

The University of the State of New York

306TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 22, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

- 21 Find, to the *nearest tenth*, the positive root of the equation $x^3 + 3x^2 - 5x - 4 = 0$ [10]
- 22 Solve completely the equation $2x^4 - 7x^3 + 5x^2 - 7x + 3 = 0$ [10]
- 23 a Newton's law of gravitation is given by the equation $F = \frac{km_1m_2}{d^2}$ where F stands for the force of attraction between the masses m_1 and m_2 which are at a distance d feet apart. Using logarithms, find F to the *nearest thousandth*, when $k = .0007$, $m_1 = 24,840$ pounds, $m_2 = 234$ pounds and $d = 149$ feet. [7]
b Solve the equation $3^x = 7$ for the value of x to the *nearest tenth*. [3]
- 24 a On the same set of axes, draw the graphs of $(x - 3)^2 + (y - 4)^2 = 25$ and $xy = 8$ [4, 4]
b From the graphs made in answer to a, estimate to the *nearest tenth* the values of x and y common to both equations. [2]
- 25 John and George ran a mile race on two different occasions. In the first race John gave George a start of 10 yards and beat him by 10 seconds. In the second race John gave George a start of 11 seconds and beat him by 5 yards. Find George's rate. [1 mile = 1760 yards] [10]
- 26 a State and prove the Remainder Theorem. [5]
b Starting with the general form of the quadratic equation $ax^2 + bx + c = 0$, derive the formula for the roots of the equation in terms of a , b and c . [5]
- 27 The roots of the equation $x^3 + 3x^2 - 6x + k = 0$ are in arithmetic progression. Find k . [10]

*28 For each of the following find the derivative of y with respect to x :

a $y = (x + 2)^7$ [3]

b $y = \sqrt{3x^2 - 4}$ [4]

c $y = \frac{2x + 4}{x^2}$ [3]

*29 a Express each of the following in polar form:

(1) -4 [2]

(2) $2i$ [1]

(3) $-3 - 3i$ [3]

b Express one of the imaginary roots of the equation $x^3 - 8 = 0$ in the form $a + bi$ [4]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

- 1 Express $\frac{5}{2-i}$ in the form $a + bi$. 1.....
- 2 Write in simplest form the *fourth* term in the expansion of $(\frac{a}{2} + \frac{2}{b})^6$ 2.....
- 3 Find the remainder when $x^{30} + 15$ is divided by $x - 1$ 3.....
- 4 Find the real root of the equation $x^{\frac{1}{3}} = 32$ 4.....
- 5 Find the product of the roots of the equation $2x^3 - 3x + 6 = 0$ 5.....
- 6 How many imaginary roots has the equation $3x^4 + 2x^2 + 1 = 0$? 6.....
- 7 Write an equation of the straight line that passes through the point (3, 4) and is parallel to the line whose equation is $y = 2x - 1$ 7.....
- 8 Find the sum of the infinite progression $3, -1, \frac{1}{3}, \dots$ 8.....
- 9 Solve the following set of equations for x : 9.....

$$\frac{2}{x} + \frac{2}{y} = 1$$

$$\frac{3}{x} - \frac{2}{y} = 4$$
- 10 When drawn on the same set of axes, in how many points will the graphs of the equations $y = 3x^2$ and $x^2 + 4y^2 = 4$ intersect? 10.....
- 11 In the equation $x^2 + kx - 3 = 0$, is there any real value which can be assigned to k so that the roots of the resulting equation will be imaginary? [Answer *yes* or *no*.] 11.....
- 12 Find the value of $f(2)$ when $f(x) = x^3 - 2x^2 + 3x - 4$ 12.....
- 13 Write the equation of lowest degree possible with real coefficients two of whose roots are $3 + i$ and -2 13.....
- 14 Transform the equation $x^3 + 3x^2 - 3x + 2 = 0$ into an equation whose roots are the roots of the given equation, each multiplied by 2. 14.....
- 15 Transform the equation $x^3 - x^2 + 4x - 12 = 0$ into an equation whose roots are the roots of the given equation, each increased by 1. 15.....
- 16 $\text{Log}(x^2 - y^2)$ is equal to (a) $2 \log x - 2 \log y$, (b) $\log(x + y) + \log(x - y)$, (c) $2 \log(x - y)$. Which is correct, a, b, or c? 16.....
- 17 Find the value of $\log_2 8$ 17.....
- 18 Find the number of combinations of 20 different things taken 18 at a time. 18.....
- 19 How many "code" words of *five* different letters each can be formed from the letters of the word RADIO, if each "code" word is to begin with the letter "R"? 19.....
- 20 If *three* letters are chosen at random from the first *five* letters of the alphabet, what is the probability that "a" will be among those chosen? 20.....