,  $q = 74.3$  and  $h = 2.34$ . [10]
- 24 a Draw the graph of  $y = x^3 - x^2 - x + 2$  from  $x = -2$  to  $x = +3$  inclusive. [6]  
b On the set of axes used in a, draw the graph of  $y = 5$ . [2]  
c From the graphs made in answer to a and b, estimate to the *nearest tenth* the real root of the equation  $x^3 - x^2 - x + 2 = 5$ . [2]
- 25 a Prove:  ${}_nC_r = {}_nC_{n-r}$  [7]  
b Find the value of  ${}_{90}C_{88}$  [3]
- 26 Three positive numbers are in arithmetic progression and their sum is 15. If 1 is added to the first, 7 to the second and 15 to the third, the resulting numbers are in geometric progression. Find the original numbers. [10]
- 27 A meat dealer has been selling  $P$  pounds of meat per week. As a result of raising the price  $R$  per cent, he finds that he loses  $S$  per cent of his former receipts. Express in terms of  $P$ ,  $S$  and  $R$  the number of pounds he now sells per week. [10]

[1]

[OVER]

- \*28 a Find the modulus of  $5 + 12i$ . [2]  
b Find to the *nearest degree* the amplitude (angle) of  $5 + 12i$ . [2]  
c Express  $4(\cos 210^\circ + i \sin 210^\circ)$  in the form of  $a + bi$ . [2]  
d Write in polar form the root of  $x^5 - 32 = 0$  that when represented graphically lies in the second quadrant. [4]
- \*29 Find the equation of the line tangent to the graph of  $y = x^3 - 8x + 6$  at the point whose abscissa is 2. [10]

\* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed.

- 1 Find the slope of the straight line that passes through the points (2, 3) and (5, 7). 1.....
- 2 Find the sum of the infinite progression  $6, -2, \frac{2}{3}, \dots$  2.....
- 3 Write in *simplest form* the fourth term in the expansion of  $\left(x^{\frac{3}{2}} + \frac{1}{y}\right)^9$  3.....
- 4 If  $(2 + 5i)(3 - 2i) = x + iy$ , find the value of  $x$ . 4.....
- 5 If  $\log_e \sqrt[3]{63} = 1.381$ , find  $\log_e 63$ . 5.....
- 6 Find the real value of  $x^{\frac{3}{2}} = \frac{8}{\sqrt[3]{7}}$  6.....
- 7 Find the value of  $k$  if  $x - 3$  is a factor of  $x^3 - 5x - k$ . 7.....
- 8 If  $f(x) = x^3 + 2x - 2$ , find  $f(x - 2)$ . 8.....
- 9 If two roots of the equation  $x^3 + px^2 + qx + r = 0$ , in which  $p, q$  and  $r$  are real, are 2 and  $3 + 2i$ , find the value of  $p$ . 9.....
- 10 Find the product of the roots of the equation  $3x^3 - 2x - 11 = 0$ . 10.....
- 11 How many imaginary roots has the equation  $x^4 - 2x^2 - 6 = 0$ ? 11.....
- 12 Transform the equation  $2x^3 - 3x^2 + 1 = 0$  into an equation whose roots are those of the original equation each multiplied by 2. 12.....
- 13 Transform the equation  $x^3 - 6x^2 + 10x - 1 = 0$  into an equation whose roots are those of the original equation each decreased by 2. 13.....

[3]

[OVER]

14 Is the following statement true or is it false? If the graph of the function  $f(x)$  crosses the  $x$ -axis three times, then the equation  $f(x) = 0$  must be of the third degree. 14.....

15 Using  $k$  as the constant of variation, write an equation representing the relationship:  $E$  varies directly as  $F$  and inversely as  $S$ . 15.....

16 A woman has 4 different pairs of gloves scattered about in a drawer. If she picks out 2 gloves in the dark, what is the probability that they will be a matching pair? 16.....

17 Find the number of permutations of 6 different things taken 4 at a time. 17.....

Directions (questions 18–20) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter  $a$ ,  $b$  or  $c$ .

18 When drawn on the same set of axes, the graphs of  $2x + 3y = 7$  and  $4x + 6y = 25$  (a)intersect (b)are parallel (c)coincide 18.....

19 When drawn on the same set of axes, the graphs of  $y = x + 3$  and  $x^2 + y^2 = 6$  (a)intersect in two distinct points (b)are tangent (c)have no points in common 19.....

20 If  $f(x) = \frac{1}{2}x^3 + x\sqrt{3} - 5$ , then the function  $f(x)$  is (a)fractional (b)rational (c)irrational 20.....