**ALGEBRA I**

Wednesday, January 25, 2023 — 1:15 to 4:15 p.m., only

Student Name Steve WatsonSchool Name JMAP

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice ...**

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

$$(2x^2 - 8) - (3x + 15)$$

1 When the expression  $2x(x - 4) - 3(x + 5)$  is written in simplest form, the result is

- (1)  $2x^2 - 11x - 15$                       (3)  $2x^2 - 3x - 19$   
 (2)  $2x^2 - 11x + 5$                       (4)  $2x^2 - 3x + 1$

Use this space for computations.

$$\begin{array}{r} 2x^2 - 8x \\ - 3x - 15 \\ \hline 2x^2 - 11x - 15 \end{array}$$

2 The point  $(3, w)$  is on the graph of  $y = 2x + 7$ . What is the value of  $w$ ?

- (1)  $-2$     (3)  $10$   
 (2)  $-4$      (4)  $13$

$$\begin{aligned} y &= 2x + 7 \\ y &= 2(3) + 7 \\ y &= 6 + 7 \\ y &= 13 \end{aligned}$$

3 Students were asked to write  $2x^3 + 3x + 4x^2 + 1$  in standard form. Four student responses are shown below.

- Alexa:  $4x^2 + 3x + 2x^3 + 1$   
 Carol:  $2x^3 + 3x + 4x^2 + 1$   
 Ryan:  $2x^3 + 4x^2 + 3x + 1$   
 Eric:  $1 + 2x^3 + 3x + 4x^2$

← exponents are in decreasing order

Which student's response is correct?

- (1) Alexa     (3) Ryan  
 (2) Carol    (4) Eric

4 Given  $f(x) = -3x^2 + 10$ , what is the value of  $f(-2)$ ?

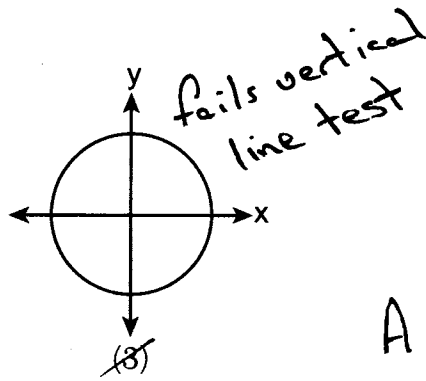
- (1)  $-26$     (3)  $22$   
 (2)  $-2$     (4)  $46$

$$\begin{aligned} f(x) &= -3x^2 + 10 \\ f(-2) &= -3(-2)^2 + 10 \\ f(-2) &= -3(4) + 10 \\ f(-2) &= -12 + 10 \\ f(-2) &= -2 \end{aligned}$$

5 Which relation is a function?

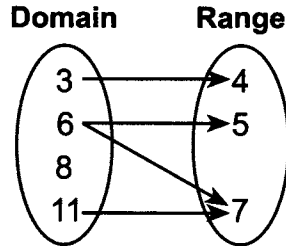
Use this space for computations.

$\{(1,3), (2,1), (3,1), (4,7)\}$



A function has one and only one value of  $y$  for each value of  $x$ .

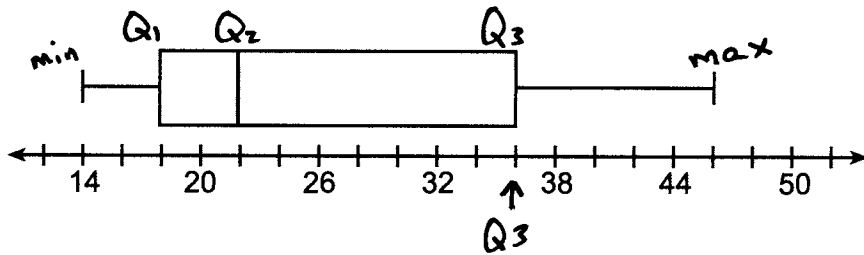
Input	Output
-6	-2
-4	2
7	3
7	5



$(7,3)$  ~~(2)~~  
 $(7,5)$

$(6,5)$  (4)  
 $(6,7)$

6 What is the value of the third quartile in the box plot shown below?



- (1) 18
- (2) 22
- (3) 36
- (4) 46

7 What is the solution to  $2 + 3(2a + 1) = 3(a + 2)$ ?

- (1)  $\frac{1}{7}$
- (2)  $\frac{1}{3}$
- (3)  $-\frac{3}{7}$
- (4)  $-\frac{1}{3}$

$$2 + 3(2a + 1) = 3(a + 2)$$

$$2 + 6a + 3 = 3a + 6$$

$$6a + 5 = 3a + 6$$

$$3a + 5 = 6$$

$$3a = 1$$

$$a = \frac{1}{3}$$

Check

$$2 + 3\left(2 \cdot \frac{1}{3} + 1\right) = 3\left(\frac{1}{3} + 2\right)$$

$$2 + 3\left(\frac{2}{3} + 1\right) = \frac{3}{3} + 6$$

$$2 + \frac{6}{3} + 3 = 7$$

$$2 + 2 + 3 = 7$$

$$7 = 7 \quad \checkmark$$

Use this space for computations.

8 One Saturday afternoon, three friends decided to keep track of the number of text messages they received each hour from 8 a.m. to noon. The results are shown below.

8-9	9-10	10-11	11-12
8	16	24	32
$x$	$2x$	$4x$	$8x$
3	10	0	15

Emily said that the number of messages she received increased by 8 each hour.

Jessica said that the number of messages she received doubled every hour.

Chris said that he received 3 messages the first hour, 10 the second hour, none the third hour, and 15 the last hour.

Which of the friends' responses best classifies the number of messages they received each hour as a linear function?

- (1) Emily, only
- (2) Jessica, only
- (3) Emily and Chris
- (4) Jessica and Chris

9 Which expression is equivalent to  $(x + 4)^2(x + 4)^3$ ?

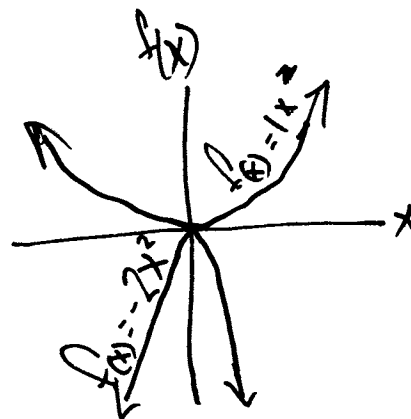
- (1)  $(x + 4)^6$
- (2)  $(x + 4)^5$
- (3)  $(x^2 + 16)^6$
- (4)  $(x^2 + 16)^5$

$$(x+4)^2(x+4)^3 = (x+4)^{2+3} = (x+4)^5$$

10 Caitlin graphs the function  $f(x) = ax^2$ , where  $a$  is a positive integer. If Caitlin multiplies  $a$  by  $-2$ , when compared to  $f(x)$ , the new graph will become

- (1) narrower and open downward
- (2) narrower and open upward
- (3) wider and open downward
- (4) wider and open upward

let  $a=1$



11 Sunny purchases a new car for \$29,873. The car depreciates 20% annually.

Which expression can be used to determine the value of the car after  $t$  years?

- (1)  $29,873(.20)^t$
- (2)  $29,873(20)^t$
- (3)  $29,873(1 - .20)^t$
- (4)  $29,873(1 + .20)^t$

$$A = P(1+r)^t$$

$$A = 29,873(1-.20)^t$$

$$A = 29,873(.8)^t$$

12 If  $f(x) = x^2 + 2x + 1$  and  $g(x) = 7x - 5$ , for which values of  $x$  is  $f(x) = g(x)$ ?

- (1)  $-1$  and  $6$
- (2)  $-6$  and  $-1$
- (3)  $-3$  and  $-2$
- (4)  $2$  and  $3$

$$x^2 + 2x + 1 = 7x - 5$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3$$

$$x = 2$$

Use this space for computations.

13 Skyler mows lawns in the summer. The function  $f(x)$  is used to model the amount of money earned, where  $x$  is the number of lawns completely mowed. A reasonable domain for this function would be

- (1) real numbers                      (3) irrational numbers  
 (2) rational numbers                (4)  natural numbers

*Completely mowed  $\rightarrow$  whole #s  
 $\rightarrow$  positive #s  
 Fractions and negative numbers are not needed*

14 Which expression is equivalent to  $2x^2 + 8x - 10$ ?

- (1)   $2(x - 1)(x + 5)$                       (3)  $2(x - 1)(x - 5)$   
 (2)  $2(x + 1)(x - 5)$                       (4)  $2(x + 1)(x + 5)$

$$2x^2 + 8x - 10$$

$$2(x^2 + 4x - 5)$$

$$2(x - 1)(x + 5)$$

15 Ian throws a ball up in the air and lets it fall to the ground. The height of the ball,  $h(t)$ , is modeled by the equation  $h(t) = -16t^2 + 6t + 3$ , with  $h(t)$  measured in feet, and time,  $t$ , measured in seconds. The number 3 in  $h(t)$  represents

- (1) the maximum height of the ball  
 (2)  the height from which the ball is thrown  
 (3) the number of seconds it takes for the ball to reach the ground  
 (4) the number of seconds it takes for the ball to reach its maximum height

16 Thirty-two teams are participating in a basketball tournament. Only the winning teams in each round advance to the next round, as shown in the table below.

$x$	<b>Number of Rounds Completed, <math>x</math></b>	0	1	2	3	4	5
$y$	<b>Number of Teams Remaining, <math>f(x)</math></b>	32	16	8	4	2	1

$$y = 32(.5)^x$$

Which function type best models the relationship between the number of rounds completed and the number of teams remaining?

- (1) absolute value                      (3) linear  
 (2)  exponential                              (4) quadratic

17 In a geometric sequence, the first term is 4 and the common ratio is  $-3$ . The fifth term of this sequence is

- (1)  324                                      (3)  $-108$   
 (2) 108                                        (4)  $-324$

term	1	2	3	4	5
value	4	-12	36	-108	324

18 The amount of energy,  $Q$ , in joules, needed to raise the temperature of  $m$  grams of a substance is given by the formula  $Q = mC(T_f - T_i)$ , where  $C$  is the specific heat capacity of the substance. If its initial temperature is  $T_i$ , an equation to find its final temperature,  $T_f$ , is

(1)  $T_f = \frac{Q}{mC} - T_i$

(3)  $T_f = \frac{T_i + Q}{mC}$

(2)  $T_f = \frac{Q}{mC} + T_i$

(4)  $T_f = \frac{Q - mC}{T_i}$

Use this space for computations.

$$Q = mC(T_f - T_i)$$

$$\frac{Q}{mC} = T_f - T_i$$

$$\frac{Q}{mC} + T_i = T_f$$

19 When using the method of completing the square, which equation is equivalent to  $x^2 - 12x - 10 = 0$ ?

(1)  $(x + 6)^2 = -26$

(3)  $(x - 6)^2 = -26$

(2)  $(x + 6)^2 = 46$

(4)  $(x - 6)^2 = 46$

$$x^2 - 12x - 10 = 0$$

$$x^2 - 12x = 10$$

$$x^2 - 12x + (-6)^2 = 10 + (-6)^2$$

$$(x - 6)^2 = 46$$

20 Which quadratic function has the *smallest* minimum value?

$f(x) = 6x^2 + 5x - 2$

$g(x) = 6(x - 2)^2 - 2$

(3)  $\min = -2$

$\min = -3.041667$

x	h(x)
0	6
1	2
2	0
3	0
4	2
5	6

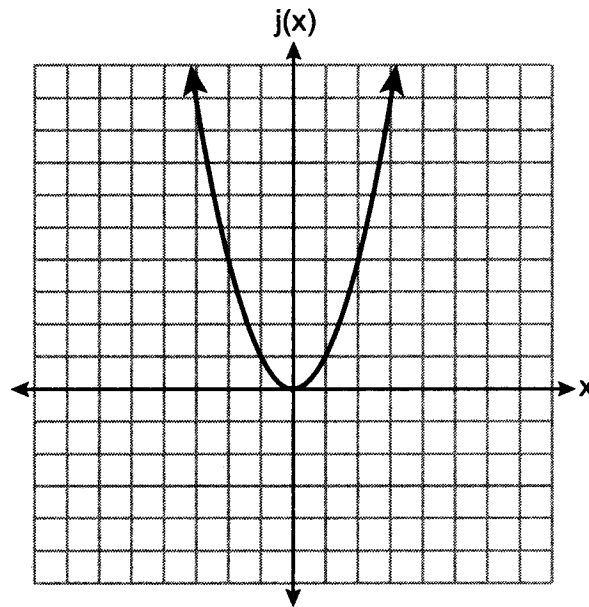
(2)

Zeros {

$$y = (x - 2)(x - 3)$$

$$y = x^2 - 5x + 6$$

$$\min = -2.25$$



(4)

$\min = 0$

Use this space for computations.

21 Which representation yields the same outcome as the sequence defined recursively below?

$$a_1 = 3$$

$$a_n = -4 + a_{n-1}$$

term	1	2	3	4	5
$-4 + a_{n-1}$	3	-1	-5	-9	-13

(1) 3, 7, 11, 15, 19, ...

(3)  $a_n = 4n - 1$

(2) 3, -1, -5, -9, -13, ...

(4)  $a_n = 4 - n$

22 If the zeros of the function  $g(x)$  are  $\{-3, 0, 4\}$ , which function could represent  $g(x)$ ?

(1)  $g(x) = (x + 3)(x - 4)$

(3)  $g(x) = x(x + 3)(x - 4)$

(2)  $g(x) = (x - 3)(x + 4)$

(4)  $g(x) = x(x - 3)(x + 4)$

23 Morgan read that a snail moves about 72 feet per day. He performs the calculation  $\frac{72 \text{ feet}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$  to convert this rate to different units. What are the units for the converted rate?

(1) hours/inch

(3) inches/hour

(2) minutes/inch

(4) inches/minute

24 During summer vacation, Ben decides to sell hot dogs and pretzels on a food cart in Manhattan. It costs Ben \$0.50 for each hot dog and \$0.40 for each pretzel. He has only \$100 to spend each day on hot dogs and pretzels. He wants to sell at least 200 items each day. If  $h$  is the number of hot dogs and  $p$  is the number of pretzels, which inequality would be part of a system of inequalities used to determine the total number of hot dogs and pretzels Ben can sell?

Price is not relevant

(1)  $h + p \leq 200$

(3)  $0.50h + 0.40p \geq 200$

(2)  $h + p \geq 200$

(4)  $0.50h + 0.40p \leq 200$

$$h + p \geq 200$$

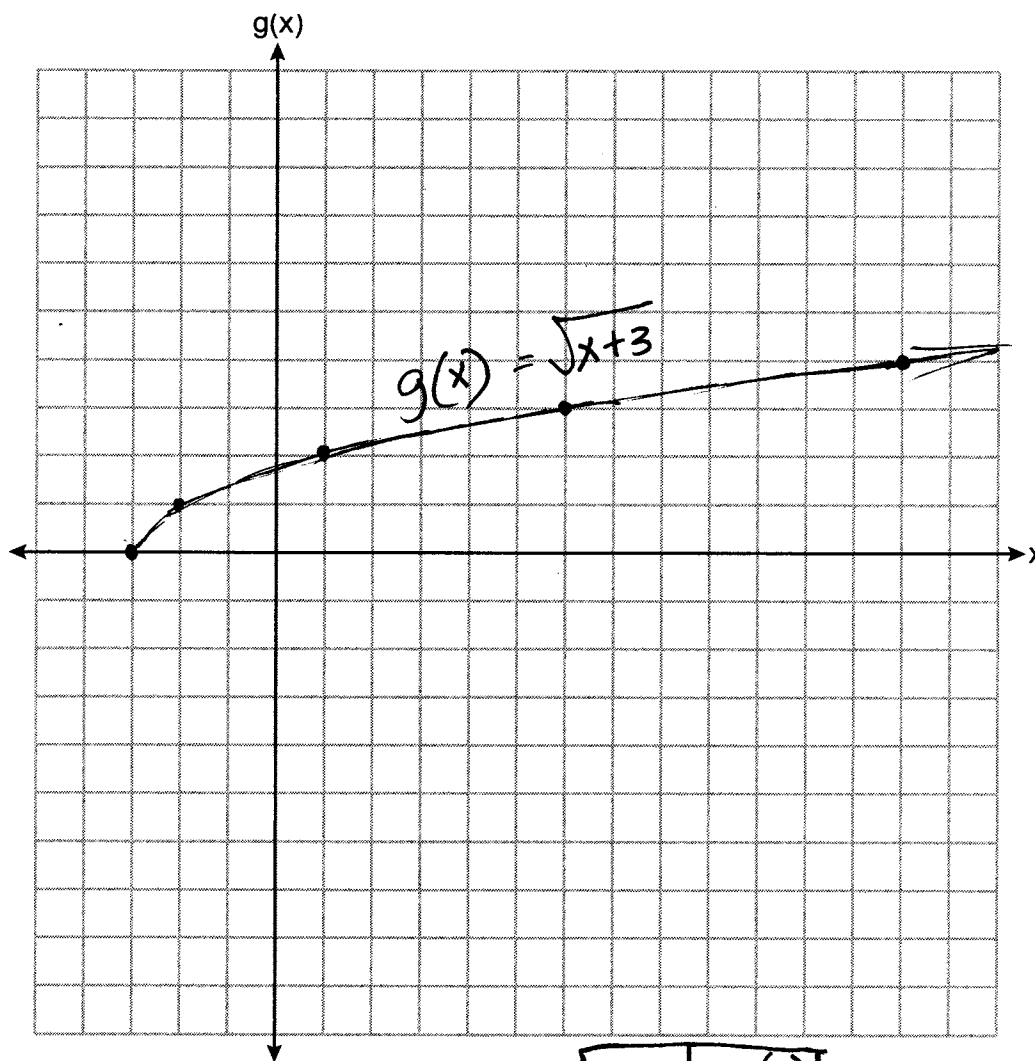
The other inequality in this system of inequalities would be:

$$.50h + .40p \leq 100$$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Graph the function  $g(x) = \sqrt{x+3}$  on the set of axes below.



*From graphing calculator*

x	g(x)
-3	0
-1	1
1	2
6	3
13	4



26 The sixth-grade classes at West Road Elementary School were asked to vote on the location of their class trip. The results are shown in the table below.

	Playland	Splashdown	Fun Central
Boys	38	53	25
Girls	39	46	37

Determine, to the nearest percent, the percentage of girls who voted for Splashdown.

$$\frac{\text{\# of girls who voted for Splashdown}}{\text{total \# of girls}} = \frac{46}{122} = .377$$

$$\text{decimal} (\times 100) = \text{percent}$$

$$.377 \times 100 = 37.7\%$$

nearest percent

$$\boxed{38\%}$$

27 Solve the inequality  $-\frac{2}{3}x + 6 > -12$  algebraically for x.

Given  $-\frac{2}{3}x + 6 > -12$

M( $\frac{3}{3}$ )  $-2x + 18 > -36$

S(18)  $-2x > -54$

D(-2)

$x < 27$
----------

28 Determine the common difference of the arithmetic sequence in which  $a_1 = 3$  and  $a_4 = 15$ .

term	1	2	3	4
$a_n$	3	7	11	15
		+4	+4	+4

The common difference is 4

29 Given:  $A = \sqrt{363}$  and  $B = \sqrt{27}$

Explain why  $A + B$  is irrational.

The sum of  $A + B$  cannot be expressed as a ratio of two integers.

Explain why  $A \cdot B$  is rational.

$$\sqrt{363} \cdot \sqrt{27} = \sqrt{9801}$$

$$\sqrt{9801} = 99$$

99 is a rational number because it can be expressed as the ratio of two integers.

$$99 = \frac{99}{1} \leftarrow \begin{array}{l} \text{1st} \\ \text{integer} \end{array}$$

$\leftarrow$  2nd integer

$$a=1 \rightarrow b=-4 \rightarrow c=1$$

30 Use the quadratic formula to solve  $x^2 - 4x + 1 = 0$  for  $x$ .

Round the solutions to the nearest hundredth.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$x = \frac{4 \pm \sqrt{12}}{2}$$

$$x = \frac{4 \pm \sqrt{4} \sqrt{3}}{2}$$

$$x = \frac{4 \pm 2\sqrt{3}}{2}$$

$$x = \frac{2 \pm \sqrt{3}}{1}$$

$$x = 2 \pm \sqrt{3}$$

$$x = 2 + \sqrt{3}$$

$$x = 2 - \sqrt{3}$$

$$x = 3.732 \dots$$

$$x = .2679$$

$$x = 3.73$$

$$x = .27$$

31 Factor completely:

$$4x^3 - 49x$$

$$4x^3 - 49x$$

$$x(4x^2 - 49)$$

$$x(2x+7)(2x-7)$$

← difference of  
perfect squares

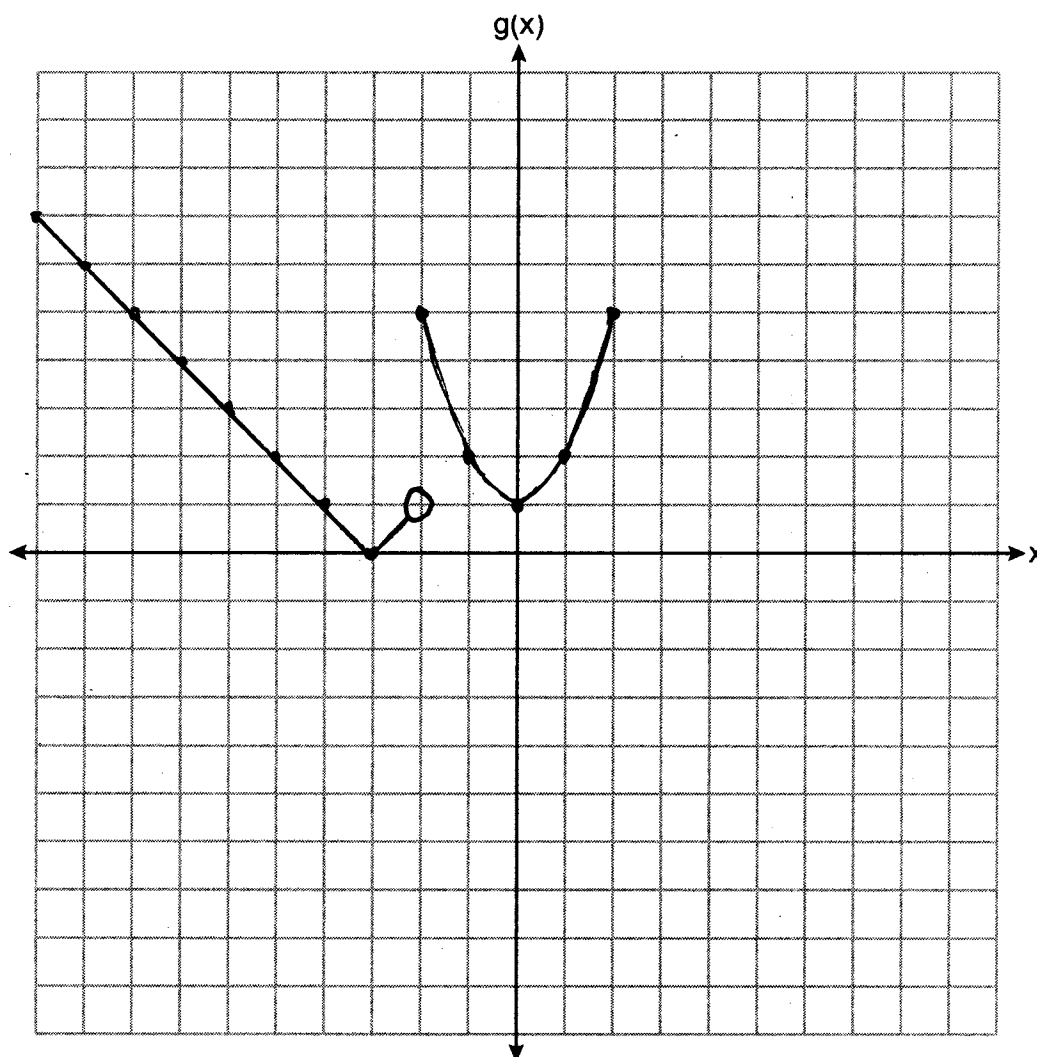
$$a^2 - b^2 = (a+b)(a-b)$$

~~0-10~~

32 The function  $g$  is defined as

$$g(x) = \begin{cases} |x + 3|, & x < -2 \\ x^2 + 1, & -2 \leq x \leq 2 \end{cases}$$

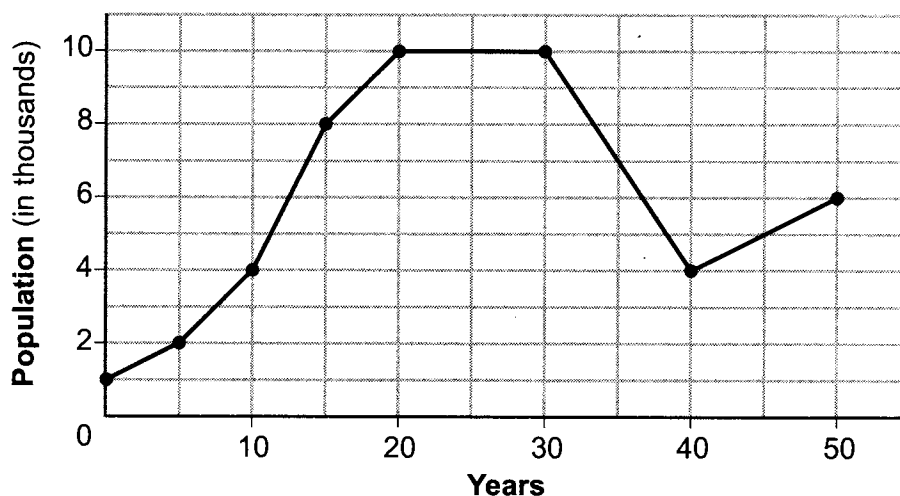
On the set of axes below, graph  $g(x)$ .



### Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Anessa is studying the changes in population in a town. The graph below shows the population over 50 years.



State the entire interval during which the population remained constant.

[20 ... 30]      20 to 30 inclusive

State the maximum population of the town over the 50-year period.

10,000

Determine the average rate of change from year 30 to year 40.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4,000 - 10,000}{40 - 30} = \frac{-6000}{10} = \frac{-600 \text{ people}}{1 \text{ year}}$$

Explain what your average rate of change means from year 30 to year 40 in the context of the problem.

The population dropped, on average, by 600 people each year during the ten year interval.



- 34 The table below shows the number of math classes missed during a school year for nine students, and their final exam scores.

<b>Number of Classes Missed (x)</b>	2	10	3	22	15	2	20	18	9
<b>Final Exam Score (y)</b>	99	72	90	35	60	80	40	43	75

Write the linear regression equation for this data set. Round all values to the nearest hundredth.

$$y = -2.81x + 97.55$$

From graphing calculator

State the correlation coefficient for your linear regression. Round your answer to the nearest hundredth.

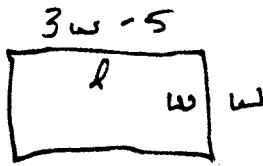
$$r = -.97$$

State what the correlation coefficient indicates about the linear fit of the data.

There is a strong negative correlation between the number of classes missed and final exam scores.

35 A fence was installed around the edge of a rectangular garden. The length,  $l$ , of the fence was 5 feet less than 3 times its width,  $w$ . The amount of fencing used was 90 feet.

Write a system of equations or write an equation using one variable that models this situation.



Perimeter of  $\square$  equals  $2l + 2w$   
 Perimeter of  $\square$  equals 90 feet

$$2l + 2w = 90$$

$$l = 3w - 5$$

$$2(3w - 5) + 2w = 90$$

Determine algebraically the dimensions, in feet, of the garden.

$$2(3w - 5) + 2w = 90$$

$$6w - 10 + 2w = 90$$

$$8w - 10 = 90$$

$$8w = 100$$

$$w = 12.5$$

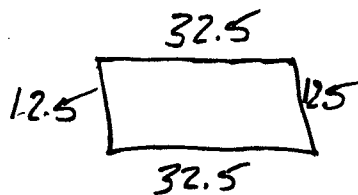
$$l = 3w - 5$$

$$l = 3(12.5) - 5$$

$$l = 37.5 - 5$$

$$l = 32.5$$

Check



$$2(32.5) + 2(12.5) = 90$$

$$65 + 25 = 90$$

$$90 = 90$$

✓

36 Given:

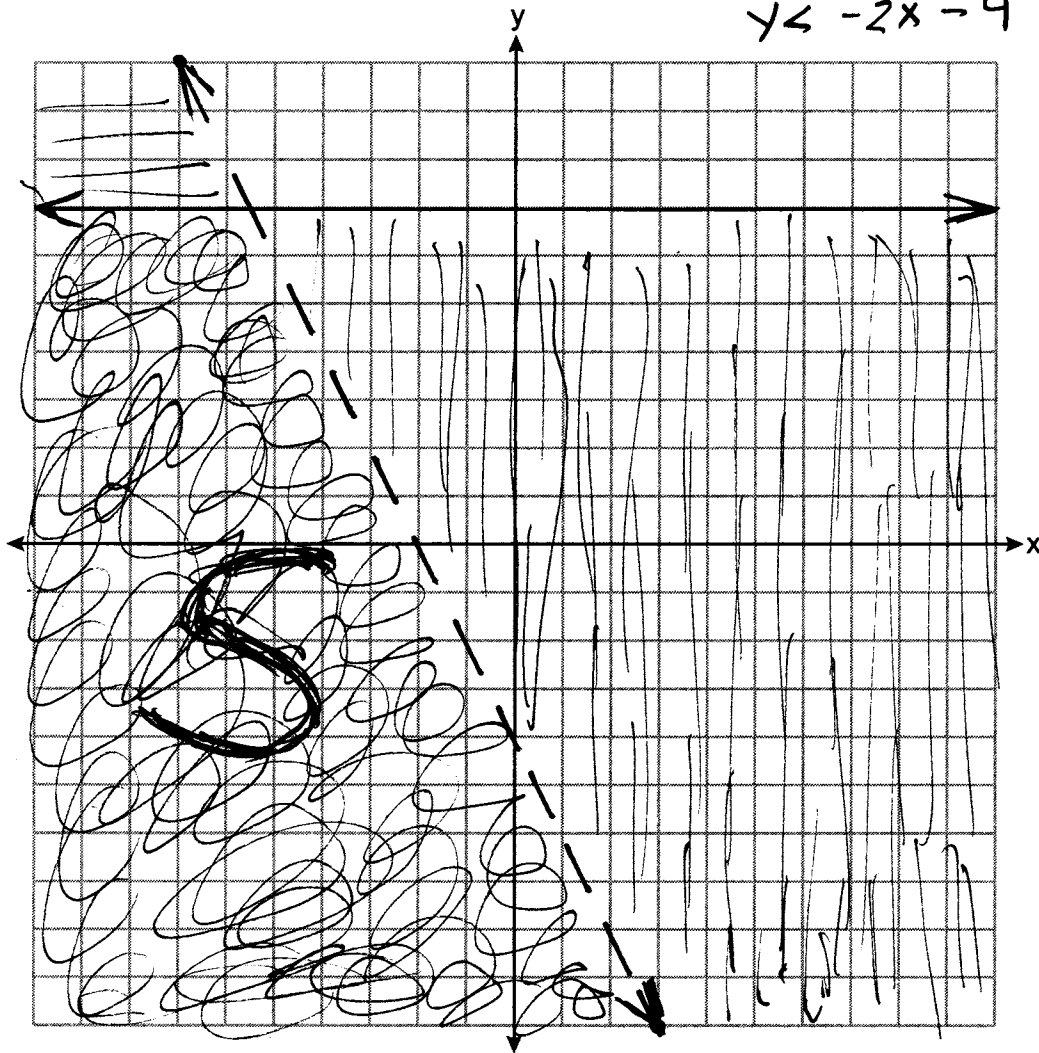
$$3y - 9 \leq 12 \rightarrow 3y \leq 12 + 9 \rightarrow 3y \leq 21 \rightarrow y \leq 7$$
$$y < -2x - 4$$

Graph the system of inequalities on the set of axes below.

Calculator Inputs

$$y \leq 0x + 7$$

$$y < -2x - 4$$



State the coordinates of a point that satisfies both inequalities. Justify your answer.

$(-5, -5)$

$$y < -2x - 4$$
$$-5 < -2(-5) - 4$$
$$-5 < 10 - 4$$
$$-5 < 6 \quad \checkmark$$

$$3y - 9 \leq 12$$
$$3(-5) - 9 \leq 12$$
$$-15 - 9 \leq 12$$
$$-24 \leq 12 \quad \checkmark$$

The point  $(-5, -5)$  satisfies both inequalities.

### Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Aidan and his sister Ella are having a race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line.

Let  $y$  represent the distance from the starting line and  $x$  represent the time elapsed, in seconds.

Write an equation to model the distance Aidan traveled.

$$y_{\text{(Aidan)}} = 10x$$

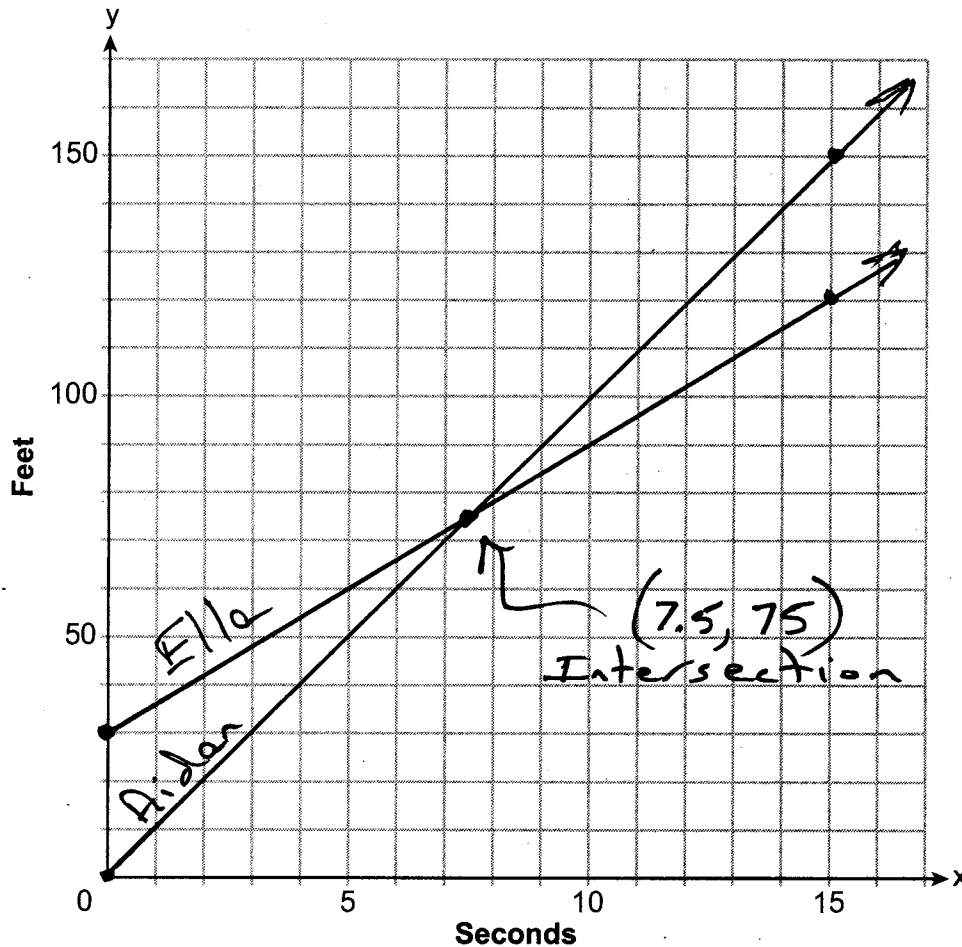
Write an equation to model the distance Ella traveled.

$$y_{\text{(Ella)}} = 6x + 30$$

Question 37 is continued on the next page.

Question 37 continued

On the set of axes below, graph your equations.



Exactly how many seconds does it take Aidan to catch up to Ella? Justify your answer.

$$7\frac{1}{2} \text{ seconds}$$

After 7.5 seconds, both Aidan and Ella are 75 feet from the starting line.

Check  $y_{\text{Aidan}} = 10x$   
 $= 10(7.5)$   
 $= 75 \checkmark$

Ella  $y_{\text{Ella}} = 6x + 30$   
 $= 6(7.5) + 30$   
 $= 45 + 30$   
 $= 75 \checkmark$