January 25, 1962

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answer on the line at the right.

1. Given $f(x) = x^2 - 2x$, express $f\left\{\frac{1}{2a}\right\}$ as a single fraction. 1_____

2. Express 0.297297 . . . in the form $\frac{a}{b}$ where a and b are integers. 2_____

3. Write an equation of the straight line perpendicular to the line $3x + 2y = 5$ and passing through the point $(-1, 2)$. 3_____

4. Find the length of the radius of the circle $x^2 + y^2 - 8x + 14y = 35$. 4_____

5. How many three-digit numbers greater than 100 may be formed using the digits 0, 1, 3, 5 if repetitions of digits are *not* allowed? 5_____

6. An equation with real coefficients has $1 - i$, 3 and 4 among its roots. What is the lowest possible degree of the equation? 6_____

7. Solve for x: $8^{2x-3} = 16^{x-2}$ 7_____

8. If $f(x) = -x^2 + x + c$, for what value of c will $f(x) = 0$ have equal roots? 8_____

9. Given: a varies directly as b and inversely as the square of c. If $a = 12$ when $b = 3$ and $c = 2$, find a when $b = \frac{1}{4}$ and $c = \frac{1}{3}$. 9_____

10. How many straight lines are determined by a set of nine points if no three points of the set are collinear? 10_____

11. Express $\frac{5 + i\sqrt{2}}{3 - i\sqrt{2}}$ as an equivalent fraction with a real denominator. 11_____

Directions (12-16): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

12. When drawn on the same set of axes, the graphs of $x^2 - 3y^2 = 4$ and $y^2 = x$ intersect *in* (1) only one point (2) only two points (3) no points (4) four points 12_____

13. If $x = \frac{y + a}{y - a}$, then $y =$ (1) $\frac{x - 1}{a - ax}$ (2) $\frac{a - ax}{x + 1}$ (3) $\frac{a + ax}{x - 1}$ (4) $\frac{a + ax}{x + 1}$ 13_____

14. Given: $\log_{10} x = M$. If b is a positive real number $\neq 1$, then M is equal to (1) $\dfrac{\log_b x}{\log_b 10}$ (2) $(\log_b x)(\log_b 10)$ (3) $\log_b x - \log_b 10$ (4) $\dfrac{\log_b 10}{\log_b x}$ 14_____

15. If $f(x) = \dfrac{x-2}{x+5}$, then the value of x for which $f(x)$ is undefined is (1) -2 (2) 2 (3) 5 (4) -5 15_____

16. The graph of $2^x = y$ has a point on the (1) positive portion of the x-axis (2) negative portion of the x-axis (3) positive portion of the y-axis (4) negative portion of the y-axis 16_____

17. Express $x^3 + 2x^2 - 5x - 6$ as an indicated product of three linear factors. 17_____

18. Solve for x to the *nearest integer*: $(1.5)^x = 25$ 18_____

19. From a deck of fifty-two cards containing four aces, one card is drawn at random. What is the probability that the card selected is *not* an ace? 19_____

20. Given: $\log k = 2 \log a - \frac{1}{3} \log b$. Express k in terms of a and b. 20_____

21. If x and y are real numbers and if $2x + 8i = 3 + iy$, find the numerical value of x. 21_____

22. Find the modulus of the complex number $-2 + 3i$. 22_____

23. Express $6i^4 - 3i^3$ in the form $a + bi$. 23_____

24. When $x^3 - 2x^2 + cx + 9$ is divided by $x - 2$, the remainder is 3. Find the value of c. 24_____

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

25. Express *one* of the imaginary roots of $x^6 - 1 = 0$ in polar form. 25_____

26. Find the coordinates of the point of inflection of the graph of $f(x) = x^3 - 6x^2$. 26_____

27. The first three terms of a geometric progression are x, y and z, in that order. Express x in terms of y and z. 27._____

28. Divide $12(\cos 110° + i \sin 110°)$ by $3(\cos 50° + i \sin 50°)$. 28._____

29. A merchant bought a articles for c cents each. If each article had cost d cents less, how many articles could be have bought for the same amount of money? 29._____

30. Write in simplest form the *fourth* term of the expansion of

$$\left\{ x + \frac{1}{x^2} \right\}^9$$

30._____

31. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are 2, 3 and 4, find the value of r. 31._____

32. For what value of k will the graph of the equation $y - 2 = 3(x - k)$ pass through the point $(4, 2)$? 32._____

33. Transform $x^2 + y^2 + 2x - y = 1$ into an equation in polar coordinates. 33._____

*34. If $\begin{vmatrix} x & 0 & 1 \\ 1 & 2 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$, find the value of x. 34._____

35. A positive root of $x^3 - 9x - 9 = 0$ lies between 3 and 4. Find this root to the *nearest integer*. 35._____

36. Find an equation of the tangent to the curve $y = x^2$ at the point $(2, 4)$. 36._____

37. Given $y = x^2 + 3x$. Find the average rate of change of y as x increases from 1 to 3. 37._____

38. If two balls are drawn at random from a bag containing 4 red balls and 4 white balls, what is the probability that both are red? 38._____

39. A motorist travels from town A to town B at the uniform rate of x miles per hour and returns over the same route at the uniform rate of y miles per hour. Express in terms of x and y the average speed at which the motorist has made the entire trip. 39._____

40. Solve the inequality: $5x + 2 \geqq 3(x - 1)$ 40._____

*41. Write in determinant form the equation of the straight line that passes through the points $(2, -3)$ and $(5, 4)$. 41._____

Directions (42-43): For each of the following *which you select*, indicate whether the statement is true for (1) all real values of a, b and c (2) some, but not all, real values of a, b and c (3) no real value of a, b and c by writing on the line at the right the *number* 1, 2 or 3.

42. If $a < b$ and $b = \frac{2}{3}c$, than $a < c$. 42._____

43. If $a < (b + c)$, then $-a < -(b + c)$. 43._____

Directions (44-48): For each of the following *which you select*, write the *number* preceding the expression that best completes the statement or answers the question.

44. All lines of the family $y = 3x + k$ have the (1) same x-intercept (2) same slope (3) same y-intercept (4) common point (2, 3) 44_____

45. What is the value of the abscissa of the minimum point of the graph of $f(x) = ax^2 + bx + c$? [Assume $a > 0$.]

(1) 1 (2) 0 (3) $\frac{-b}{2a}$ (4) $\sqrt{b^2 - 4ac}$ 45_____

46. Which is *not* a possible root of $4x^3 - px^2 + qx - 6 = 0$?

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{3}{2}$ (4) 4 46_____

47. The equation $\sqrt{2x + 3} + x = 0$ has (1) no real root (2) only one positive root (3) only one negative root (4) one positive and one negative root 47_____

*48. If r and θ denote polar coordinates, then the graph of $r = \cos \theta$ is a (1) straight line perpendicular to the polar axis (2) straight line parallel to the polar axis (3) circle whose center is at the pole (4) circle whose center is not at the pole 48_____

FOR TEACHERS ONLY

12A

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS 12A (Advanced Algebra)

Thursday, January 25, 1962 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 12–16, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{1 - 4a}{4a^2}$

(2) $\frac{297}{999}$ *or* $\frac{33}{111}$ *or* $\frac{11}{37}$

(3) $2x - 3y = -8$

(4) 10

(5) 18

(6) 4

(7) $\frac{1}{2}$

(8) $-\frac{1}{4}$

(9) 36

(10) 36

(11) $\frac{13 + 8i\sqrt{2}}{11}$

(12) 2

(13) 3

(14) 1

(15) 4

(16) 3

(17) $(x + 1)(x + 3)(x - 2)$

(18) 8

(19) $\frac{48}{52}$ *or* $\frac{12}{13}$

(20) $\frac{a^2}{\sqrt[3]{b}}$

(21) $\frac{3}{2}$

(22) $\sqrt{13}$

(23) $6 + 3i$

(24) -3

[OVER]

Part II

Allow 2½ credits for each of not more than 16 correct answers; allow no partial credit. If more than sixteen questions have been answered, only the first sixteen of these should be considered. For questions 44–48, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4. Do not allow credit for No. 41 unless an equation is written.

(25) $1\ (\cos 60° + i \sin 60°)$ *or* $1\ (\cos 120° + i \sin 120°)$ *or* $1\ (\cos 240° + i \sin 240°)$ *or* $1\ (\cos 300° + i \sin 300°)$

(26) $(2, -16)$

(27) $\frac{y^2}{z}$

(28) $4\ (\cos 60° + i \sin 60°)$ *or* $2 + 2i\sqrt{3}$

(29) $\frac{ac}{c - d}$

(30) 84

(31) -24

(32) 4

(33) $r^2 + 2r \cos \theta - r \sin \theta = 1$

(34) -1

(35) 3

(36) $y = 4x - 4$

(37) 7

(38) $\frac{3}{14}$

(39) $\frac{2xy}{x + y}$

(40) $x \geqq -\frac{5}{2}$

(41) $\begin{vmatrix} x & y & 1 \\ 2 & -3 & 1 \\ 5 & 4 & 1 \end{vmatrix} = 0$

(42) 1

(43) 3

(44) 2

(45) 3

(46) 4

(47) 3

(48) 4