The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Friday, June 14, 2013 — 1:15 to 4:15 p.m., only

Student Name: *Mr. Sibol*

School Name: *JMAP*

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 27 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [54]

Use this space for computations.

1 A market research firm needs to collect data on viewer preferences for local news programming in Buffalo. Which method of data collection is most appropriate?
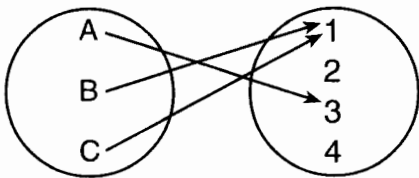
(1) census

(2) survey

(3) observation

(4) controlled experiment

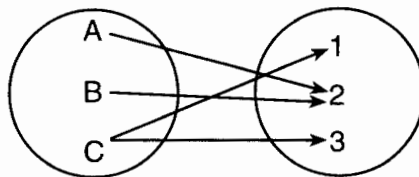2 What is the number of degrees in an angle whose radian measure is $\frac{8\pi}{5}$?

(1) 576

(2) 288

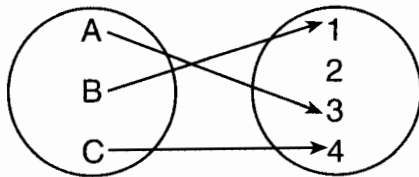(3) 225

(4) 113

$\frac{8\pi}{5} \cdot \frac{180}{\pi} = 288$

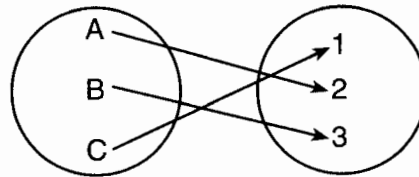3 Which diagram represents a relation that is both one-to-one and onto?

(1)

(3)

(2)

(4)

4 The sum of the first eight terms of the series $3 - 12 + 48 - 192 + \ldots$ is

(1) $-13,107$ (3) $-39,321$

(2) $-21,845$ (4) $-65,535$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{3(1-(-4)^8)}{1-(-4)} = -39,321$$

5 The simplest form of $\dfrac{1 - \dfrac{4}{x}}{1 - \dfrac{2}{x} - \dfrac{8}{x^2}}$ is

$$\cdot \frac{x^2}{x^2} = \frac{x^2-4x}{x^2-2x-8} = \frac{x(x-4)}{(x-4)(x+2)} = \frac{x}{x+2}$$
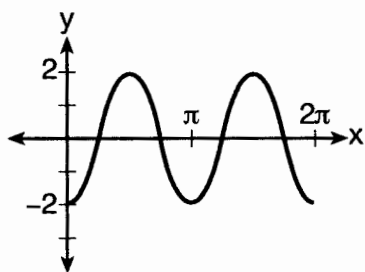
(1) $\dfrac{1}{2}$ (3) $\dfrac{x}{3}$

(2) $\dfrac{x}{x + 2}$ (4) $-\dfrac{x}{x - 2}$

6 Which equation represents the graph below?
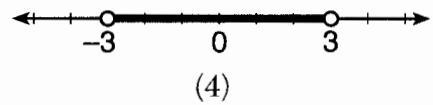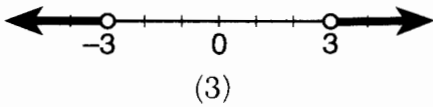

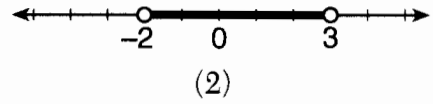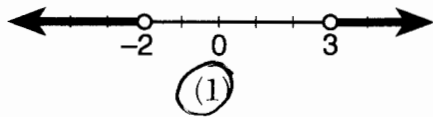(1) $y = -2 \sin 2x$ (3) $y = -2 \cos 2x$

(2) $y = -2 \sin \dfrac{1}{2}x$ (4) $y = -2 \cos \dfrac{1}{2}x$

7 What is the graph of the solution set of $|2x - 1| > 5$?

(1)

(2)

(3)

(4)

$2x-1>5$

$2x>6$

$x>3$

$2x-1<-5$

$2x<-4$

$x<-2$

8 What is the range of the function shown below?

(1) $x \leq 0$

(2) $x \geq 0$

(3) $y \leq 0$

(4) $y \geq 0$

9 The expression $\sin(\theta + 90)°$ is equivalent to

(1) $-\sin\theta$ (3) $\sin\theta$

(2) $-\cos\theta$ (4) $\cos\theta$

Handwritten:
$\sin(\theta+90) = \sin\theta\cos 90 + \cos\theta\sin 90$
$= \sin\theta\,(0) + \cos\theta\,(1)$
$= \cos\theta$

10 The points $(2,3)$, $(4,\frac{3}{4})$, and $(6,d)$ lie on the graph of a function.
If y is inversely proportional to the square of x, what is the value of d?

(1) 1   (3) 3

(2) $\frac{1}{3}$ (4) 27

Handwritten:
$2^2 \cdot 3$ $4^2 \cdot \frac{3}{4}$
12   12

$6^2 d = 12$
$36 d = 12$
$\frac{36d}{36} = \frac{12}{36} = \frac{1}{3}$

11 In the right triangle shown below, what is the measure of angle S, to the *nearest minute*?

Handwritten:
$\sin S = \frac{8}{17}$
$S = \sin^{-1}\left(\frac{8}{17}\right) \approx 28°4'$

(1) 28°1′   (3) 61°56′
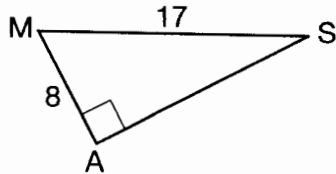
(2) 28°4′   (4) 61°93′

12 Which ordered pair is in the solution set of the system of equations shown below?

$$y^2 - x^2 + 32 = 0$$
$$3y - x = 0$$

(1) $(2,6)$ (3) $(-1,-3)$

(2) $(3,1)$ (4) $(-6,-2)$

Handwritten:
$y^2 - (3y)^2 + 32 = 0$
$y^2 - 9y^2 = -32$
$-8y^2 = -32$
$y^2 = 4$
$y = \pm 2$
$x = 3(-2) = -6$

13 Susie invests $500 in an account that is compounded continuously at an annual interest rate of 5%, according to the formula $A = Pe^{rt}$, where A is the amount accrued, P is the principal, r is the rate of interest, and t is the time, in years. Approximately how many years will it take for Susie's money to double?

(1) 1.4

(3) 13.9

(2) 6.0

(4) 14.7

$$\frac{1000}{500} = \frac{500e^{.05t}}{500}$$

$$\ln 2 = \ln e^{.05t}$$

$$13.9 \approx \frac{\ln 2}{.05} = \frac{.05t}{.05}$$

14 If n is a negative integer, then which statement is always true?

(1) $6n^{-2} < 4n^{-1}$

(3) $6n^{-1} < 4n^{-1}$

(2) $\frac{n}{4} > -6n^{-1}$

(4) $4n^{-1} > (6n)^{-1}$

$$\frac{6}{n} < \frac{4}{n} \quad \text{if } n < 0$$

15 The expression $4 + \sum\limits_{k=2}^{5} 3(k - x)$ is equal to

(1) $58 - 4x$

(3) $58 - 12x$

(2) $46 - 4x$

(4) $46 - 12x$

$$4 + 3(2-x) + 3(3-x) + 3(4-x) + 3(5-x)$$
$$4 + 6 - 3x + 9 - 3x + 12 - 3x + 15 - 3x$$
$$46 - 12x$$

16 Which value of r represents data with a strong positive linear correlation between two variables?

(1) 0.89

(3) 1.04

(2) 0.34

(4) 0.01

17 Which problem involves evaluating $_6P_4$?

(1) How many different four-digit ID numbers can be formed using 1, 2, 3, 4, 5, and 6 without repetition?

(2) How many different subcommittees of four can be chosen from a committee having six members?

(3) How many different outfits can be made using six shirts and four pairs of pants?

(4) How many different ways can one boy and one girl be selected from a group of four boys and six girls?

18 Which equation is represented by the graph below?
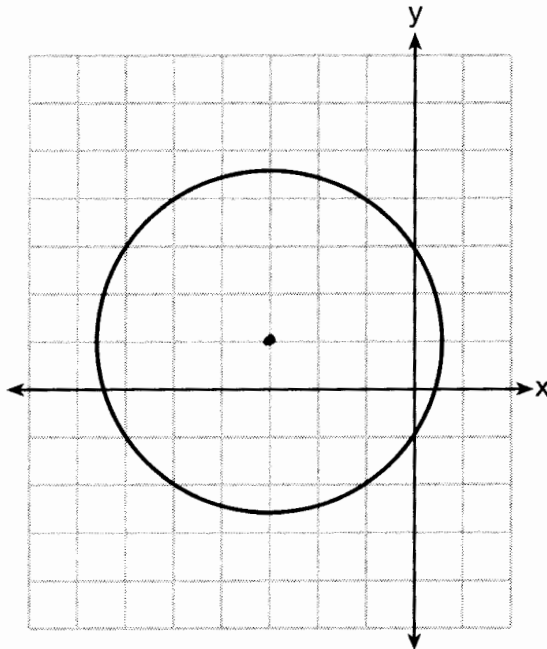
(1) $(x - 3)^2 + (y + 1)^2 = 5$

(2) $(x + 3)^2 + (y - 1)^2 = 5$

(3) $(x - 1)^2 + (y + 3)^2 = 13$

(4) $(x + 3)^2 + (y - 1)^2 = 13$

19 If $x = 3i$, $y = 2i$, and $z = m + i$, the expression xy^2z equals

(1) $-12 - 12mi$ (3) $12 - 12mi$

(2) $-6 - 6mi$ (4) $6 - 6mi$

$(3i)(2i)^2(m+i)$

$(3i)(4i^2)(m+i)$

$(3i)(-4)(m+i)$

$-12i(m+i)$

20 An angle, P, drawn in standard position, terminates in Quadrant II if

(1) $\cos P < 0$ and $\csc P < 0$ (3) $\csc P > 0$ and $\cot P < 0$

(2) $\sin P > 0$ and $\cos P > 0$ (4) $\tan P < 0$ and $\sec P > 0$

$-12mi - 12i^2$

$-12mi + 12$

If $\csc P$ is positive, $\sin P$ is positive

If $\cot P$ is negative & $\sin P$ is positive, $\cos P$ must be negative.

21 The expression $\log 4m^2$ is equivalent to

(1) $2(\log 4 + \log m)$ (3) $\log 4 + 2\log m$

(2) $2\log 4 + \log m$ (4) $\log 16 + 2\log m$

$\log 4 + \log m^2$

$\log 4 + 2\log m$

22 In $\triangle PQR$, p equals

(1) $\dfrac{r\sin P}{\sin Q}$ (3) $\dfrac{r\sin R}{\sin P}$

(2) $\dfrac{r\sin P}{\sin R}$ (4) $\dfrac{q\sin R}{\sin Q}$

$\dfrac{p}{\sin P} = \dfrac{r}{\sin R}$

$p = \dfrac{r\sin P}{\sin R}$

23 If $\tan\left(\text{Arc}\cos\dfrac{\sqrt{3}}{k}\right) = \dfrac{\sqrt{3}}{3}$, then k is

(1) 1 (3) $\sqrt{2}$

(2) 2 (4) $3\sqrt{2}$

$\tan 30 = \dfrac{\sqrt{3}}{3}$

$\text{Arc}\cos\dfrac{\sqrt{3}}{k} = 30$

$\dfrac{\sqrt{3}}{k} = \cos 30$

$k = 2$

24 Which expression is equivalent to $\dfrac{2x^{-2}y^{-2}}{4y^{-5}}$?

(1) $\dfrac{y^3}{2x^2}$ ⟵ circled

(3) $\dfrac{2x^2}{y^3}$

(2) $\dfrac{2y^3}{x^2}$

(4) $\dfrac{x^2}{2y^3}$

25 Expressed with a rational denominator and in simplest form,

$\dfrac{x}{x-\sqrt{x}}$ is

(1) $\dfrac{x^2 + x\sqrt{x}}{x^2 - x}$

(3) $\dfrac{x + \sqrt{x}}{1 - x}$

(2) $-\sqrt{x}$

(4) $\dfrac{x + \sqrt{x}}{x - 1}$ ⟵ circled

Handwritten: $\dfrac{x}{x-\sqrt{x}} \cdot \dfrac{x+\sqrt{x}}{x+\sqrt{x}} = \dfrac{x^2 + x\sqrt{x}}{x^2 - x}$

$\dfrac{x(x+\sqrt{x})}{x(x-1)}$

26 What is the common ratio of the sequence
$\dfrac{1}{64}a^5b^3,\ -\dfrac{3}{32}a^3b^4,\ \dfrac{9}{16}ab^5,\ldots$?

(1) $-\dfrac{3b}{2a^2}$

(3) $-\dfrac{3a^2}{b}$

(2) $-\dfrac{6b}{a^2}$ ⟵ circled

(4) $-\dfrac{6a^2}{b}$

Handwritten: $\dfrac{-\dfrac{3}{32}a^3b^4}{\dfrac{1}{64}\cdot a^5 b^3}$

27 In $\triangle KLM$, $KL = 20$, $LM = 13$, and $m\angle K = 40$. The measure of $\angle M$

(1) must be between $0°$ and $90°$

(2) must equal $90°$

(3) must be between $90°$ and $180°$

(4) is ambiguous ⟵ circled

Handwritten: $\dfrac{13}{\sin 40} = \dfrac{20}{\sin M}$

$M \approx 81, 99$

$81 + 40 < 180 \qquad 99 + 40 < 180$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 Determine the sum and the product of the roots of the equation $12x^2 + x - 6 = 0$.

$$SUM = \frac{-B}{A} = \frac{-1}{12}$$

$$product = \frac{C}{A} = \frac{-1}{2}$$

29 Solve algebraically for x:

$$\log_{27}(2x - 1) = \tfrac{4}{3}$$

$$2x - 1 = 27^{\frac{4}{3}}$$

$$2x - 1 = 81$$

$$2x = 82$$

$$x = 41$$

30 Find the number of possible different 10-letter arrangements using the letters of the word "STATISTICS."

$$\frac{10!}{3!\,3!\,2!} = 50,400$$

31 Express the product of cos 30° and sin 45° in simplest radical form.

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4}$$

32 Find, algebraically, the measure of the obtuse angle, to the *nearest degree*, that satisfies the equation $5 \csc \theta = 8$.

$$\csc \theta = \frac{8}{5}$$

$$\sin \theta = \frac{5}{8}$$

$$\theta \approx 39, \boxed{141}$$

33 If $g(x) = \left(ax\sqrt{1-x}\right)^2$, express $g(10)$ in simplest form.

$$g(10) = \left(10a\sqrt{1-10}\right)^2$$
$$= 100a^2(-9)$$
$$= -900a^2$$

34 Express $\dfrac{\cot x \sin x}{\sec x}$ as a single trigonometric function, in simplest form, for all values of x for which it is defined.

$$\frac{\dfrac{\cos x}{\sin x} \cdot \sin x}{\dfrac{1}{\cos x}} = \cos^2 x$$

35 On a multiple-choice test, Abby randomly guesses on all seven questions. Each question has four choices. Find the probability, to the *nearest thousandth*, that Abby gets *exactly* three questions correct.

$$_7C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^4$$

$$35 \left(\frac{1}{64}\right)\left(\frac{81}{256}\right)$$

$$\frac{2835}{16384} \approx .173$$

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$X\left(\frac{13}{x} = 10 - x\right)$$

$$13 = 10x - x^2$$

$$x^2 - 10x + 13 = 0$$

$$X = \frac{10 \pm \sqrt{100 - 4(13)}}{2(1)}$$

$$X = \frac{10 \pm \sqrt{48}}{2}$$

$$X = \frac{10 \pm 4\sqrt{3}}{2}$$

$$X = 5 \pm 2\sqrt{3}$$

37 A ranch in the Australian Outback is shaped like triangle ACE, with $m\angle A = 42$, $m\angle E = 103$, and $AC = 15$ miles. Find the area of the ranch, to the *nearest square mile*.

$$\frac{15}{\sin 103} = \frac{a}{\sin 42}$$

$$a \approx 10.3$$

$$Area = \frac{1}{2}(15)(10.3) \sin 35 \approx 44$$

38 Ten teams competed in a cheerleading competition at a local high school. Their scores were 29, 28, 39, 37, 45, 40, 41, 38, 37, and 48.

How many scores are within one population standard deviation from the mean?

$$\sigma_x \approx 5.9$$

$$\bar{x} \approx 38.2$$

$$32.3 - 44.1$$

$$6$$

For these data, what is the interquartile range?

$$Q_3 - Q_1 = 41 - 37 = 4$$

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

$$x(x^3 + 4x^2 + 4x + 16) = 0$$

$$x(x^2(x+4) + 4(x+4)) = 0$$

$$x(x^2+4)(x+4) = 0$$

$$0 \qquad \pm\sqrt{-4} \qquad\qquad -4$$

$$\pm 2i$$