

The University of the State of New York

275TH HIGH SCHOOL EXAMINATION

**ADVANCED ALGEBRA**

Wednesday, June 21, 1939 -- 9.15 a. m. to 12.15 p. m., only

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**Instructions**

*Do not open this sheet until the signal is given.*

**Group I**

*This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.*

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

**Group II**

Write at top of first page of answer paper to group II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

## Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

## Group I

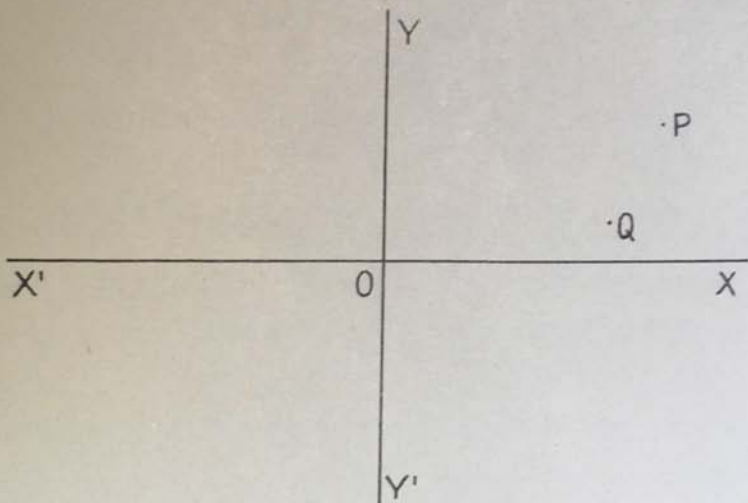
Answer all questions in this group. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 If  $f(x) = x^3 - 2x^2 - x - 1$ , what is the value of  $f(-2)$ ? 1.....
- 2 Write as an equation the following statement: The illumination  $I$  of an object varies inversely as the square of its distance  $d$  from the source of light. 2.....
- 3 Write the equation of the straight line which is parallel to the  $x$  axis and which passes through the point whose coordinates are  $(0, 3)$ . 3.....
- 4 Write the equation of lowest possible degree, with rational coefficients, two of whose roots are 3 and  $i$ . 4.....
- 5 Transform the equation  $2x^4 - x - 1 = 0$  into an equation whose roots are numerically equal but opposite in sign to the roots of the original equation. 5.....
- 6 Transform the equation  $x^3 - 3x^2 + 1 = 0$  into an equation whose roots are less by 1 than the roots of the original equation. 6.....
- 7 How many negative roots has the equation  $x^4 + x^3 - x^2 - 1 = 0$ ? 7.....
- 8 Find, correct to the nearest hundredth, the value of  $\sqrt[3]{.5327}$  8.....
- 9 Given  $10^x = 45$ ; find  $x$  correct to the nearest hundredth. 9.....
- 10 Write the first two terms of the expansion  $(\frac{1}{x^2} + x)^6$  10.....
- 11 Find the positive root of the equation  $x^{\frac{3}{2}} = 4$  11.....
- 12 Express the repeating decimal .313131... as a common fraction. 12.....
- 13 Write the equation of the circle whose center is at the origin and which passes through the point whose coordinates are  $(3, 4)$ . 13.....
- 14 The numbers  $x$ ,  $y$  and  $z$ , in order, form a geometric progression; express  $z$  as a function of  $x$  and  $y$ . 14.....
- 15 Is the following statement true: If  $(a + bi)^4$  equals  $P + Qi$ , then  $(a - bi)^4$  equals  $P - Qi$ ? [Answer Yes or No.] 15.....
- 16 A baseball league is made up of 6 teams. If each team is to play each of the other teams twice, how many games are to be played? 16.....
- 17 A ball is to be drawn at random from a bag containing 5 white balls and a certain number of black balls. If in one trial the probability of drawing a white ball is to be 1 to 15, how many black balls must the bag contain? 17.....
- 18 The graph of  $y = \log_{10}x$  (a) is symmetrical with respect to the  $x$  axis, (b) is symmetrical with respect to the  $y$  axis, or (c) is not symmetrical with respect to either axis. Which is correct, (a), (b) or (c)? 18.....

19 What positive value of  $k$  will make the roots of the equation  $kx^2 - 6x + k = 0$  real and equal?

19.....

20 In the drawing below the complex numbers  $a + bi$  and  $c + di$  are represented graphically by the points  $P$  and  $Q$  respectively. Locate the point  $R$  which represents the complex number  $(a + bi) - (c + di)$ .



See instructions for group II on page 1.

## Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Solve completely:  $2x^4 - x^3 + 3x^2 - 2x - 2 = 0$  [10]

22 Find, correct to the nearest tenth, the real root of the equation  $x^3 - 3x - 4 = 0$  [10]

23 Using the same set of axes, draw the graphs of the following equations and from the graphs determine, correct to the nearest tenth, the solutions common to the two equations:

$$y^2 - x^2 = 4$$

$$y = 2x \quad [6, 2, 2]$$

24 The number  $n$  of vibrations per second made by a stretched string is given by the formula

$$n = \frac{1}{2L} \sqrt{\frac{Mg}{m}}$$

where  $L$  is the length in centimeters,  $M$  the attached weight in grams,  $m$  the weight in grams of one centimeter of the string, and  $g$  the gravitational constant. Using logarithms, find, correct to the nearest integer, the value of  $n$  if  $L = 78.5$ ,  $M = 5468$ ,  $m = .0065$  and  $g = 980$  [10]

25 a Prove:  ${}_nC_r = {}_nC_{n-r}$  [7]

b Find the value of  $n$  for which  ${}_nC_2 = 120$  [3]

26 Three numbers are in arithmetic progression. Prove that the reciprocals of these numbers can not be in arithmetic progression. [10]

27 A rectangular cardboard box has a square base and no top. The total amount of cardboard in the box is 27 square inches.

a Express the volume  $V$  of the box as a function of its base edge  $x$ . [7]

b Find the value of  $x$  for which  $V = 13\frac{1}{2}$  cubic inches. [3]

\*28 Differentiate each of the following functions:

a  $y = x^2 - 8$  [2]

b  $y = \sqrt{x^2 - x - 1}$  [4]

c  $y = \frac{x}{x^2 - 5}$  [4]

\*29 a Express  $5(\cos 180^\circ + i \sin 180^\circ)$  in the form  $a + bi$ . [3]

b Express  $\sqrt{3} - i$  in polar form. [3]

c Give the modulus and the amplitude of one of the roots of the equation  $x^4 - 16 = 0$  [2, 2]

\* This question is based on one of the optional topics in the syllabus.