

ADVANCED ALGEBRA

Monday, September 8, 1919—9.15 a. m. to 12.15 p. m., only

*Answer eight questions. Each answer should be reduced to its simplest form. Papers entitled to less than 75 credits will not be accepted.*

1 Find to two decimal places the positive root of the equation  $x^4 - 8x^3 + 14x^2 + 4x - 8 = 0$  which lies between 2 and 3.

2 a Write a rational integral equation of the sixth degree which will have for three of its roots  $-1 + \sqrt{3}$ ,  $2 + \sqrt{-2}$ , and  $3 - \sqrt{-1}$ .

b Change the equation  $2x^4 - 3x^3 + 4x^2 - x + 2 = 0$  into another, the coefficient of whose highest term is unity and the coefficients of the other terms integers.

3 a Plot the graph of  $2x^3 - 3x^2 - 12x + 4 = y$

b If  $2x^3 - 3x^2 - 12x + 4 = 0$ , find from the graph the roots of the equation correct to the nearest tenth.

4 From 4 Americans, 4 Englishmen, 3 Italians and 5 Frenchmen, how many committees of 8 members can be formed, each nation to be represented by two men?

5 A man invests \$3100 in Liberty Bonds of the 1st, 2d and 3d issues. Later he converts his  $3\frac{1}{2}\%$  bonds into those of the 2d issue paying 4% and in this way increases his yearly income by \$5. Still later he exchanges all his 4% bonds for the  $4\frac{1}{4}\%$  bonds of the 3d issue, thus receiving \$9.75 more each year than he would have received from his original investment. How much did he invest originally in each of the three issues?

6 By means of graphs find the sum of

a  $\frac{-2 - \sqrt{-3}}{2}$  and  $\frac{-2 + \sqrt{-3}}{2}$

b  $3 - \sqrt{-2}$ ,  $-5$ , and  $2 - 2\sqrt{-2}$

7 Write the derivatives of  $x^4 - 3x^3 + 2x^2 - 7x - 5$

8 The hypotenuse of a right-angled triangle is 13 and its area is 30; find the three sides of the triangle.

9 Solve for  $H$  the equation  $3b = \frac{4\sqrt{H} - 2b^2}{H}$

10 Given the equation  $x^5 - 3x^4 + 2x^3 + x^2 - x - 12 = 0$  Without solving the equation, fill out the following statements by placing the proper number in each parenthesis; justify each statement by quoting the appropriate theorem or principle:

a The equation has ( ) roots.

b It has at most ( ) positive roots and at most ( ) negative roots.

c It has at least ( ) real roots.

d It has ( ) rational fractional roots.

e The sum of the roots is ( ).

f The roots are factors of ( ).