The University of the State of New York

323D HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, January 25, 1955 — 9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part. Show all work, unless otherwise directed.

26 *a* Solve for x and check: $\frac{x}{x-3} - \frac{x+5}{x^2-9} = 1$ [5]

b Perform the indicated operation and reduce to lowest terms: [5]

$$\frac{x-7}{3x^2-8x+4} \div \frac{3x-21}{6x^2-24}$$

27 *a* Draw the graph of $y = x^2 - 2x - 4$ from $x = -2$ to $x = 4$ inclusive. [6]

b From the graph, estimate to the *nearest tenth* the roots of $x^2 - 2x - 4 = 0$. [2]

c On the same set of axes used in *a*, draw the graph $y = -6$. [1]

d From the graphs made in answer to *a* and *c* what conclusion can you draw about the roots of $x^2 - 2x - 4 = -6$? [1]

28 Solve the following set of equations for x, y and z and check: [8, 2]

$$2x - y + z = 5$$
$$4x - 3y = 5$$
$$6x + 2y + 2z = 7$$

29 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [*Solution of equations is not required.*]

a The sum of the digits of a two-digit number is 13. If the digits are reversed, the new number is 9 less than the original number. Find the number. [5]

b How much water must be added to 20 ounces of a 10% solution of boric acid to reduce it to a 4% solution? [5]

*30 The sides of a triangle are 162, 143 and 107. Using the formula $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, find to the *nearest ten minutes* the largest angle of the triangle. [10]

*31 Solve the following equation for all values of x greater than 0° but less than 180°: [10]

$$\frac{\sin 4x - \sin 2x}{\cos 4x - \cos 2x} = \sqrt{3}$$

*** These problems, 30 and 31, are based upon optional topics in the syllabus and one of them may be substituted for any one problem in either part II or part III. Therefore one, but not both, of these problems may be included in the total of 5 required problems from parts II and III.**

Part III

Answer at least two questions from this part. Show all work.

32 *a* Solve the following equation for the positive value of sin x to the *nearest tenth:* [9]

$$2 \sin^2 x + 5 \sin x = 4$$

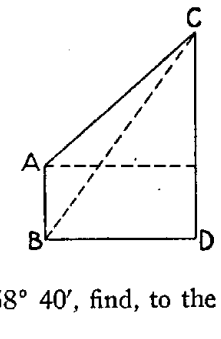
b How many values of x between 0° and 360° satisfy the equation in *a*? [1]

33 *a* Starting with the formula for sin $(x + y)$, *derive* the formula for sin $(x - y)$. [3]

b Without the use of tables, find sin 75°. [Answer may be left in radical form.] [2]

c Prove the identity $\dfrac{1 - \cos 2x}{\sec^2 x - \tan^2 x} = 2 \sin^2 x$. [5]

34 In the diagram at the right, AB represents a tower and CD a monument standing on level ground. From A, the angle of elevation of C is 42° 40′. From B, the angle of elevation of C is 54° 30′. If the tower is 34 feet high, find to the *nearest foot* [10]



a the distance BC

b the height of the monument

35 If two forces of 12 pounds and 17 pounds act upon a body at an angle of 58° 40′, find, to the *nearest pound,* the resultant of the two forces. [10]

Be sure you have answered a total of 5 questions from Parts II and III.

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the value of $2x^0 + x^{\frac{2}{3}}$ when $x = 27$. 1.....

2 Simplify the complex fraction $\dfrac{2 - \dfrac{1}{x}}{2 + \dfrac{1}{x}}$. 2.....

3 Express $\dfrac{1}{\sqrt{6}+1}$ as an equivalent fraction with a rational denominator. 3.....

4 Find the product of the roots of the equation $5x^2 + 2x - 1 = 0$. 4.....

5 Express $2\sqrt{-25}$ in terms of i. 5.....

6 If v varies directly as b^2 and $v = 45$ when $b = 3$, find the value of v when $b = 6$. 6.....

7 Find log tan 49° 44′. 7.....

8 Find to the *nearest minute* the positive acute angle whose cosine is equal to 0.8709. 8.....

9 Express sin (—250°) as a function of a positive acute angle. 9.....

10 Solve for x the equation $\sqrt{2x-1} = 3$. 10.....

11 What is the name of the graph of the equation $4x^2 - 25y^2 = 100$? 11.....

12 If $\cos C = m$ and angle C is in the first quadrant, write the value of $\sin \frac{1}{2} C$ in terms of m. 12.....

13 Write the equation which expresses the relationship between x and y shown in the following table:

x	—1	0	3	5
y	—5	—3	3	7

13.....

[OVER]

14 In a circle whose radius is 6, a central angle of 2 radians intercepts an arc AB. Find the length of arc AB. 14.....

15 Express tan $(A - B)$ in terms of tan A and tan B. 15.....

16 Find the area of triangle RST if $r = 12$, $s = 8$ and angle $T = 30°$. 16.....

17 The first term of an arithmetic progression is $\frac{2}{3}$ and the 12th term is 19. Find the common difference. 17.....

18 Find two numbers that, when inserted between 4 and 108, form with these numbers a geometric progression. 18.....

Directions (19–25): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

19 For values of x between 0 and π radians, the graphs of $y = 2 \sin x$ and $y = 1$, when drawn on the same axes, (*a*) intersect in two points (*b*) intersect in only one point (*c*) do not intersect 19.....

20 The period of the function $3 \cos 2x$ is (*a*) 180° (*b*) 360° (*c*) 720° 20.....

21 The expression $\log x - \log y$ is equal to (*a*) $\log (x - y)$ (*b*) $\log \frac{x}{y}$ (*c*) $\frac{\log x}{\log y}$ 21.....

22 If the data angle $A = 63°$, $a = 10$ and $c = 12$ are used, (*a*) no triangle can be constructed (*b*) only one triangle can be constructed (*c*) two triangles can be constructed 22.....

23 The equation of the axis of symmetry of the graph $y = 2x^2 - 8x - 7$ is (*a*) $x = 8$ (*b*) $x = 4$ (*c*) $x = 2$ 23.....

24 If $A = \sin^{-1} \frac{4}{5}$ and A is in the first quadrant, then tan A is (*a*) $\frac{3}{5}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{4}{3}$ 24.....

25 If a quadratic equation has roots which are real, rational and unequal, its discriminant may be (*a*) -4 (*b*) 0 (*c*) 9 25.....

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

Tuesday, January 25, 1955 — 9.15 a.m. to 12.15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–25, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) 11

(2) $\frac{2x-1}{2x+1}$

(3) $\frac{\sqrt{6}-1}{5}$

(4) $-\frac{1}{5}$

(5) $10i$

(6) 180

(7) 0.0721

(8) $29^\circ\ 26'$

(9) $\sin 70^\circ$ *or* $\cos 20^\circ$

(10) 5

(11) hyperbola

(12) $\sqrt{\frac{1-m}{2}}$

(13) $y = 2x - 3$

(14) 12

(15) $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

(16) 24

(17) $1\frac{2}{3}$

(18) 12, 36

(19) *a*

(20) *a*

(21) *b*

(22) *a*

(23) *c*

(24) *c*

(25) *c*