

ADVANCED ALGEBRA

Wednesday, June 18, 1958—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

1. Express $\frac{5}{3+2i}$ as an equivalent fraction with a real denominator. 1.....
2. If $(3 + i\sqrt{2})(5 - i\sqrt{2})$ is expressed in the form $x + iy$, in which x and y are real numbers, find the value of x . 2.....
3. Write in *simplest form* the fourth term *only* in the expansion of $(\sqrt{x} + 1)^7$. 3.....
4. Find the real value of $\left(\frac{3^9}{27^{2/3}}\right)^{-1}$. 4.....
5. Express in terms of k the discriminant of the equation $x^2 + kx + k - 5 = 0$. 5.....
6. Solve for x : $9^x = \frac{1}{27}$ 6.....
7. If $f(x) = x^2 + 2x$, find $f(1 - b)$. 7.....
8. Find the remainder when $6x^{11} + 5x^6 - 8$ is divided by $x + 1$. 8.....
9. If $\log \frac{x}{3} = 9.4874 - 10$, find x to the *nearest hundredth*. 9.....
10. Find the real value of x which satisfies the equation $x^{3/4} = 64$. 10.....
11. Solve the following set of equations for x : 11.....

$$\frac{3}{x} + \frac{1}{y} = \frac{1}{4}$$

$$\frac{2}{x} - \frac{1}{y} = \frac{3}{8}$$
12. Using k as the constant of variation, write an equation representing the relationship: x varies inversely as the product of r and t . 12.....

13. Find the sum of the infinite geometric series

$$2 + \frac{2}{3} + \frac{2}{9} + \dots \quad 13 \dots \dots \dots$$

14. The first three terms of an arithmetic progression are represented by $2x - 14$, x and $5x - 1$, respectively. Find x . 14.....

15. The sum of the first five terms of a geometric progression is 341. If the common ratio is 2, find the first term of the progression. 15.....

16. An article sells for d dollars at a profit of $r\%$ of the selling price. Express in terms of d and r the cost of the article. 16.....

17. Find the sum of the roots of the equation $3x^4 - 5x^3 + 7x - 9 = 0$. 17.....

18. Express in simplest form the complex fraction

$$\frac{2x}{1 - \frac{4-x}{4+x}} \quad 18 \dots \dots \dots$$

19. Points $(-2, 3)$ and $(1, 6)$ lie on a straight line L_1 , and points $(7, 2)$ and $(a, 5)$ lie on a second line L_2 . If the lines are drawn on the same set of axes, for what value of a will L_1 be parallel to L_2 ? 19.....

20. Write an equation of the straight line that passes through the origin and makes an angle of 45° with the positive direction of the x -axis. 20.....

21. The equation of the axis of symmetry of the graph of $y = x^2 - 5x + 8$ is $x = k$. Find the value of k . 21.....

22. If $V = \pi r^2 h$ and $S = 2\pi r h$, express V in terms of r and S . 22.....

23. How many distinct permutations can be made from the letters of the word *WEEKEND* if they are all used every time? 23.....

24. A man has 3 striped and 4 solid-color ties. If he selects one of the ties at random, what is the probability that it will be a striped one? 24.....

25. From a group of 4 boys and 7 girls, how many different committees can be formed, each consisting of 1 boy and 2 girls? 25.....

Directions (26-29): Indicate the correct completion for each of the following by writing on the line at the right the letter a , b , c or d .

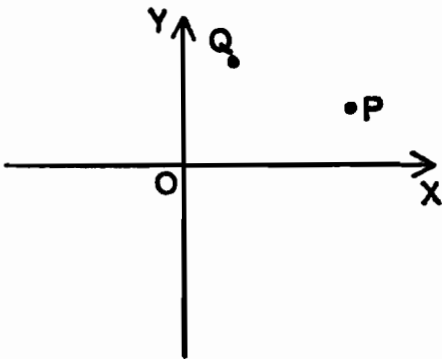
26. Given the equation $2x^4 + px^3 + qx + 6 = 0$ in which p and q are integers. Which of the following can *not* be a root of the equation? (a) $1/2$ (b) $2/3$ (c) 3 (d) $3/2$ 26.....

27. The equation $\sqrt{8-x^2} + x = 0$ has (a) one positive root only (b) one negative root only (c) one positive and one negative root (d) no roots 27.....

28. When drawn on the same set of axes, the graphs of $x^2 + 4y^2 = 16$ and $y = 3x^2$ (a) do not intersect (b) intersect in one point (c) intersect in two points (d) intersect in four points 28.....

29. Which of the following is a rational number?
 (a) $-\frac{5}{7}$ (b) $\sqrt{2}$ (c) π (d) $\sqrt{-4}$ 29.....

30. In the drawing, OX is the real axis and OY is the imaginary axis. The points P and Q represent $a + bi$ and $c + di$, respectively. On the drawing, indicate by the letter S the point which represents $(a + bi) + (c + di)$.



Part II

Answer ten questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

31. Write an equation whose roots are the roots of $x^2 - 5x^2 + 3x - 1 = 0$, each decreased by 3. 31.....

32. Write an equation whose roots are the roots of $x^4 - 3x^2 + x + 2 = 0$, each multiplied by m . 32.....

33. How many negative real roots has the equation $x^2 - 4x^2 + 7x + 9 = 0$? 33.....

34. The roots of the equation $x^3 - 6x^2 - x + 30 = 0$ are (a) 3, 1, -10 (b) 3, -2, 5 (c) 3, 2, -5 (d) 3, -3, 6 34.....

35. Find the rational fractional root of the equation $3x^2 + 4x^2 + 13x + 4 = 0$. 35.....

36. Between what two consecutive integers does the positive root of the equation $x^2 - 5x - 3 = 0$ lie? 36.....

37. A positive root of the equation $x^2 - 3x + 1 = 0$ lies between 0.3 and 0.4. Find this root to the nearest tenth. 37.....

38. Find m if $x^2 - mx^2 + 3mx - 20$ is exactly divisible by $x - 2$. 38.....

39. If a function of x is divided by $x - 2$, the quotient is $2x^2 - 3$ and the remainder is 7. Find the function. 39.....

40. The graph of the parabola $y = ax^2 + bx$ passes through the points $(-1, -2)$ and $(1, 6)$. Find the value of a . 40.....
41. Express the product of $\left(x + 1 - \frac{x^2 - 1}{x}\right)$ and $\left(1 + \frac{1}{x + 1}\right)$ as a fraction in *lowest terms*. 41.....
42. If ${}_nC_{18} = {}_nC_4$, find n . 42.....
- *43. The volume V of a box is given by the equation $V = 12x^3 - x^4$ where x represents the height in inches of the box. Find the value of x for which V is a maximum. 43.....
- *44. The distance S in feet through which a body moves in t seconds is given by the formula $S = 4t^2 - 3t + 10$. Find the velocity of the body in feet per second when $t = 5$. 44.....
- *45. If $y = 4x^3 - 5x^2 + 3$, find the second derivative of y with respect to x . 45.....

Part III

*Answer ten questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

46. Find to the *nearest tenth* the value of $(17.3)^{1.2}$. 46.....
47. $\log x^2y = 0.3457$ and $\log x = 0.2815$. Find $\log y$. 47.....
48. Given $\log 2.6 = 0.4150$ and $\log 7.5 = 0.8751$, solve the equation $2.6^x = 7.5$ for x to the *nearest tenth*. 48.....
49. Express $\log a \sqrt{\frac{b}{c}}$ in terms of $\log a$, $\log b$ and $\log c$. 49.....
50. Two roots of the equation $x^3 + px^2 + qx + r = 0$ are 3 and $1 + i$. If p , q and r are real numbers, find the value of q . 50.....
51. It takes b bricklayers d days to build a wall. If c more bricklayers had been put on the job and they all worked at the same rate, how many days would it have taken to build the wall? 51.....
52. The rate of a boat upstream is $\frac{2}{3}$ the rate of the boat downstream. Find the ratio of the rate of the stream to the rate of the boat in still water. 52.....
53. A builder has two piles of a mixture of sand and gravel. The ratio of the sand to the gravel is 1:3 in the first pile and 3:5 in the second. If 24 cubic yards from the first pile are mixed with 40 cubic yards from the second, how many cubic yards of sand are there in the resulting mixture? 53.....

Directions (54-57): Indicate the correct completion for each of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

54. The graph of $y = \log_{10}x$ (a) intersects both coordinate axes (b) intersects the *y*-axis only (c) intersects the *x*-axis only (d) does not intersect either axis 54.....

55. The graphs of $xy = 8$ and $8y = x$, when drawn on the same set of axes, (a) intersect in one point (b) intersect in two distinct points (c) are tangent (d) do not intersect 55.....

56. The graph of $(x + 2)^2 + (y - 5)^2 = 16$ lies in quadrants (a) I and II (b) II and III (c) III and IV (d) I and IV 56.....

57. When drawn on the same set of axes, the graphs of $y = x^2 - 2x - 3$ and $y = -x^2 + 2x + 3$ (a) have no points in common (b) are tangent (c) have the same *y*-intercept (d) have the same *x*-intercepts 57.....

*58. Find the modulus of $-2 + 3i$. 58.....

*59. If the number $2(\cos 330^\circ + i \sin 330^\circ)$ is expressed in the form $a + bi$, find the value of *b*. 59.....

*60. Express one of the roots of $x^2 + 36 = 0$ in polar form. 60.....