ADVANCED ALGEBRA

Wednesday, June 23, 1954—9.15 a. m. to 12.15 p. m., **only**

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Write the *third term* in the expansion of $(\frac{1}{a} - b)^9$. 1.....

2. If $(2 - i)(3 + 2i) = a + bi$, find the value of a. 2.....

3. Find the slope of the straight line that passes through the points (3,2) and (8,4). 3.....

4. Write an equation of the straight line passing through the point (—1,2) and parallel to the line $3y = 2x - 6$. 4.....

5. Write an equation of the straight line passing through the point (2,—3) and perpendicular to the x-axis. 5.....

6. For what value of k will the graph of $y = kx^2 - 2x + 3$ be tangent to the x-axis? 6.....

7. Find the value of m for which $x + 2$ is a factor of $x^3 + x^2 + mx - 4$. 7.....

8. Using k as the constant of variation, write an equation representing the following relationship: P varies directly as the product of R and S and inversely as the square of T. 8.....

9. If $f(x) = 3x - 2$, find $f(2 - h)$. 9.....

10. An equation with real coefficients has i and $2i$ among its roots. What is the lowest possible degree of the equation? 10.....

11. Solve the following equation for x: $4^{2x} = 8^{2x-1}$ 11.....

12. If $\log_b \sqrt{N} = 0.3961$, find $\log_b N$. 12.....

13. If n is a negative integer, then (*a*) $\frac{6}{n^2}$ is less than $\frac{4}{n}$ (*b*) $\frac{n}{4}$ is greater than $\frac{-6}{n}$ (*c*) $\frac{6}{n}$ is less than $\frac{4}{n}$ [Which is correct (*a*), (*b*) or (*c*)?] 13.....

14. How many different committees each consisting of two teachers and three students can be formed from a group of four teachers and seven students? 14.....

15. The order in which four students, A, B, C and D, are to speak at an assembly program is to be determined by lot. What is the probability that A will speak first and B second? 15.....

16. Find the sum of the infinite progression 2, 1.2, 0.72, ... 16.....

17. Solve the equation $x^2 = \frac{1 - y}{1 + y}$ for y in terms of x. 17.....

18. How many real roots has the equation $x^6 + 2x^2 + 4 = 0$? 18.....

19. Transform the equation $x^3 - 4x^2 + 6x - 4 = 0$ into an equation whose roots are those of the original equation each diminished by 1. 19.....

20. Transform the equation $x^3 - 6x^2 - 9x + 54 = 0$ into an equation whose roots are those of the original equation each multiplied by 1/3. 20.....

Part II

Answer five questions from part II. All work, including computation, should be shown.

21 Solve the following equation: $2x^3 + x^2 - 13x + 6 = 0$. [10]

22. Find to the *nearest tenth* the smaller positive root of $x^3 - 6x^2 + 8x + 2 = 0$. [10]

23. *a. Derive* the formula for the roots of the equation $ax^2 + bx + c = 0$ in terms of a, b, and c. [7]

b. Write in the form $ax^2 + bx + c = 0$ an equation whose roots are $\frac{2 \pm \sqrt{3}}{2}$. [3]

24. Given $D = 0.4\sqrt[3]{\frac{P^2}{R^5}}$. Find D to the *nearest thousandth* when $P = 18.4$ and $R = 5.89$. [10]

25. *a.* Draw the graph of $y = \log_2 x$. [Use the following values for x: ¼, ½, 1, 2, 4.] [5]

b. On the same set of axes used in answer to *a*, draw the graph of $(x - 1)^2 + y^2 = 4$. [3]

c. From the graphs drawn in answer to *a* and *b*, estimate to the *nearest tenth* the values of x and y common to both equations. [2]

26. Three numbers are in geometric progression and their sum is 19. If 1 is subtracted from the first number, the numbers then are in arithmetic progression. Find the original numbers. [10]

27. A park superintendent has a 25-pound sack of grass seed containing 70% permanent grasses. How much of this seed should he replace by seed containing 53% permanent grasses in order to make 25 pounds of seed containing 60% permanent grasses? [Give your answer to the *nearest tenth* of a pound.] [10]

*28. Answer *either a* or *b*:

a. (1) Find the modulus of $3 - 2i$. [2]

(2) Find to the *nearest degree* the amplitude (angle) of $3 - 2i$. [3]

(3) Express $2(\cos 150° + i \sin 150°)$ in the form $a + bi$. [2]

(4) Write in polar form one of the real fourth roots of 4. [3]

b. Given the equation $y = 2x^3 - 3x^2 - 12x + 10$.

(1) Find the coordinates of

(*a*) the maximum point [3]

(*b*) the minimum point [3]

(*c*) the point of inflection [2]

(2) Find the slope of the line tangent to the curve at the point where the curve crosses the y axis. [2]

*This question is based upon optional topics in the syllabus.

The University of the State of New York

321ST HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

(Advanced Algebra)

Wednesday, June 23, 1954—9.15 a. m. to 12.15 p. m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A*, *B* or *C*. (Write answers on the regular question paper opposite the questions you are replacing.)

A Solve the following equation for x: $\log x - \log (x - 3) = \log 2$

B A circle with its center at the point $(3, 0)$ passes through the point $(3, 4)$. Write an equation of the circle.

C Between what two values of x is $x^2 - 5x - 6$ less than zero?

Part II

Directions: The following question is based on optional topics of the twelfth year syllabus and may be used in place of *any one* of the questions on part II of the examination in advanced algebra.

29 Answer *either a* or *b*:

a Using determinants, solve the following system of equations for x: [10]

$$\begin{aligned} x + y - z &= -3 \\ 2x - y + z &= 9 \\ x - 3y + 2z &= 13 \end{aligned}$$

b (1) Transform the equation $x^2 + y^2 - 4y = 0$ from rectangular coordinates to polar coordinates. [5]

(2) Transform the equation $r = \dfrac{2}{\sin\theta - \cos\theta}$ from polar coordinates to rectangular coordinates. [5]

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA
and
TWELFTH YEAR MATHEMATICS
(Advanced Algebra)

Wednesday, June 23, 1954 — 9.15 a. m. to 12.15 p. m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For question 13, allow credit if the pupil has written the correct answer instead of the letter *c*.

(1) $\frac{36b^2}{a^7}$
(2) 8
(3) $\frac{2}{5}$
(4) $3y = 2x + 8$
(5) $x = 2$
(6) $\frac{1}{3}$
(7) -4
(8) $P = \frac{kRS}{T^2}$
(9) $4 - 3h$
(10) fourth
(11) $\frac{3}{2}$
(12) 0.7922
(13) *c*
(14) 210
(15) $\frac{1}{1\frac{1}{2}}$
(16) 5
(17) $\frac{1 - x^2}{1 + x^2}$
(18) none
(19) $y^3 - y^2 + y - 1 = 0$
(20) $y^3 - 2y^2 - y + 2 = 0$

Twelfth Year Mathematics (Advanced Algebra)

A 6
B $(x - 3)^2 + y^2 = 16$
C —1 and 6