

The University of the State of New York

266TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 17, 1936 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Is the sum of two complex conjugate numbers always a real number?
[Answer Yes or No.] Ans.....

2 Express $\frac{10(3+i)}{3-i}$ in the form $a + bi$ Ans.....

3 For what value of m , other than zero, will the roots of the equation $x^2 - mx + 4m = 0$ be equal? Ans.....

4 Write in simplest form the eighth term in the expansion $(2x - \frac{y}{2})^8$ Ans.....

5 In how many points do the graphs of $x^2 + y^2 = 9$ and $y^2 = 4x$ intersect? Ans.....

6 What is the numerical value of the remainder when $x^{51} + 51$ is divided by $x - 1$? Ans.....

7 If $f(x) = (10)^x$, write the value of $f(2)$. Ans.....

8 The two lines represented by the equations $2x + 3y = 5$ and $4x + 6y = 10$ (a) are parallel, (b) are coincident or (c) intersect. Which is correct (a), (b) or (c)? Ans.....

9 Write the equation of the line whose x -intercept is 6 and which is parallel to the line whose equation is $\frac{x}{3} + \frac{y}{4} = 1$ Ans.....

10 How many numbers of two digits each can be written with the digits 0, 3, 5, 7, if repetition in any of the numbers is not allowed? Ans.....

11 In choosing a committee of three from a group of four boys and three girls, what is the probability that the three members of the committee will be boys? Ans.....

12 Write the numerical value of N when $\log N = \log 2a^2 - 2 \log a$ Ans.....

13 Given $S = r^n - 1$; express n in terms of S and r . Ans.....

14 If i is one root of $x^4 + px + q = 0$, in which p and q are real, what must be another root? Ans.....

15 Transform the equation $x^3 + 2x^2 + 4x + 8 = 0$ into an equation whose roots are one half the roots of the given equation. Ans.....

16 Transform the equation $x^3 - 6x^2 + 12x - 9 = 0$ into an equation whose roots are less by 2 than the roots of the given equation. Ans.....

17 What is the only possible rational fractional root in the equation $5x^3 + px^2 + qx + 1 = 0$, where p and q are positive integers?

Ans.....

18 One root of the equation $2x^4 - x^3 - 14x^2 + 19x - 6 = 0$ is $\frac{1}{2}$; what is the sum of the other three roots?

Ans.....

19 May the equation $a_0x^6 - a_2x^4 + a_4x^2 + a_6 = 0$, where a_0, a_2, a_4 and a_6 are positive integers, have four complex roots? [Answer *Yes* or *No*.]

Ans.....

20 Given $f(x) = 0$; if $f(2)$ and $f(3)$ are both positive, is it possible for $f(x) = 0$ to have real roots between $x = 2$ and $x = 3$? [Answer *Yes* or *No*.]

Ans.....

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See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the nearest tenth, the real root of the equation

$$x^3 - 3x^2 - 12 = 0 \quad [10]$$

22 Given the equation $x^3 + 3x^2 - 16x + k = 0$; if one of the roots is twice another and the three roots are integers, determine the value of k . [10]

23 In order to discharge q cubic feet of water per second, under a head of h feet, a water pipe l feet long must have a diameter of d inches computed from the following semi-empirical formula:

$$d = 5.8 \sqrt[5]{\frac{0.02q^2l}{h}}$$

Use logarithms to find d correct to the nearest tenth of an inch when $q = 8.5$ cubic feet, $l = 250$ feet and $h = 11.55$ feet. [10]

24 A debt of \$500 on an automobile is to be paid off in monthly instalments of \$25 plus interest at 12% per annum on unpaid balances. Therefore the first payment one month hence will be \$30. Find the total amount that will be paid. [10]

25 Two cities, P and Q , are 40 miles apart. A leaves P for Q at the same time that B leaves Q for P . They meet at the end of 3 hours. If B had left Q two hours later, he would have met A at the end of $2\frac{1}{2}$ hours. Find the rate of B . [7, 3]

26 a State and prove the Remainder Theorem. [7]

b State the Factor Theorem. [3]

27 Obtain graphically, correct to the nearest tenth, the values of x and y common to the two equations $2x - 3y + 12 = 0$ and $x^2 - 2y + 4 = 0$ [2, 6, 2]

*28 Given the function $y = \frac{x^3}{3} - x^2 - 3x + 4$

a Write the first and second derivatives of y . [2]

b Find the maximum and minimum points and the point of inflection. [3]

c Sketch the curve. [5]

* This question is based on one of the optional topics in the syllabus.