

The University of the State of New York

320TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, January 26, 1954—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Solve the following system of equations and check: [8, 2]

$$3x^2 - xy = 3$$

$$6x - y = 10$$

27 Find to the nearest degree the value of x between 0° and 90° that satisfies the equation $5 \tan^2 x + 6 \tan x - 2 = 0$. [10]

28 a Draw the graph of $y = x^2 - 4x + 6$ from $x = -1$ to $x = 5$ inclusive. [6]

b On the graph made in answer to a indicate by letters A and B the points whose abscissas are the roots of the equation $x^2 - 4x + 6 = 8$. [2]

c From the graph determine the least value of k for which the roots of $x^2 - 4x + 6 = k$ are real. [2]

29 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.] [5, 5]

a A man has \$2,000 invested in bonds that pay 5% interest. How much should he invest in bonds that pay $2\frac{1}{2}\%$ interest in order to get a return equal to $3\frac{1}{2}\%$ on his total investment?

b Mr. Jones bought a machine costing \$240. He arranged to pay \$60 down and the rest in monthly installments. If he paid \$33 the first month and each payment thereafter was \$3 less than the preceding one, in how many months did Mr. Jones complete payment for the machine?

Part III

Answer three questions from part III.

30 a Draw the graphs of $y = 1$ and $y = -1$. [2]

b On the same set of axes used in a, sketch the graph of $y = 2 \cos x$ as x varies from 0° to 360° . [6]

c From the graphs made in answer to a and b, determine the number of values of x between 0° and 360° that satisfy the equations $2 \cos x = \pm 1$ [2]

31 a Starting with a formula for $\cos 2A$, derive the formula for $\sin \frac{1}{2}x$ in terms of $\cos x$. [6]

b Prove that the following statement is an identity : $\tan (45^\circ + A) = \frac{\cos A + \sin A}{\cos A - \sin A}$ [4]

ELEVENTH YEAR MATHEMATICS

32 In the formula $a = \sqrt{\frac{2K \sin A}{\sin B \sin C}}$, A , B and C are the angles of triangle ABC , K is its area and a is a side. Using logarithms, find a to the nearest tenth if $K = 462$, $A = 76^\circ 50'$ and $B = 36^\circ 40'$. [10]

33 Airport A is 350 miles due north of airport B . Their radio stations receive signals of distress from ship C located S 72° E from A and N 46° E from B . How long will it take an airplane from A , traveling at the rate of 190 miles an hour, to reach the ship? [5, 5]

34 A triangular plot of ground measures 18 rods on one side and 24 rods on another side. The angle included between these sides is 72° . Find to the nearest rod the amount of fencing required to enclose the plot. [10]

*35 In triangle ABC , $BC = 38.3$, $BA = 57.2$ and angle $B = 44^\circ$. Using the law of tangents, find angle A to the nearest minute. [10]

* This question is based upon one of the optional topics in the syllabus.

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

- 1 Express as a single term the sum of $2\sqrt{-9}$ and $5i$. 1.....
- 2 Solve the equation $\tan^2 x - 5 \tan x - 14 = 0$ for the positive value of $\tan x$. 2.....
- 3 Write the fraction $\frac{2}{\sqrt{7}-2}$ as an equivalent fraction with a rational denominator. 3.....
- 4 Simplify the complex fraction $\frac{\frac{\sin x}{\sin y} - \frac{\sin y}{\sin x}}{\frac{1}{\sin x \cdot \sin y}}$ 4.....
- 5 Find the value of $4^{-\frac{3}{2}} + 8^0$ 5.....
- 6 If x varies directly as y and $y = 27$ when $x = 6$, find y when $x = 8$. 6.....
- 7 Solve the following formula for P : $A = P + PRT$ 7.....
- 8 Find n if $\log n = 9.6791 - 10$ 8.....
- 9 Find $\cos 31^\circ 18'$ 9.....
- 10 Write an equation that expresses the relationship between x and y shown in the following table:

x	-2	-1	0	2
y	-6	-2	2	10

10.....
- 11 Solve the equation $\sqrt{4 \sin x + 2} - 2 = 0$ for the smallest positive value of x . 11.....
- 12 Express $\cot 320^\circ$ as a function of a positive acute angle. 12.....
- 13 Find the principal value of $\tan^{-1} (0.8391)$. 13.....

ELEVENTH YEAR MATHEMATICS

14 The first term of an arithmetic progression is -4 and the fifth term is 20. Find the second term. 14.....

15 Insert the positive geometric mean between a and $\frac{1}{a}$. 15.....

16 In a circle whose radius is 4 inches find the length of the arc intercepted by a central angle of $2\frac{1}{2}$ radians. 16.....

17 If $\cos x = a$ and x is in the first quadrant, express $\cos \frac{1}{2}x$ in terms of a . 17.....

18 Find the value of $\sin^2 50^\circ + \cos^2 50^\circ$. 18.....

19 A and B are positive acute angles. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{1}{\sqrt{2}}$, find $\cos(A + B)$. 19.....

Directions (20-25): Indicate the correct completion for each of the following by writing on the line at the right the letter a , b or c .

20 If the discriminant of a quadratic equation is -25 , the roots of the equation are (a) real and equal (b) real and unequal (c) imaginary 20.....

21 In the quadratic equation $x^2 - 5x + 2 = 0$, the sum of the roots exceeds the product of the roots by (a) -3 (b) 3 (c) 7 21.....

22 An equation of an ellipse may be (a) $3x^2 + y = 4$ (b) $3x^2 + y^2 = 4$ (c) $3x^2 - y^2 = 4$ 22.....

23 In triangle ABC , if $a = 4$, $b = 5$ and $c = 6$, then $\cos A$ equals (a) $\frac{5}{6}$ (b) $\frac{3}{4}$ (c) $\frac{9}{16}$ 23.....

24 In triangle ABC , if $A = 60^\circ$ and $B = 45^\circ$, then $\frac{a}{b}$ is equal to

(a) $\frac{60}{45}$ (b) $\frac{3}{2}$ (c) $\frac{\sqrt{6}}{2}$ 24.....

25 If the data $A = 23^\circ 40'$, $b = 6$, $a = 2.5$ are used, (a) no triangle can be constructed (b) only one triangle can be constructed (c) two triangles can be constructed 25.....