

The University of the State of New York  
 REGENTS HIGH SCHOOL EXAMINATION  
**TWELFTH YEAR MATHEMATICS**

12A (Advanced Algebra)

Monday, January 25, 1965—1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

- 1 Given that  $y$  varies directly as  $w$  and inversely as  $z$ . When  $w = z$ , the value of  $y$  is 1. Find  $y$  when  $w = 25$  and  $z = 10$ .
- 2 Given:  $f(x) = 2x^0 + 8x^{-\frac{3}{2}} - 1^x$   
Find:  $f(27)$ .
- 3 Find the slope of the line tangent to the curve whose equation is  $y = \frac{x^3}{3} - \frac{x^2}{2} - 6x + 5$  at the point where  $x = 3$ .
- 4 For what negative value of  $k$  will the equation  $x^2 + kx - 2k = 0$  have equal roots?
- 5 How many pints of water must be evaporated from 15 pints of an 8% salt solution to obtain a 20% salt solution?
- 6 How many permutations are possible from the letters of the word "ellipse" if all of the letters are used every time?
- 7 Three cards are to be drawn from a deck of 52 cards containing 4 aces. The first two cards drawn are aces. These are put aside and a third card drawn from the remaining cards. What is the probability that this third card drawn will be an ace?
- 8 Solve for  $x$ :  $x - 2 < 3(x - 5)$
- 9 If  $y = mx + b$  is the equation of a family of lines all of which pass through the origin, what is the value of  $b$ ?
- 10 If the expression  $x^3 - 3x^2 + 5x^2 - 7x + k$ , in which  $k$  is a constant, is divided by  $x - 2$ , the remainder is 22. Find the value of  $k$ .
- 11 If  $(1 + i)^5$  is written in the form  $r(\cos \theta + i \sin \theta)$ , what is the value of  $\theta$  if  $\theta$  is greater than 0 and less than  $360^\circ$ ?
- 12 Express the repeating decimal  $0.0313131\dots$ , in which the digits 3 and 1 are repeated endlessly as indicated, in the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers.
- 13 Find in simplest form the 4th term only of the expansion of  $\left(x - \frac{y}{2}\right)^9$ .
- 14 Write an equation of the straight line through the point  $(-5, 4)$  and perpendicular to the line whose equation is  $5x - 4y + 7 = 0$ .
- 15 The sum of the first four terms of an arithmetic progression is 42. If the fourth term is  $\frac{1}{6}$  of the first term, find the value of the first term.
- 16 Between what two consecutive integers does the positive root of  $x^4 + x^3 - 4x^2 - 5x - 5 = 0$  lie?
- 17 Write in the form  $x^2 + ax^2 + bx + c = 0$  ( $a, b, c$  real numbers) the equation two of whose roots are  $1 + i$  and  $1 - i$ .
- 18 Solve for  $x$ :  $8^{x-2} = 16^{\frac{x}{2}}$

- 19 One of the roots of the equation  $x^3 + x^2 + 4x + 4 = 0$  is  $-1$ . Find the other two roots.

Directions (20-24): Write in the space provided on the separate answer sheet the number preceding the expression that best completes the statement or answers the question.

20 The expression  $\frac{3-4i}{5+3i}$  is equivalent to

- (1)  $\frac{27-29i}{34}$  (3)  $\frac{3-29i}{34}$   
 (2)  $\frac{27-29i}{16}$  (4)  $-\frac{1}{4}$

21 Which is a rational integral function of  $x$ ?

- (1)  $\frac{x+1}{x}$  (3)  $\sqrt{x^2}$   
 (2)  $x^3 + x\sqrt{2}$  (4)  $x^2 + 7$

22 If  $\log_a 2 = x$  and  $\log_a 3 = y$ , then  $\log_a 102$  is equal to

- (1)  $4xy$  (3)  $2(x+y)$   
 (2)  $10xy$  (4)  $10(x+y)$

23 If the complex number representing the sum of  $a + bi$  and  $c + di$  falls in the third quadrant, then

- (1)  $a + c > 0$  and  $b + d > 0$   
 (2)  $a + c > 0$  and  $b + d < 0$   
 (3)  $a + c < 0$  and  $b + d > 0$   
 (4)  $a + c < 0$  and  $b + d < 0$

24 If, in the equation  $y = 2^x$ , the value of  $x$  is increased by 2, then the value of  $y$  is

- (1) divided by 2 (3) squared  
 (2) multiplied by 2 (4) multiplied by 4

## Part II

Answer sixteen questions from this part, 25-48. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Questions marked \* are based upon optional topics in the syllabus. Write your answers on the separate answer sheet.

25 Find a nonintegral rational root of  $3x^3 - 10x^2 - 51x + 18 = 0$ .

26 Find the coordinates of the center of the circle whose equation is  $x^2 + y^2 - 4x + 8y + 11 = 0$ .

27 Find the common ratio of the geometric progression  $\log 81, \log 9, \log 3, \dots$

28 Find the abscissa of the point of inflection of the graph of  $y = 2x^3 - 6x^2 + x - 5$ .

29 If  $f(x) = x + \frac{1}{x}$ , express  $f(c) - f\left(\frac{1}{c}\right)$  in simplest form.

30 In how many ways can a committee of four be selected from nine men so as always to include a particular man?

31 Two balls are drawn at random at the same time from a box containing 3 red balls and 8 white ones. What is the probability that both balls will be white?

32 If the number of permutations of  $n$  things taken two at a time is 22 more than the number of permutations of  $(n-1)$  things taken two at a time, what is the value of  $n$ ?

33 Find to the nearest tenth the value of  $\log_3 7$ .

\*34 The area of a triangle is  $12\frac{1}{2}$  and is numerically equal to

$$\frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 2 & 4 & 1 \\ -7 & k & 1 \end{vmatrix}. \text{ Find the positive value of } k.$$

35 A body moves along a straight line in such a way that its distance  $s$  in feet from a fixed point on the line at the end of  $t$  seconds is given by the equation  $s = 2t^3 - 4t^2 + 3$ . Find the acceleration in feet per second per second when  $t = 5$ .

36 Find the cube root of  $27(\cos 30^\circ + i \sin 30^\circ)$  which, when represented graphically, lies in the second quadrant.

37 If  $m_1$  machines take  $d_1$  days to do a job and if  $m_2$  machines take  $d_2$  days to do the same job, express  $d_2$  in terms of  $m_1, d_1$  and  $m_2$ .

38 A root of  $x^2 + 4x^2 - x - 14 = 0$  lies between 1 and 2. Find this root to the nearest integer.

39 If  $y = x^2 - 2x^2 + x - 3$ , find the average rate of change of  $y$  with respect to  $x$  as  $x$  increases from 1 to 4.

*Directions (40-43):* For each of those chosen, write in the space provided on the separate answer sheet the number preceding the expression that best completes the statement.

40 If  $y = (x - 2)^2$  and  $4x^2 + 9y^2 = 36$  are graphed on the same set of axes, the total number of points of intersection is

- (1) 1 (3) 3  
(2) 2 (4) 4

41 The volume of a circular cylinder varies jointly as the altitude and the square of the radius. The altitude varies

- (1) directly as the volume and inversely as the square of the radius  
(2) directly as the square of the radius and inversely as the volume  
(3) jointly as the volume and the square of the radius  
(4) directly as the square of the quotient of the volume and the radius

\*42 The graph of  $r = 2 \sec \theta$  is a

- (1) circle whose center is at the pole  
(2) circle whose center is not at the pole  
(3) straight line perpendicular to the polar axis  
(4) straight line parallel to the polar axis

\*43 The point  $(3, -\frac{\pi}{5})$  may also be represented by the coordinates

- (1)  $(3, -\frac{6\pi}{5})$  (3)  $(3, \frac{6\pi}{5})$   
(2)  $(3, -\frac{9\pi}{5})$  (4)  $(-3, -\frac{9\pi}{5})$

44 Find that number which exceeds its square by the greatest amount.

45 If the solution of the inequality  $x^2 + 3x - 4 < 0$  has the form  $a < x < b$ , what is the value of  $a$ ?

46 Solve for the positive value of  $x$ :  
 $\log x + \log (x - 48) = \log 100$

47 Find the area of the triangle bounded by the  $x$ -axis and the lines whose equations are  $2x - y + 4 = 0$  and  $2x + y - 8 = 0$ .

48 Sketch the graph of  $y = 2^x$  from  $x = -1$  to  $x = 2$ .



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Percent:.....
Rater's Initials:.....
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FOR TEACHERS  
12A

The University of the State of New York

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ANSWER SHEET

Pupil.....Teacher.....

School.....

All of your answers should be recorded on this answer sheet.

Part I

Answer all questions in this part.

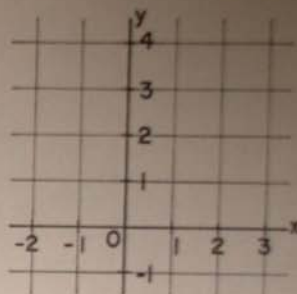
- |        |         |         |
|--------|---------|---------|
| 1..... | 9.....  | 17..... |
| 2..... | 10..... | 18..... |
| 3..... | 11..... | 19..... |
| 4..... | 12..... | 20..... |
| 5..... | 13..... | 21..... |
| 6..... | 14..... | 22..... |
| 7..... | 15..... | 23..... |
| 8..... | 16..... | 24..... |

Your answers for part II should be placed in the proper spaces on the back of this sheet.

Part II

Answer only sixteen questions from this part. Be sure to write in the properly numbered spaces the answers to the questions you have chosen. Leave blank the spaces for questions you do not choose to answer.

- |         |         |         |
|---------|---------|---------|
| 25..... | 33..... | 41..... |
| 26..... | 34..... | 42..... |
| 27..... | 35..... | 43..... |
| 28..... | 36..... | 44..... |
| 29..... | 37..... | 45..... |
| 30..... | 38..... | 46..... |
| 31..... | 39..... | 47..... |
| 32..... | 40..... | 48..... |



I do so declare.....  
(Signature)

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# FOR TEACHERS ONLY

# 12A

## SCORING KEY

### TWELFTH YEAR MATHEMATICS

### 12A (Advanced Algebra)

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

#### Part I

Allow  $2\frac{1}{2}$  credits for each correct answer; allow no partial credit. For questions 20–24, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1)  $2\frac{1}{2}$

(2)  $\frac{17}{9}$

(3) 0

(4) -8

(5) 9

(6) 1260

(7)  $\frac{1}{25}$

(8)  $x > 6\frac{1}{2}$

(9) 0

(10) 8

(11)  $225^\circ$

(12)  $\frac{31}{990}$

(13)  $-\frac{21}{2}x^8y^3$

(14)  $4x + 5y = 0$

(15) 18

(16) 2 and 3

(17)  $x^5 - 3x^2 + 4x - 2 = 0$

(18) 9

(19)  $2i$  and  $-2i$

(20) 3

(21) 2

(22) 3

(23) 4

(24) 4

[OVER]

195

## Part II

Allow  $2\frac{1}{2}$  credits for each of not more than 16 correct answers; allow no partial credit. If more than sixteen questions have been answered, only the first sixteen of these should be considered. For questions 40–43, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(25)  $\frac{1}{2}$

(33) 1.8

(41) 1

(26) (2, -4)

\*(34) 3

\*(42) 3

(27)  $\frac{1}{2}$

(35) 52

\*(43) 2

(28) 1

(36)  $3(\cos 130^\circ + i \sin 130^\circ)$

(44)  $\frac{1}{2}$

(29) 0

(37)  $\frac{m_1 d_1}{m_2}$

(45) -4

(30) 56

(46) 50

(38) 2

(47) 18

(31)  $\frac{28}{55}$

(39) 12

(48)

(32) 12

(40) 2

