

ADVANCED ALGEBRA

Monday, June 15, 1914—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) intermediate algebra, (3) advanced algebra.

The minimum time requirement is five recitations a week in algebra for two school years.

Answer eight questions. Credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form.

1 For what values of k does the equation $(x+k)x - (k-3) = 0$ have its roots equal? Give a value of k that makes both roots rational.

2 Determine k and m so that the equation

$$x^2 + 2kx + 3mx - k + 2m + 7 = 0$$
 shall have both roots equal to zero.

3 What is the value of $x^3 + 3x^2 - 7x + 10$ when $x = 1 - i$?
 $[i = \sqrt{-1}]$

4 In the system of equations $\begin{cases} 2x + 3y - z + 3 = 0 \\ 2x - 3y + 3z = 2 \\ -x + 2y + 5z = 5 \end{cases}$

find the value of z by the use of determinants.

5 a In how many ways can 7 boys stand in line, only 2 being willing to stand at the extremities of the line?

b Of 8 books of the same size, a shelf will hold 5; how many different arrangements may be made on the shelf?

6 a Prove that if all of the elements of a row of a determinant are zero, the value of the determinant is zero. $[4\frac{1}{2}]$

b Solve $x(x+1)(x+2)(x+3) = 24$ $[8]$

7 a What information regarding the roots of the equation $x^5 - 4x^4 + 2x + 1 = 0$ is obtainable by the application of Descartes' rule?

b From a sketch of the graph of the equation

$$f(x) = x^5 - 4x^4 + 2x + 1 = y$$

estimate the values of the real roots of $f(x) = 0$

8 It is desired to double the capacity of a tank $3 \times 4 \times 5$ feet by making equal elongations of its dimensions; find the elongation of each dimension.

9 Prove that if a rational integral equation with real coefficients has the complex number $c + id$ for a root, it must also have the number $c - id$ for a root.

10 Compute by Horner's method, to two decimal places, one root of the equation $x^4 + 2x^2 - 2x - 4 = 0$