

**REGENTS EXAMINATION IN
ALGEBRA 2/TRIGONOMETRY
TEST SAMPLER
FALL 2009**



The University of the State of New York
THE STATE EDUCATION DEPARTMENT
Office of Standards, Assessments and Reporting
Albany, New York 12234
www.emsc.nysed.gov/osa/



David Abrams, Assistant Commissioner
Office of Standards, Assessment and Reporting

October 2009

Dear Colleagues:

Thank you for your support as we begin the third year of the transition to the new Regents Examinations in mathematics. The new Regents Examination in Algebra 2/Trigonometry will be administered for the first time in June 2010. That administration will be the last step in the transition from Mathematics A and Mathematics B to Integrated Algebra, Geometry, and Algebra 2/Trigonometry that will take place over the next year.

The Regents Examination in Algebra 2/Trigonometry is being developed to evaluate student achievement of the Mathematics Learning Standard 3 and the core curriculum, revised 2005. This Regents Examination in Algebra 2/Trigonometry Test Sampler consists of the types of questions, the formatting, and the scoring guides that are being developed for the examination. It also includes examples of student work from field tests. This Test Sampler may be printed and duplicated for use in classroom instruction.

The Department is proud of its tradition of involving New York State teachers in a variety of curriculum guidance initiatives. Over the years, thousands of teachers have worked with us, and the expertise of diverse educators representing New York State's diverse student population is essential in guiding this important work.

If you would like to become one of the teachers involved in test development and standard-setting activities, please download and complete the Department's application for Item Writer Orientation found at:

<http://www.emsc.nysed.gov/osa/app-itw.htm>

Thank you for all the work that you do on behalf of the students in New York State.

Sincerely,

David Abrams

Contents

| | |
|--|-----------|
| Introduction | iv |
| General Directions to the Student | 1 |
| Test Questions | 2 |
| Part I | 2 |
| Part II | 12 |
| Part III | 16 |
| Part IV | 19 |
| Reference Sheet | 21 |
| Scrap Graph Paper | 23 |
| Sample Answer Sheet | 25 |
| Scoring Guide | 27 |
| Part I | 27 |
| Part II | 28 |
| Part III | 57 |
| Part IV | 76 |
| Appendices | 84 |
| Appendix A: Specifications for the Regents Examination in Algebra 2/Trigonometry . . . | 84 |
| Appendix B: Map to Core Curriculum | 85 |

Introduction

In March 2005, the Board of Regents adopted a new Learning Standard for Mathematics and issued a revised Mathematics Core Curriculum, resulting in the need for the development and phasing in of three new Regents Examinations in mathematics: Integrated Algebra, Geometry, and Algebra 2/Trigonometry. These new Regents Examinations in mathematics will replace the Regents Examinations in Mathematics A and Mathematics B. Students must pass any one of these new commencement-level Regents Examinations in order to fulfill the mathematics Regents Examination requirement for graduation. The first administration of the Regents Examination in Integrated Algebra took place in June 2008 and the first administration of the Regents Examination in Geometry took place in June 2009. The first administration of the Regents Examination in Algebra 2/Trigonometry will take place in June 2010. The Regents Examination in Algebra 2/Trigonometry will be based on the content of the Mathematics Core Curriculum (Revised 2005).

The Regents Examination in Algebra 2/Trigonometry Test Sampler provides examples of the format and types of questions that will comprise the operational examination. The scoring guide in the sampler includes examples of student responses from field testing and the credit allowed for each response.

The reference sheet included in the test sampler will also be provided as part of the operational examination booklet. A straightedge (ruler) and a graphing calculator must be available for the exclusive use of each student taking the examination. For the operational examination, the memory of any calculator with programming capability must be cleared, reset, or disabled when students enter the testing room. If the memory of a student's calculator is password-protected and cannot be cleared, the calculator must not be used. Students may not use calculators that are capable of symbol manipulation or that can communicate with other calculators through infrared sensors, nor may students use operating manuals, instruction or formula cards, or other information concerning the operation of calculators during the examination.

The sampler may be duplicated for use in your classroom.

ALGEBRA 2/TRIGONOMETRY TEST SAMPLER FALL 2009

GENERAL DIRECTIONS TO THE STUDENT

Answer all 39 questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. No partial credit will be allowed on the multiple-choice section.

For Parts II, III, and IV, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in these parts, a correct numerical answer with no work shown will receive only 1 credit.

A reference sheet that you may need to answer some questions in this examination is included.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this examination as scrap paper. Scrap graph paper is provided at the end of this examination for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

Note: A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [54]

Use this space for
computations.

1 The expression $(3 - 7i)^2$ is equivalent to

(1) $-40 + 0i$

(3) $58 + 0i$

(2) $-40 - 42i$

(4) $58 - 42i$

2 If $f(x) = \frac{1}{2}x - 3$ and $g(x) = 2x + 5$, what is the value of $(g \circ f)(4)$?

(1) -13

(3) 3

(2) 3.5

(4) 6

3 What are the values of θ in the interval $0^\circ \leq \theta < 360^\circ$ that satisfy the equation $\tan \theta - \sqrt{3} = 0$?

(1) $60^\circ, 240^\circ$

(2) $72^\circ, 252^\circ$

(3) $72^\circ, 108^\circ, 252^\circ, 288^\circ$

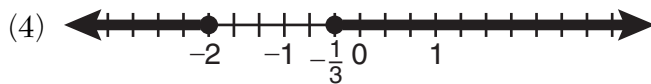
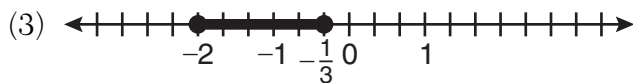
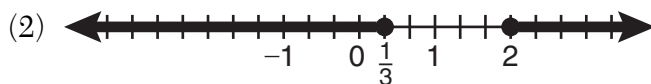
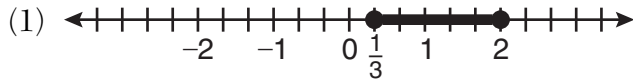
(4) $60^\circ, 120^\circ, 240^\circ, 300^\circ$

Use this space for computations.

4 A survey completed at a large university asked 2,000 students to estimate the average number of hours they spend studying each week. Every tenth student entering the library was surveyed. The data showed that the mean number of hours that students spend studying was 15.7 per week. Which characteristic of the survey could create a bias in the results?

- (1) the size of the sample
- (2) the size of the population
- (3) the method of analyzing the data
- (4) the method of choosing the students who were surveyed

5 Which graph represents the solution set of $|6x - 7| \leq 5$?



Use this space for computations.

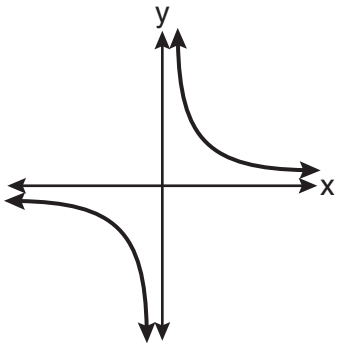
6 Which function is *not* one-to-one?

- (1) $\{(0,1), (1,2), (2,3), (3,4)\}$
- (2) $\{(0,0), (1,1), (2,2), (3,3)\}$
- (3) $\{(0,1), (1,0), (2,3), (3,2)\}$
- (4) $\{(0,1), (1,0), (2,0), (3,2)\}$

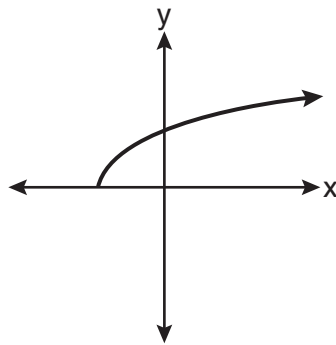
7 In $\triangle ABC$, $m\angle A = 120$, $b = 10$, and $c = 18$. What is the area of $\triangle ABC$ to the nearest square inch?

- (1) 52
- (2) 78
- (3) 90
- (4) 156

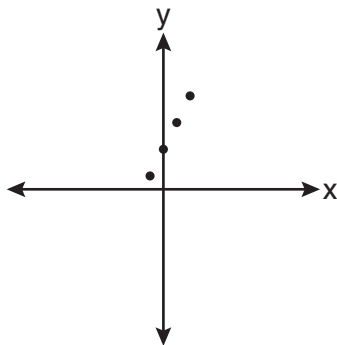
8 Which graph does *not* represent a function?



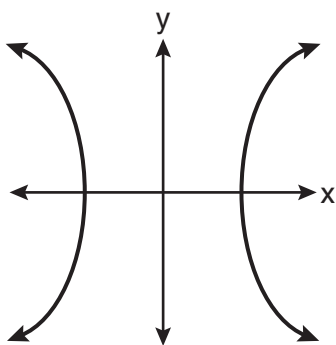
(1)



(3)



(2)



(4)

**Use this space for
computations.**

9 The expression $\log_8 64$ is equivalent to

(1) 8

(3) $\frac{1}{2}$

(2) 2

(4) $\frac{1}{8}$

10 The expression $\cos 4x \cos 3x + \sin 4x \sin 3x$ is equivalent to

(1) $\sin x$

(3) $\cos x$

(2) $\sin 7x$

(4) $\cos 7x$

11 The value of the expression $2 \sum_{n=0}^2 (n^2 + 2^n)$ is

(1) 12

(3) 24

(2) 22

(4) 26

12 For which equation does the sum of the roots equal $\frac{3}{4}$ and the product of the roots equal -2 ?

(1) $4x^2 - 8x + 3 = 0$

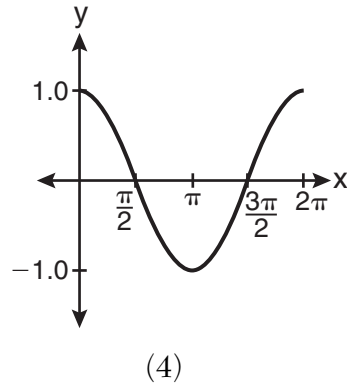
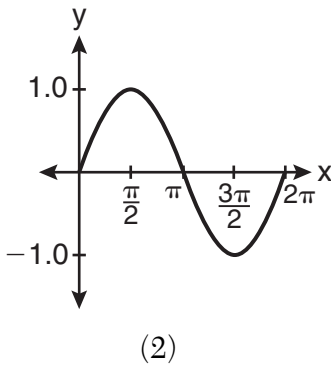
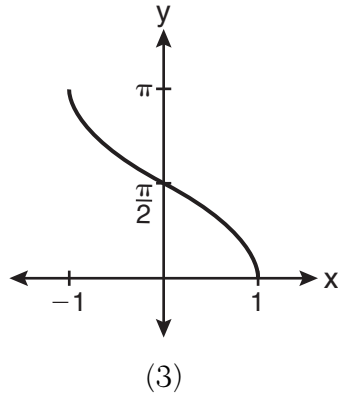
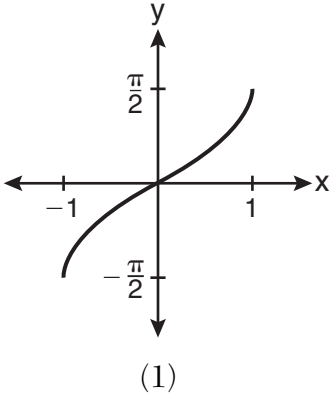
(2) $4x^2 + 8x + 3 = 0$

(3) $4x^2 - 3x - 8 = 0$

(4) $4x^2 + 3x - 2 = 0$

Use this space for computations.

13 Which graph represents the equation $y = \cos^{-1} x$?



14 The expression $\frac{a^2 b^{-3}}{a^{-4} b^2}$ is equivalent to

(1) $\frac{a^6}{b^5}$

(3) $\frac{a^2}{b}$

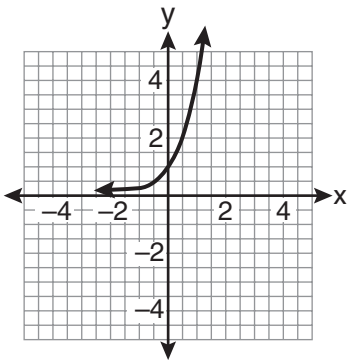
(2) $\frac{b^5}{a^6}$

(4) $a^{-2} b^{-1}$

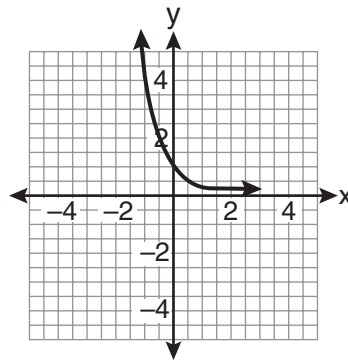
Use this space for computations.

- 15 The lengths of 100 pipes have a normal distribution with a mean of 102.4 inches and a standard deviation of 0.2 inch. If one of the pipes measures exactly 102.1 inches, its length lies
- (1) below the 16th percentile
 - (2) between the 16th and 50th percentiles
 - (3) between the 50th and 84th percentiles
 - (4) above the 84th percentile

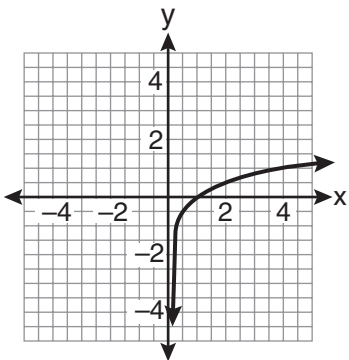
- 16 If a function is defined by the equation $f(x) = 4^x$, which graph represents the inverse of this function?



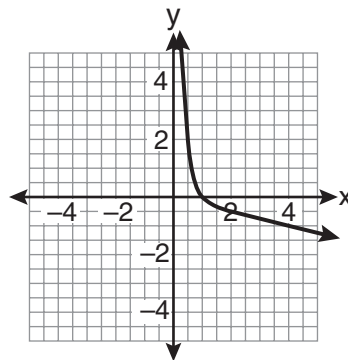
(1)



(3)



(2)



(4)

17 Factored completely, the expression $6x - x^3 - x^2$ is equivalent to

- (1) $x(x + 3)(x - 2)$
- (2) $x(x - 3)(x + 2)$
- (3) $-x(x - 3)(x + 2)$
- (4) $-x(x + 3)(x - 2)$

18 The expression $4ab\sqrt{2b} - 3a\sqrt{18b^3} + 7ab\sqrt{6b}$ is equivalent to

- (1) $2ab\sqrt{6b}$
- (2) $16ab\sqrt{2b}$
- (3) $-5ab + 7ab\sqrt{6b}$
- (4) $-5ab\sqrt{2b} + 7ab\sqrt{6b}$

19 What is the fourth term in the expansion of $(3x - 2)^5$?

- (1) $-720x^2$
- (2) $-240x$
- (3) $720x^2$
- (4) $1,080x^3$

20 Written in simplest form, the expression $\frac{\frac{x}{4} - \frac{1}{x}}{\frac{1}{2x} + \frac{1}{4}}$ is equivalent to

- (1) $x - 1$
- (2) $x - 2$
- (3) $\frac{x - 2}{2}$
- (4) $\frac{x^2 - 4}{x + 2}$

21 What is the solution of the equation $2\log_4(5x) = 3$?

- (1) 6.4
- (2) 2.56
- (3) $\frac{9}{5}$
- (4) $\frac{8}{5}$

Use this space for computations.

25 Which formula can be used to determine the total number of different eight-letter arrangements that can be formed using the letters in the word *DEADLINE*?

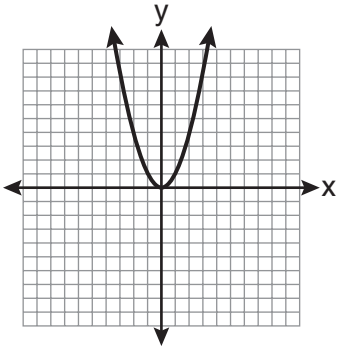
(1) $8!$

(3) $\frac{8!}{2! + 2!}$

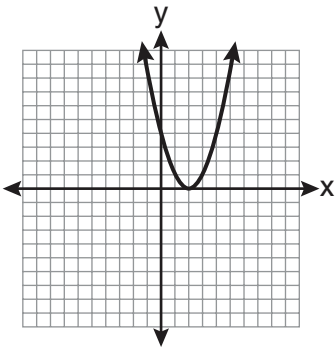
(2) $\frac{8!}{4!}$

(4) $\frac{8!}{2! \cdot 2!}$

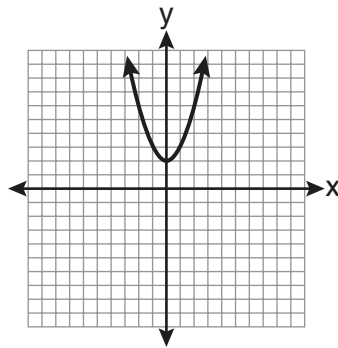
26 The graph below shows the function $f(x)$.



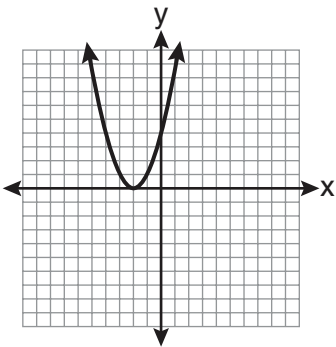
Which graph represents the function $f(x + 2)$?



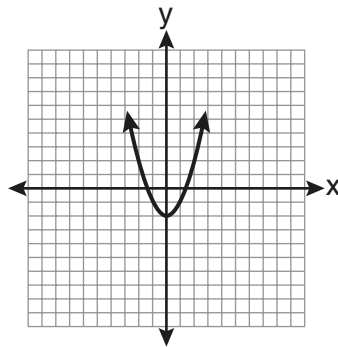
(1)



(3)



(2)



(4)

**Use this space for
computations.**

27 The equation $y - 2\sin\theta = 3$ may be rewritten as

(1) $f(y) = 2\sin x + 3$

(2) $f(y) = 2\sin\theta + 3$

(3) $f(x) = 2\sin\theta + 3$

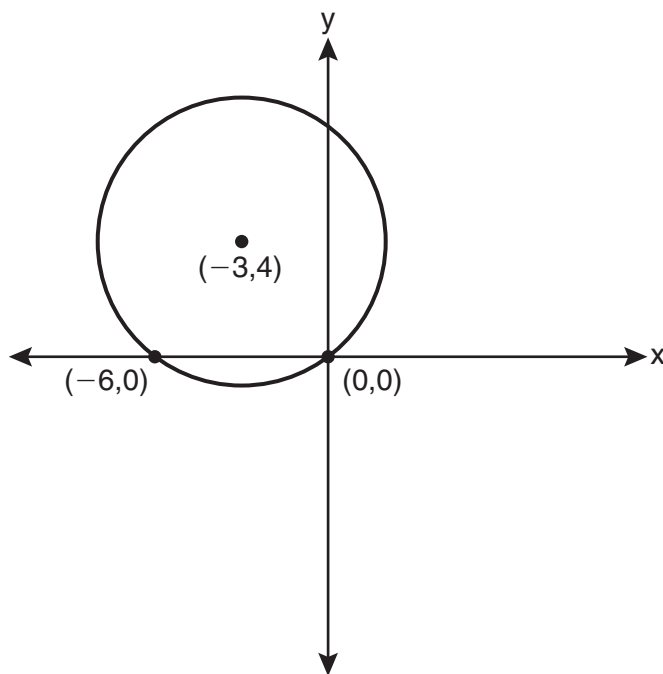
(4) $f(\theta) = 2\sin\theta + 3$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [16]

28 Express $\frac{5}{3 - \sqrt{2}}$ with a rational denominator, in simplest radical form.

29 Write an equation of the circle shown in the graph below.



30 Solve for x : $\frac{4x}{x-3} = 2 + \frac{12}{x-3}$

31 Find, to the *nearest minute*, the angle whose measure is 3.45 radians.

- 32** Matt places \$1,200 in an investment account earning an annual rate of 6.5%, compounded continuously. Using the formula $V = Pe^{rt}$, where V is the value of the account in t years, P is the principal initially invested, e is the base of a natural logarithm, and r is the rate of interest, determine the amount of money, to the *nearest cent*, that Matt will have in the account after 10 years.
- 33** If θ is an angle in standard position and its terminal side passes through the point $(-3,2)$, find the exact value of $\csc \theta$.

34 Find the first four terms of the recursive sequence defined below.

$$a_1 = -3$$

$$a_n = a_{(n-1)} - n$$

35 A committee of 5 members is to be randomly selected from a group of 9 teachers and 20 students. Determine how many different committees can be formed if 2 members must be teachers and 3 members must be students.

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

36 Solve $2x^2 - 12x + 4 = 0$ by completing the square, expressing the result in simplest radical form.

37 Solve the equation $8x^3 + 4x^2 - 18x - 9 = 0$ algebraically for all values of x .

38 The table below shows the results of an experiment involving the growth of bacteria.

| | | | | | | |
|-------------------------------|---|----|----|-----|-----|-----|
| Time (x) (in minutes) | 1 | 3 | 5 | 7 | 9 | 11 |
| Number of Bacteria (y) | 2 | 25 | 81 | 175 | 310 | 497 |

Write a power regression equation for this set of data, rounding all values to *three decimal places*.

Using this equation, predict the bacteria's growth, to the *nearest integer*, after 15 minutes.

Part IV

Answer the question in this part. The correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. [6]

39 Two forces of 25 newtons and 85 newtons acting on a body form an angle of 55° .

Find the magnitude of the resultant force, to the *nearest hundredth of a newton*.

Find the measure, to the *nearest degree*, of the angle formed between the resultant and the larger force.

Algebra 2/Trigonometry Reference Sheet

Area of a Triangle

$$K = \frac{1}{2} ab \sin C$$

Functions of the Sum of Two Angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Sum of a Finite Arithmetic Series

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Binomial Theorem

$$(a + b)^n = {}_n C_0 a^n b^0 + {}_n C_1 a^{n-1} b^1 + {}_n C_2 a^{n-2} b^2 + \dots + {}_n C_n a^0 b^n$$

$$(a + b)^n = \sum_{r=0}^n {}_n C_r a^{n-r} b^r$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \frac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

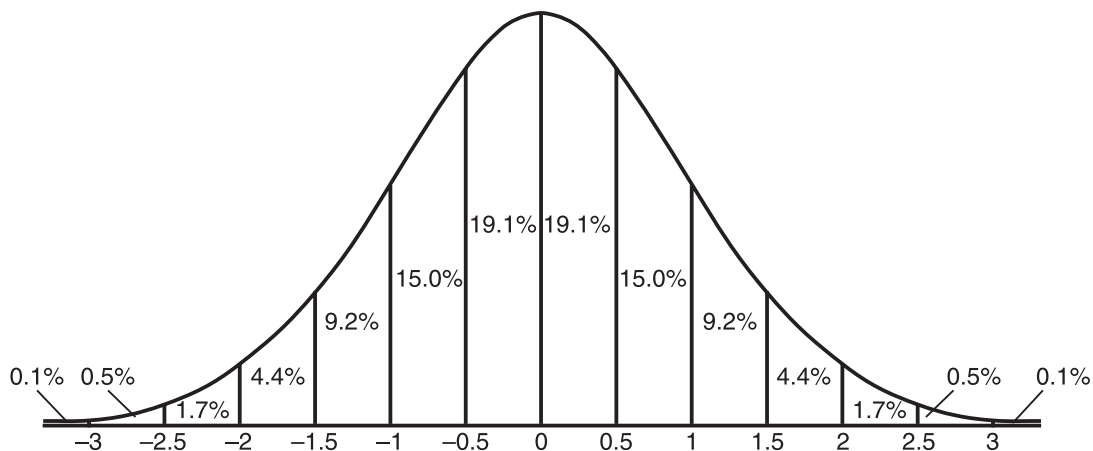
$$\cos \frac{1}{2} A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

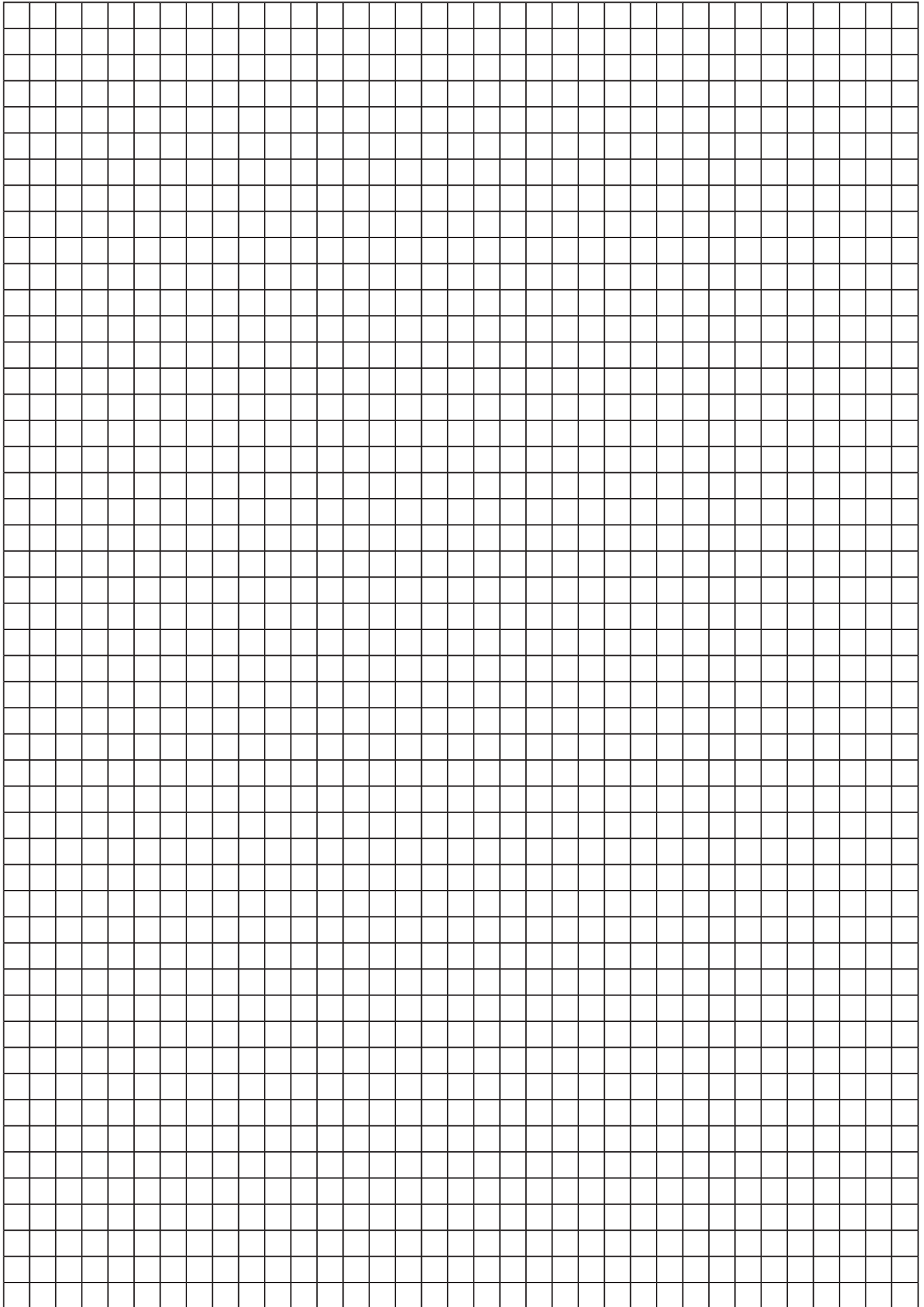
Sum of a Finite Geometric Series

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

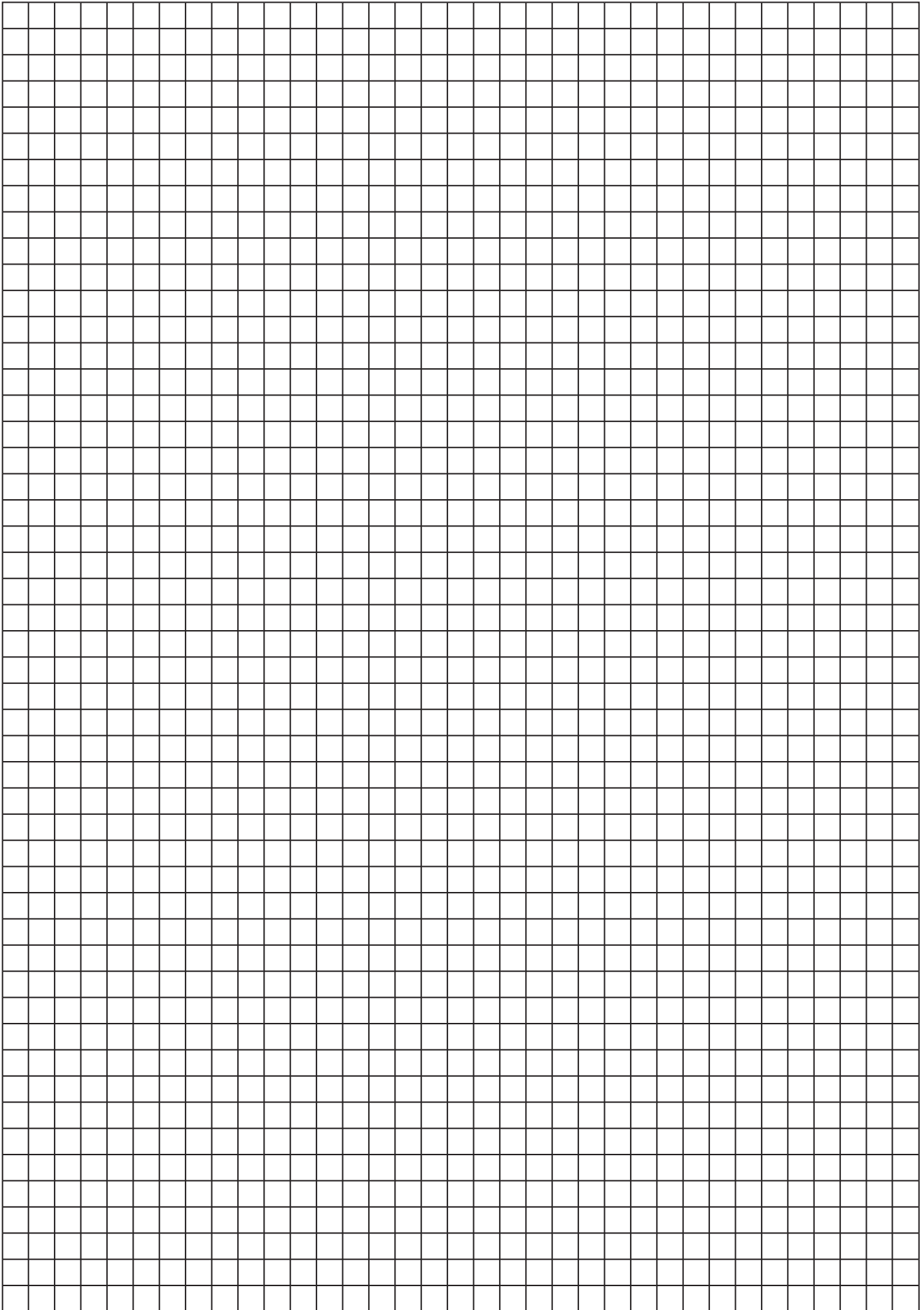
Normal Curve Standard Deviation



Scrap Graph Paper — This sheet will *not* be scored.



Scrap Graph Paper — This sheet will *not* be scored.



ALGEBRA 2/TRIGONOMETRY TEST SAMPLER

Fall 2009

ANSWER SHEET

Student Sex: Male Female Grade

Teacher School

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 27 questions in this part.

- | | | | |
|---------|----------|----------|----------|
| 1 | 8 | 15 | 22 |
| 2 | 9 | 16 | 23 |
| 3 | 10 | 17 | 24 |
| 4 | 11 | 18 | 25 |
| 5 | 12 | 19 | 26 |
| 6 | 13 | 20 | 27 |
| 7 | 14 | 21 | |

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Scoring Guide for the Algebra 2/Trigonometry Test Sampler

Answers to multiple-choice questions 1 through 27, and the specific rubrics for open-ended questions 28 through 39, are provided on the following pages. A complete and correct student response is provided for each open-ended question. The response shows one example of how to solve the problem. In most cases there are other acceptable solutions. Other student responses are shown for each score level.

The maximum raw score for the Regents Examination in Algebra 2/Trigonometry is allocated as follows:

| | | |
|----------|---|------------|
| Part I | 27 two-credit multiple-choice questions | 54 credits |
| Part II | 8 two-credit open-ended questions | 16 credits |
| Part III | 3 four-credit open-ended questions | 12 credits |
| Part IV | 1 six-credit open-ended question | 6 credits |

Part I

| | | | |
|-------|--------|--------|--------|
| (1) 2 | (8) 4 | (15) 1 | (22) 4 |
| (2) 3 | (9) 2 | (16) 2 | (23) 3 |
| (3) 1 | (10) 3 | (17) 4 | (24) 3 |
| (4) 4 | (11) 3 | (18) 4 | (25) 4 |
| (5) 1 | (12) 3 | (19) 1 | (26) 2 |
| (6) 4 | (13) 3 | (20) 2 | (27) 4 |
| (7) 2 | (14) 1 | (21) 4 | |

Part II

(28) Express $\frac{5}{3-\sqrt{2}}$ with a rational denominator, in simplest radical form.

Rubric

[2] $\frac{5(3+\sqrt{2})}{7}$ or an equivalent answer, and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made, such as not expressing the answer in simplest radical form.

or

[1] $\frac{5(3+\sqrt{2})}{7}$ or an equivalent answer, but no work is shown.

[0] The answer is expressed as a decimal.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 28 – Score 2

$$\frac{5}{3-\sqrt{2}}$$

$$\frac{5}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$\frac{15+5\sqrt{2}}{9-2}$$

$$\frac{15+5\sqrt{2}}{7}$$

$$\frac{5(3+\sqrt{2})}{7}$$

Student work for Item 28 – Score 1

$$\begin{aligned}\frac{5}{(3-\sqrt{2})(3+\sqrt{2})} &= \frac{15+5\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}+\sqrt{4}} \\ &= \frac{15+5\sqrt{2}}{9+\sqrt{4}} \\ &= \frac{15+5\sqrt{2}}{11}\end{aligned}$$

Student work for Item 28 – Score 0

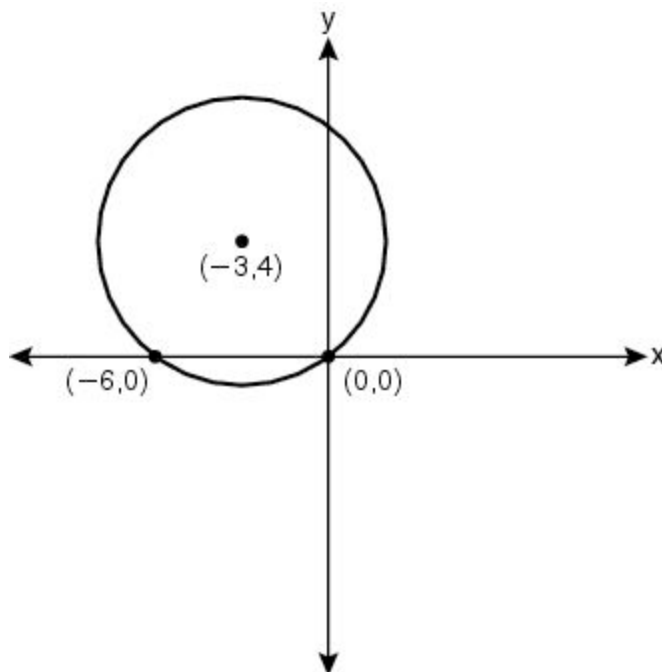
$$\frac{5}{3-\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{5\sqrt{2}}{3-2}$$

$$\frac{5\sqrt{2}}{1}$$

$$5\sqrt{2}$$

(29) Write an equation of the circle shown in the graph below.



Rubric

[2] $(x + 3)^2 + (y - 4)^2 = 25$ or an equivalent equation, and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

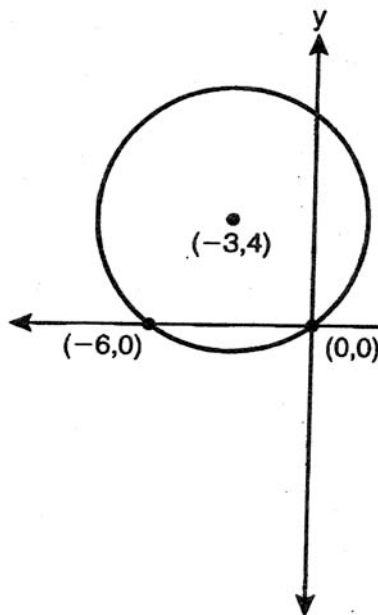
[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $(x + 3)^2 + (y - 4)^2 = 25$ or an equivalent equation, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 29 – Score 2



$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-3 - 6)^2 + (4 - 0)^2}$$

$$= \sqrt{8^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$x = \sqrt{25}$$

$$d = 5$$

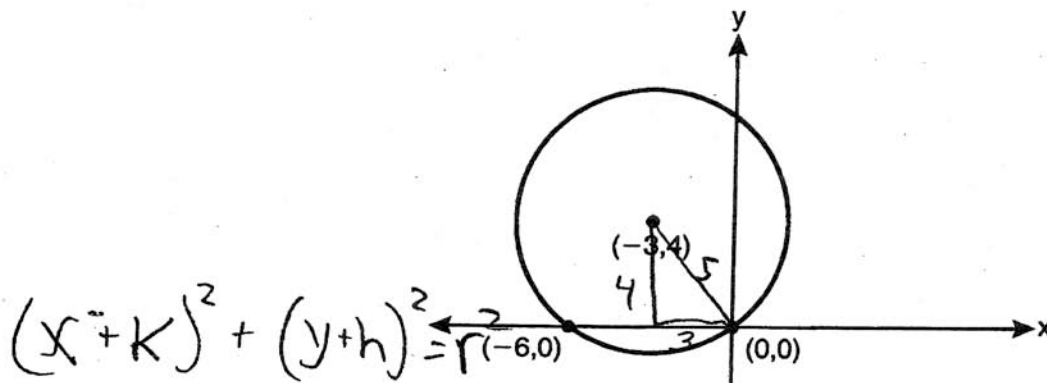
$$(x - h)^2 + (y - k)^2 = r^2$$

center = $-3, 4$

$$(x + 3)^2 + (y - 4)^2 = 5^2$$

$$(x + 3)^2 + (y - 4)^2 = 25$$

Student work for Item 29 – Score 2



center = $-3, 4$

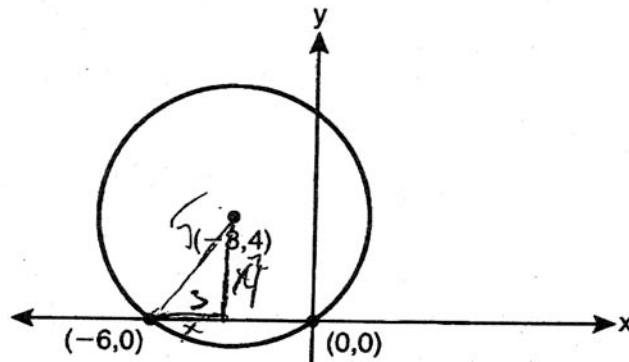
$k = 3$

$h = -4$

radius = 5

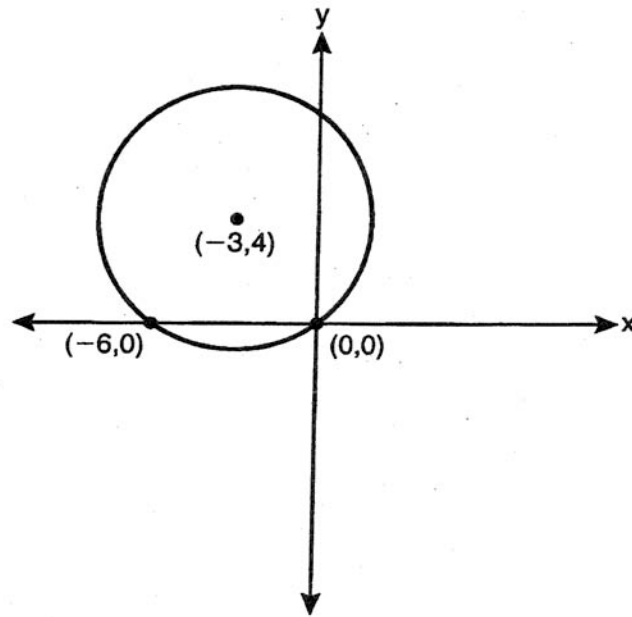
$(x+3)^2 + (y-4)^2 = 25$

Student work for Item 29 – Score 1



$$(x^2 + 3) + (y^2 - 4) = 25$$

Student work for Item 29 – Score 0



$$(x + 3)^2 - (y - 4)^2 = 36$$

(30) Solve for x : $\frac{4x}{x-3} = 2 + \frac{12}{x-3}$

Rubric

[2] \emptyset , $\{ \}$, or no solution, and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] The equation $4x = 2(x - 3) + 12$ is written, but no further correct work is shown.

or

[1] 3, and appropriate work is shown.

or

[1] \emptyset , $\{ \}$, or no solution, but no work is shown.

[0] $\{ \emptyset \}$, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 30 – Score 2

Solve for x: $\frac{4x}{\cancel{x-3}} = \frac{2}{1} + \frac{12}{\cancel{x-3}}$

$$4x = 2x - 6 + 12$$

$$4x = 2x + 6$$

$$2x = 6$$

$$x \neq 3$$

No solution

Student work for Item 30 – Score 1

$$\frac{4x}{x-3} = \left(\frac{2}{1}\right) + \frac{12}{x-3} \rightarrow \frac{2x-6+12}{x-3}$$

~~$$\frac{4x}{x-3} = \frac{2x+6}{x-3}$$~~

$$(x-3)(2x+6)$$

$$4x^2 - 12x = 2x^2 + 6x - 6x - 18$$

$$4x^2 - 12x = 2x^2 - 18$$

$$-2x^2 + 18 = -2x^2 + 18$$

$$2x^2 - 12x + 18 = 0$$

$$2(x^2 - 6x + 9) = 0$$

$$2(x-3)(x-3) = 0$$

$$x=3 \quad | \quad x=3$$

$$\boxed{x=3}$$

undefined

$$\frac{4(3)}{(3)-3} = 2 + \frac{12}{(3)-3}$$

$$\frac{12}{0} = 2 + \frac{12}{0}$$

$$\frac{4x}{x-3} = \frac{2}{1} + \frac{12}{x-3}$$

$$\frac{4x}{x-3} = \frac{2+12}{x-3}$$

$$(x-3)(14) = (4x)(x-3)$$

$$14x - 42 = 4x^2 - 12x$$

$$= 14x + 42 = 4x^2 - 12x + 42$$

$$4x^2 - 26x + 42 = 0$$

$$2(2x^2 - 13x + 21)$$

$$2 = 0 \quad (2x - 7)(x - 3) = 0$$

| | |
|--|------------------------|
| $2x - 7 = 0$ $2x = 7$ $x = \frac{7}{2}$ $x = 3.5$ | $x - 3 = 0$ $x = 3$ |
|--|------------------------|

(31) Find, to the *nearest minute*, the angle whose measure is 3.45 radians.

Rubric

[2] $197^{\circ}40'$ or $11,860'$, and appropriate work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made, such as stating the angle to be 197.67° .

or

[1] A correct formula, substitution, or conversion is written, but no further correct work is shown.

or

[1] $197^{\circ}40'$ or $11,860'$, but no work is shown.

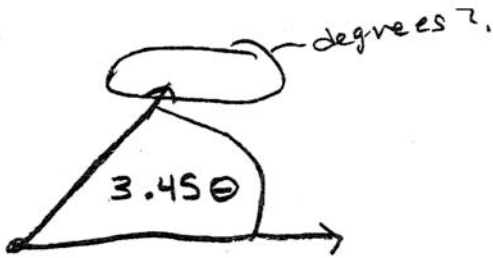
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 31 – Score 2

$$\begin{array}{l} \frac{\text{Deg.}}{\text{Radians}} \times \frac{180}{\pi} \\ \frac{x}{3.45} \times \frac{180}{\pi} \end{array}$$
$$\frac{621}{\pi} = \frac{\pi x}{\pi}$$

$x = 197^{\circ}40'$

Student work for Item 31 – Score 1



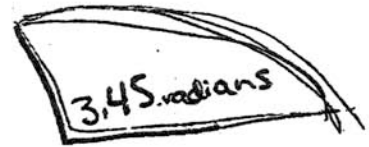
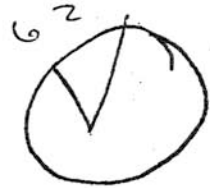
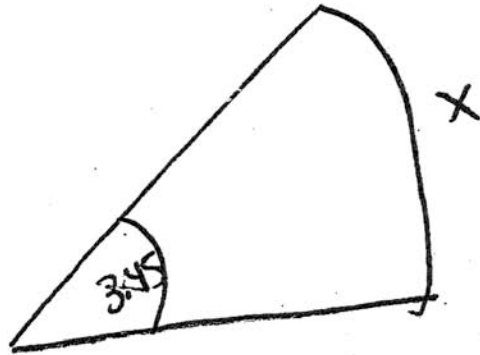
$$\frac{3.45}{1} \cdot \frac{180}{\pi} = 621 - 360 = 261^\circ$$

$$\frac{90}{15} = 6 \quad \frac{261}{6} = 43.5$$

$$60 - 43.5 = 16.5$$



Student work for Item 31 – Score 0



S



197.77

$$\theta = s/r$$
$$3.45 = x$$

198
minutes

- (32) Matt places \$1,200 in an investment account earning an annual rate of 6.5% compounded continuously. Using the formula $V = Pe^{rt}$, where V is the value of the account in t years, P is the principal initially invested, e is the base of a natural logarithm, and r is the rate of interest, determine the amount of money, to the *nearest cent*, that Matt will have in the account after 10 years.

Rubric

[2] 2,298.65, and appropriate work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $V = 1200e^{0.065(10)}$, but no further correct work is shown.

or

[1] 2,298.65, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 32 – Score 2

$$V = Pe^{rt}$$

$$V = 1200e^{0.065(10)}$$

$$V = 1200e^{0.65}$$

$$V \approx \$2298.648995$$

The amount will be about \$2298.65 after 10 years.

$$V = Pe^{rt}$$

$$P = 1,200$$

$$t = 10 \text{ years}$$

$$r = \underline{\underline{6.5\%}}$$

$$V = 1,200 e^{(.065)(10)}$$

$$\underline{V} = 1,200 e^{.65}$$

$$V = 1,200 (2.718281828)^{.65}$$

$$V = 2120.259826$$

$$V = \$2120.26$$

Student work for item 32 – Score 0

$$v = 1200(e)^{0.6 \cdot 5(10)}$$

$$v = 1003.385475(e)$$

- (33) If θ is an angle in standard position and its terminal side passes through the point $(-3,2)$, find the exact value of $\csc \theta$.

Rubric

[2] $\frac{\sqrt{13}}{2}$, and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Appropriate work is shown, but the answer is not stated as an exact value.

or

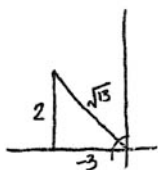
[1] $\frac{\sqrt{13}}{2}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 33 – Score 2

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\csc \theta = \frac{\sqrt{13}}{2}$$



Student work for Item 33 – Score 1

| |
|-----|
| S/A |
| T/C |

$$\sin \theta = \frac{y}{r} = \frac{2}{\sqrt{12}}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\csc \theta = \frac{1}{\frac{2}{\sqrt{12}}} \cdot \frac{\sqrt{12}}{\sqrt{12}}$$

$$= \frac{\sqrt{12}}{2}$$

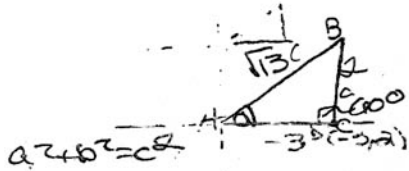
$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-3)^2 + 2^2}$$

$$r = \sqrt{13}$$

Student work for Item 33 – Score 0

$$\csc = \sin$$



$$\frac{\sin \theta}{2} = \frac{\sin 90}{\sqrt{3}}$$

$$\sin \theta = 33.69006752 \cdot \frac{150}{\pi}$$

$$\frac{33.69 \pi}{180}$$

(34) Find the first four terms of the recursive sequence defined below.

$$a_1 = -3$$
$$a_n = a_{(n-1)} - n$$

Rubric

[2] -3, -5, -8, and -12, and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] -3, -5, -8, and -12, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 34 – Score 2

$$a_2 = a_1 - 2$$

$$a_2 = -3 - 2$$

$$a_2 = -5$$

$-3, -5, -8, -12$

$$a_3 = a_2 - 3$$

$$a_3 = -5 - 3$$

$$a_3 = -8$$

$$a_4 = a_3 - 4$$

$$a_4 = -8 - 4$$

$$a_4 = -12$$

Student work for Item 34 – Score 1

$$a_2 = a_1 - 2$$

$$a_2 = -3 - 2$$

$$a_2 = -5$$

$$a_3 = a_2 - 2$$

$$a_3 = -5 - 2$$

$$a_3 = -7$$

$$a_4 = a_3 - 2$$

$$a_4 = -7 - 2$$

$$a_4 = -9$$

$$\{-3, -5, -7, -9\}$$

Student work for Item 34 – Score 0

$$-3_0 = -3_0^{-1-0} =$$

$$-3 = -3 - 1^{-0} =$$

$$a_2 = -4$$

$$a_3 = -5$$

$$a_4 = -6$$

$$\begin{pmatrix} -3 \\ -4 \end{pmatrix} = \frac{-4n}{-4} \quad \left. \vphantom{\begin{pmatrix} -3 \\ -4 \end{pmatrix}} \right\} \text{1st term}$$

$$a_{-4} = 0_{-4}^{-1-(-4)} \quad \left. \vphantom{a_{-4}} \right\} \text{2nd term}$$

$$a_{-4} = (-5)$$

$$a_{-5} = 0_{-5}^{-1-(-5)} = (-6) \quad \left. \vphantom{a_{-5}} \right\} \text{3rd term}$$

$$a_{-6} = 0_{-6}^{-1-(-6)} = (-7) \quad \left. \vphantom{a_{-6}} \right\} \text{4th term}$$

- (35) A committee of 5 members is to be randomly selected from a group of 9 teachers and 20 students. Determine how many different committees can be formed if 2 members must be teachers and 3 members must be students.

Rubric

[2] 41,040, and appropriate work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] ${}^9C_2 \cdot {}^{20}C_3$, but no further correct work is shown.

or

[1] 41,040, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 35 – Score 2

$${}^9C_2 \cdot {}^{20}C_3$$

$$36 \cdot 1140 = 41040$$

Student work for Item 35– Score 1

$$9C_2 + 20C_3 = 1176$$

$$\begin{array}{r} 9C_2 + 20C_3 \\ 36 + 1140 = 1176 \end{array}$$

Student work for Item 35 – Score 0

$$\begin{aligned} & 20P_2 \qquad 20P_3 \\ & 72 + 6840 \\ & 6912 \end{aligned}$$

Part III

(36) Solve $2x^2 - 12x + 4 = 0$ by completing the square, expressing the result in simplest radical form.

Rubric

[4] $3 \pm \sqrt{7}$, and appropriate work is shown.

[3] Appropriate work is shown, but one computational error is made, or the answer is not expressed in simplest radical form.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $3 \pm \sqrt{7}$, but the answer is found by a method other than completing the square.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or

[1] $3 \pm \sqrt{7}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 36 – Score 4

$$\frac{2x^2 - 12x + 4 = 0}{2}$$

$$x^2 - 6x + 2 = 0$$

$$x^2 - 6x + 9 = -2 + 9$$

$$\sqrt{(x-3)^2} = \sqrt{7}$$

$$x - 3 = \pm\sqrt{7}$$

$$x = 3 \pm \sqrt{7}$$

$$-b/2 = -3^2 = 9$$

$$2x^2 - 12x + 4 = 0$$

$$2(x^2 - 6x + 2) = 0$$

$$x^2 - 6x + 2 = 0$$

$$x^2 - 6x = -2$$

$$(x - 3)(x - 3) = -2 + 9$$

$$(x - 3)^2 = 7$$

$$x - 3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

Student work for Item 36 – Score 2

$$\frac{2x^2 - 12x + 4}{2} = \frac{0}{2}$$

$$x^2 - 6x + 2 = 0$$

$$x^2 - 6x + 9 = 2 + 9$$

$$(x-3)(x-3) = 11$$

$$(x-3)^2 = 11$$

$$x-3 = \sqrt{11}$$

$$x = 3 + \sqrt{11}$$

Student work for Item 36 – Score 1

$$\cancel{x^2 - 12x + 8 = 0}$$

~~x~~

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{(12)^2 - 4 \cdot 2 \cdot 4}}{4}$$

$$12 \pm \sqrt{\frac{112}{4}} = \sqrt{16} \checkmark$$

| | | |
|----------|-----|------------|
| $12 \pm$ | 4 | $\sqrt{7}$ |
| 4 | | |

$$\begin{array}{l} a = 2 \\ b = -12 \\ c = 4 \end{array} \quad \begin{array}{l} 8 \\ 1 \\ 2 \end{array} \quad \begin{array}{l} 8 \\ 8 \\ 4 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{12 \pm \sqrt{144 - 32}}{4}$$

$$x = \frac{12 \pm \sqrt{112}}{4}$$

$$x = 3 \pm \sqrt{112} \leftarrow \begin{array}{l} \sqrt{16} \\ \sqrt{7} \end{array}$$

$$\boxed{x = 3 \pm 4\sqrt{7}}$$

(37) Solve the equation $8x^3 + 4x^2 - 18x - 9 = 0$ algebraically for all values of x .

Rubric

[4] $\pm \frac{3}{2}, -\frac{1}{2}$ or an equivalent answer, and appropriate algebraic work is shown.

[3] Appropriate work is shown, but one computational or factoring error is made.

[2] Appropriate work is shown, but two or more computational or factoring errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $(4x^2 - 9)(2x + 1) = 0$ is found, but no further correct work is shown.

or

[2] $\pm \frac{3}{2}, -\frac{1}{2}$, but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual error and one computational or factoring error are made.

or

[1] $4x^2(2x + 1) - 9(2x + 1) = 0$ is found, but no further correct work is shown.

or

[1] $\pm \frac{3}{2}, -\frac{1}{2}$ or an equivalent answer, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

$$8x^3 + 4x^2 - 18x - 9 = 0$$

$$4x^2(2x+1) - 9(2x+1) = 0$$

$$(4x^2 - 9)(2x+1) = 0$$

$$(2x+1)(2x+3)(2x-3) = 0$$

$$2x+1=0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$2x+3=0$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

$$2x-3=0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$4x^2(2x+1) - 9(2x+1) = 0$$

$$(4x^2 - 9)(2x+1) = 0$$

$$4x^2 - 9 = 0$$

$$4x^2 = 9$$

$$x^2 = \frac{9}{4}$$

$$x = \sqrt{\frac{9}{4}}$$

$$x = \frac{3}{2}$$

$$2x+1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$8x^3 + 4x^2 = 18x + 9$$

$$4x^2(\cancel{2x+1}) = 9(\cancel{2x+1})$$

$$4x^2 = 9$$

$$x^2 = \frac{9}{4}$$

$$x = \pm \frac{3}{2}$$

$$8x^3 + 4x^2 - 18x - 9 = 0$$

$$4x^2(2x+1) - 9(2x+1) = 0$$

Student work for Item 37 – Score 0

$$8x^3 + 4x^2 - 18x - 9 = 0$$

$$2x(4x^2 + 2x - 9) - 9 = 0$$

(38) The table below shows the results of an experiment involving the growth of bacteria.

| | | | | | | |
|-------------------------------|---|----|----|-----|-----|-----|
| Time (x) (in minutes) | 1 | 3 | 5 | 7 | 9 | 11 |
| Number of Bacteria (y) | 2 | 25 | 81 | 175 | 310 | 497 |

Write a power regression equation for this set of data, rounding all values to *three decimal places*.

Using this equation, predict the bacteria's growth, to the *nearest integer*, after 15 minutes.

Rubric for Item 38

[4] $y = 2.001x^{2.298}$ and 1,009, and appropriate work is shown, such as substituting $x = 15$ into the regression equation.

[3] Appropriate work is shown, but one computational or rounding error is made.

or

[3] $y = 2.001x^{2.298}$ and 1,009, but no substitution is shown.

or

[3] The expression $2.001x^{2.298}$ is written, and appropriate work is shown to find 1,009.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $y = 2.001x^{2.298}$, but no further correct work is shown.

or

[2] The expression $2.001x^{2.298}$ is written and 1,009, but no substitution is shown.

or

[2] An incorrect regression equation of equal difficulty is solved appropriately for the bacteria's growth, and appropriate work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or

[1] An incorrect regression equation of a lesser degree of difficulty is solved appropriately for the number of bacteria at fifteen minutes, and appropriate work is shown.

or

[1] The expression $2.001x^{2.298}$ is written, but no further correct work is shown.

or

[1] 1,009, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 38 – Score 4

$$y = ax^b$$

$$a = 2.000877106$$

$$b = 2.248056258$$

$$a) y = 2.001x^{2.248}$$

$$b) y = 2.001(15)^{2.248}$$

$$y = 1009.03168$$

$$y = 1009$$

Student work for Item 38 – Score 3

power reg
 $y = ax^b$

$$a = 2.000877106$$

$$b = 2.29856758$$

$$r^2 = .999967547$$

$$r = .999983773$$

$$(2.001)(x)^{(2.298)}$$

$$(2.001)(15)^{(2.298)}$$

$$1009.03168$$

The power regression equation for this set of data, rounding all values to three decimal places is $(2.001)(x)^{(2.298)}$.

The bacteria's growth, to the nearest integer, after 15 minutes is 1009.

Student work for Item 38 – Score 2

$$y = a \times n^b$$

$$a = 2.000877106$$

$$b = 2.298056758$$

$$y = 2.0 \times 2.29$$

$$y = 2.0 (5)^{2.29}$$

~~Handwritten scribble~~

$$A = 986.913$$

Student work for Item 38 – Score 1

$$a = 2.001$$
$$b = 2.298$$

$$(a)(x^b)$$
$$(2.001)(x^{2.298})$$

Student work for Item 38 – Score 0

$$y = 48.9x - 111.8$$

~~$$y = 48.9x$$~~

$$x = 15$$

$$y = 623$$

Part IV

- (39) Two forces of 25 newtons and 85 newtons acting on a body form an angle of 55° .

Find the magnitude of the resultant force, to the *nearest hundredth of a newton*.

Find the measure, to the *nearest degree*, of the angle formed between the resultant and the larger force.

Rubric

[6] 101.43 and 12, and appropriate work is shown.

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] Appropriate work is shown, but two computational or rounding errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made.

or

[4] The magnitude of the resultant force is found correctly, but no further correct work is shown.

[3] Appropriate work is shown, but three or more computational or rounding errors are made.

or

[3] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] 101.43 and 12, but no work is shown.

[1] Appropriate work is shown, but two conceptual errors and one computational or rounding error are made.

or

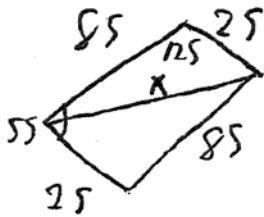
[1] A correct substitution is made into the Law of Cosines, but no further correct work is shown.

or

[1] 101.43 or 12, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Student work for Item 39 – Score 6



$$x^2 = (c)^2 + (b)^2 - 2(c)(b) \cos(C)$$

$$x^2 = (85)^2 + (25)^2 - 2(85)(25) \cos 125$$

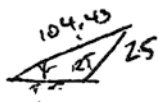
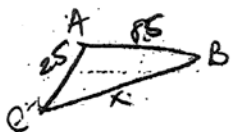
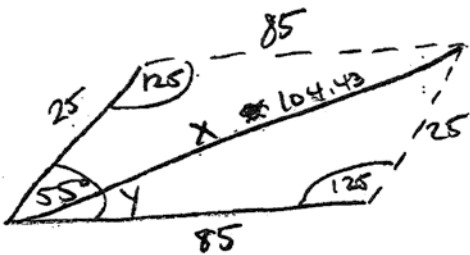
$$x = 101.43 \text{ rounded}$$

$$\frac{\sin(125)}{101.43} = \frac{\sin(x)}{25}$$

$$\sin(x) = \frac{25 \sin(125)}{101.43}$$

$$x = 12^\circ$$

Student work for Item 39 – Score 5



$$X^2 = 25^2 + 85^2 - 2(25 \cdot 85) \cos 125$$

$$7285 = 10287.5 - 6875 \cos Y$$

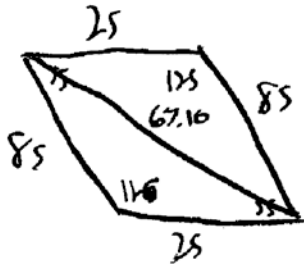
$$X \approx 101.43 \text{ Newtons}$$

$$\frac{\sin Y}{25} = \frac{\sin 125}{104.43}$$

$$\sin Y = .1961007479$$

$$Y \approx 11^\circ$$

Student work for Item 39 – Score 4



67.10N

$$c^2 = 85^2 + 25^2 - 2(85)(25)\cos(115^\circ)$$

$$c^2 = 1502.21$$

$$c = 67.10N$$

17.8°

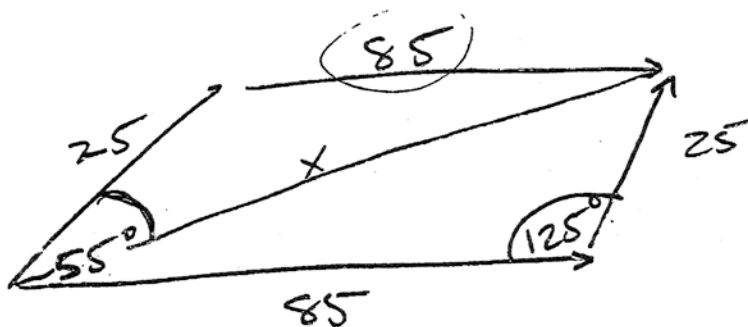
~~$$\frac{\sin 115^\circ}{67.10} = \frac{\sin x}{25}$$~~

$$\frac{\sin 115^\circ}{67.10} = \frac{\sin x}{25}$$

$$\sin x = .365$$

$$x = 19.8^\circ$$

Student work for Item 39 – Score 3



$$\begin{array}{r} -180 \\ 55 \\ \hline 125 \end{array}$$

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos 125 \\ a^2 &= 25^2 + 85^2 - 2(25)(85) \cos 125 \\ a^2 &= 625 + 7225 - 60 \cdot 85 \cos 125 \\ a^2 &= 7850 - 4250 \cos 125 \end{aligned}$$

(-.57)

$$a^2 = 7850 + 2422.5$$

$$a^2 = \sqrt{10272.5}$$

$$a = 101.353$$

$$\boxed{a = 101.35 \text{ newtons}}$$

$$\frac{101.35}{\sin 125} = \frac{85}{\sin X}$$

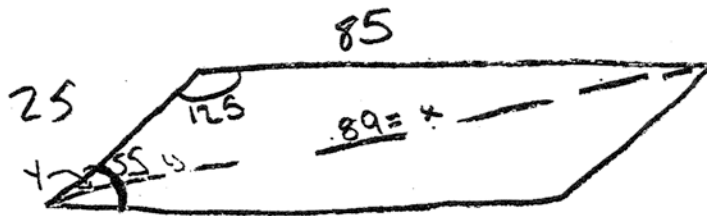
$$.8191 \cdot 85 = 101.35 \sin X$$

$$\frac{69.627}{101.35} = \frac{101.35 \sin X}{101.35}$$

$$\sin X = .687$$

$$X = 43.39$$

$$\boxed{X = 43^\circ}$$



85

$$x^2 = 85^2 + 25^2 - 2(25)(85)\cos 125$$

$$x^2 = 7878.678822$$

$$x = 88.8$$

$$x = 89$$

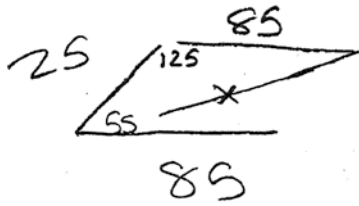
$$\frac{\sin 125}{89} = \frac{\sin y}{85}$$

$$\frac{85 \sin 125}{89} = \sin y$$

$$.7823362221 = \sin y$$

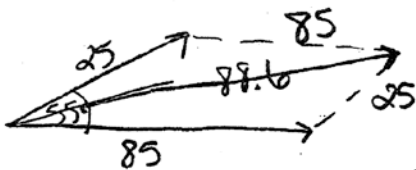
$$51 = y$$

Student work for Item 39 – Score 1



$$\begin{array}{r} 1710 \\ \cancel{1800} \\ - 35 \\ \hline 125 \end{array}$$

$$x^2 = 25^2 + 85^2 - 2(25)(85)\cos 125$$

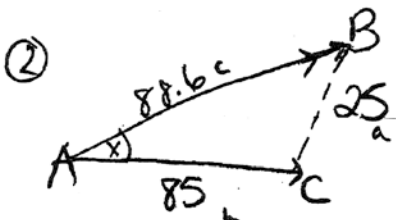


$$25^2 + 85^2 = x^2$$

$$\sqrt{7850} = x^2$$

$$x = 88.6$$

① 88.6 newtons



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$25^2 = 85^2 + 88.6^2 - 2(85)(88.6) \cos(x)$$

$$625 = 15074.96 - 15062 \cos(x)$$

$$.0415 = -15062 \cos(x)$$

$$-.00000275 = \cos(x)$$

$$\cos(1.57) =$$

2°

Appendix A

**The University of the State of New York
THE STATE EDUCATION DEPARTMENT
Albany, New York 12234**

**Specifications for the Regents Examination in Algebra 2/Trigonometry
(First Administration – June 2010)**

The questions on the Regents Examination in Algebra 2/Trigonometry will assess both the content and the process strands of New York State Mathematics Standard 3. Each question will be aligned to one content performance indicator but will also be aligned to one or more process performance indicators, as appropriate for the concepts embodied in the task. As a result of the alignment to both content and process performance strands, the examination will assess students' conceptual understanding, procedural fluency, and problem-solving abilities rather than assessing knowledge of isolated skills and facts.

There will be 39 questions on the Regents Examination in Algebra 2/Trigonometry. The table below shows the percentage of total credits that will be aligned with each content strand.

| Content Strand | % of Total Credits |
|--------------------------------|--------------------|
| 1) Number Sense and Operations | 6–10% |
| 2) Algebra | 70–75% |
| 4) Measurement | 2–5% |
| 5) Statistics and Probability | 13–17% |

Question Types

The Regents Examination in Algebra 2/Trigonometry will include the following types and numbers of questions.

| Question Type | Number of Questions |
|-------------------------------------|---------------------|
| Multiple choice (2 credits each) | 27 |
| 2-credit open ended | 8 |
| 4-credit open ended | 3 |
| 6-credit open ended | 1 |
| Total Credits | 88 |

Calculators

Schools must make a graphing calculator available for the exclusive use of each student while that student takes the Regents Examination in Algebra 2/Trigonometry.

Appendix B

Map to Core Curriculum

The table below shows which content strand each item is aligned to. The numbers in the table represent the question numbers on the test.

| Content Strand | Multiple-Choice Item Number | 2-Credit Item Number | 4-Credit Item Number | 6-Credit Item Number |
|-----------------------------|---|-----------------------------|-----------------------------|-----------------------------|
| Number Sense and Operations | 1, 11 | 28 | | |
| Algebra | 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27 | 29, 30, 32, 33, 34 | 36, 37 | 39 |
| Measurement | | 31 | | |
| Statistics and Probability | 4, 15, 24, 25 | 35 | 38 | |