The University of the State of New York

291st High School Examination

ADVANCED ALGEBRA

Wednesday, June 21, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find *all* the roots of $3x^3 + 2x^2 - 19x + 6 = 0$ [10]

22 Find, correct to the *nearest tenth*, the positive root of $x^3 + x^2 - 4x - 2 = 0$ [10]

23 *a* Given: $y = \frac{1}{k}\sqrt{\frac{cg}{m}}$; using logarithms, find y correct to the *nearest hundredth* if $k = 157$, $c = 5468$, $g = 980$ and $m = 65$ [7]

b Given $r^x = t^{x+1}$; express x in terms of the logarithms of r and t. [3]

24 *a* Construct the graph of $y = x^3 - 5x - 3$ for values of x from -3 to $+3$ inclusive. [7]

b From the graph made in answer to *a*, estimate to the *nearest tenth*

(1) The roots of the equation $x^3 - 5x - 3 = 0$ [2]

(2) The real root of the equation $x^3 - 5x - 3 = 4$ [1]

25 *a* State and prove the Remainder Theorem. [1, 5]

b Find the remainder when $x^n - 4$ is divided by $x - 1$. [2]

c State the Factor Theorem. [1]

d Using the Factor Theorem, show that $x - 2$ is a factor of $x^5 - 32$. [1]

26 Fifteen gallons of pure alcohol are added to 25 gallons of 15% alcohol. How much water must then be added to obtain a mixture which is 20% alcohol? [10]

27 Given the equation $y = x^3 - 3x^2 - 9x + k$

Find the values of k for which the graph of the equation will be tangent to the x axis. [10]

*28 *a* Explain the meaning of $\frac{\Delta y}{\Delta x}$ when used in connection with a straight line graph. [2]

b Explain the meaning of $\frac{dy}{dx}$ when used in connection with a curve. [2]

c The distance s, in feet, through which a certain body moves in t seconds, is given by the formula $s = 16t^2 - 20t + 10$. Find (1) the velocity of the body when $t = 2$, (2) the acceleration of the body. [2, 2]

d The velocity V, in feet per second, of a moving body is given by the equation $V = 36t^2 - 2t^3$ in which t is the time in seconds. Find the value of t for which V is a maximum. [2]

*29 *a* Write in polar form the number $-1 + i$ [3]

b Write in rectangular form the number $4(\cos 30^\circ + i \sin 30^\circ)$. [3]

c Write in polar form *two* of the imaginary roots of the equation $x^5 - 32 = 0$ [4]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If the complex number $-3 - 4i$ is represented graphically by point P, how far is P from the origin? 1.....

2 Write $\dfrac{1}{-3 + 4i}$ in the form $a + bi$. 2.....

3 Find, correct to the *nearest hundredth*, the value of $\sqrt[5]{578}$ 3.....

4 Solve for x correct to the *nearest thousandth*:

$$\log \frac{x}{3} = 9.5611 - 10$$ 4.....

5 If the roots of the equation $x^3 - 7x + 6 = 0$ are represented by a, b and c, write the equation whose roots would be represented by $-a$, $-b$ and $-c$. 5.....

6 Given the equation $x^3 - 2x^2 + x - 3 = 0$, write the equation whose roots are the roots of the given equation each multiplied by 3. 6.....

7 Given the equation $x^3 - 2x^2 + x - 3 = 0$, write the equation whose roots are less by 2 than the roots of the given equation. 7.....

8 Write the equation of lowest degree with real coefficients, two of whose roots are 0 and $-1 + \sqrt{-5}$ 8.....

9 Given $f(x) = x^3 - 2x^2 + mx + 9$. Find the value of m if $f(x)$ is exactly divisible by $x - 3$. 9.....

10 The product of two of the roots of $x^3 + px^2 + qx + 12 = 0$ is -6. What is the third root? 10.....

11 How many imaginary roots has the equation $x^7 + x^5 + 5x + 7 = 0$? 11.....

12 Express 0.00029 as the product of 2.9 and a power of 10. 12.....

13 Write in simplest form the *fourth* term of the expansion of $(x - \frac{1}{2}x)^8$ 13.....

14 Given $V = e^3$ and $d = e\sqrt{3}$. Express V as a function of d. 14.....

15 Write the equation of the line which passes through the point $(-4, 3)$ and is parallel to the line whose equation is $2x - y = 6$ 15.....

16 Given the line whose equation is $y = \frac{1}{3}x + 2$; find, correct to the *nearest degree*, the acute angle which this line forms with the x axis. 16.....

17 The electrical resistance R of a wire of fixed length is inversely proportional to the cross section A of the wire. Express this relation by means of an equation. 17.....

18 From 10 men, how many different committees of 4 men each can be formed? 18.....

19 If five persons are arranged in a straight line, what is the probability that two particular persons, A and B, are next to each other? 19.....

20 Express the repeating decimal 0.1313... as a common fraction. 20.....