The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, January 21, 1958 — 9:15 a.m. to 12:15 p.m., only

Name of pupil...Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\tan^3 x - \tan x$ as the product of three factors. 1................

2 Combine into a single fraction: $\frac{5}{1-x} - \frac{1}{x}$ 2................

3 Express in *simplest form* the value of $\frac{\sin^4 A - \cos^4 A}{\sin^2 A - \cos^2 A}$. 3................

4 Find the value of $(x-1)^{\frac{2}{3}} + 2x^0$ when $x = 9$. 4................

5 Express $\frac{3}{2-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 5................

6 Solve for *sin x*: $\sqrt{8 - 3\sin x} = 3$ 6................

7 Solve the following equation for x: $2^{(4x-2)} = 8^x$ 7................

8 Find the value of the discriminant of the equation $3x^2 = 2x + 6$. 8................

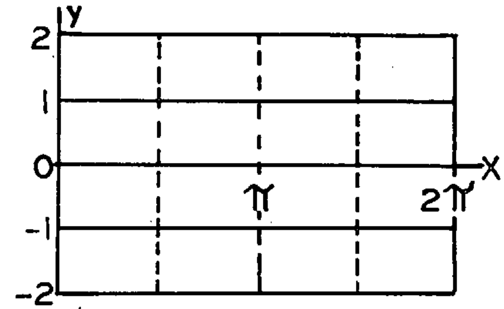
9 Find the sum of the roots of the equation $3x^2 - 2x - 5 = 0$. 9................

10 Write an equation of the line which is parallel to the line $2y - x = 4$ and which passes through the point (0, 3). 10................

11 Write an equation of the axis of symmetry of the curve $y = 3x^2 - 2x$. 11................

12 Find the antilogarithm of 9.5974 — 10. 12................

13 On the axes shown at the right sketch the graph of $y = \sin x$ for values of x from 0 to 2π radians.



14 Find cosecant (arc sin $\frac{1}{2}$). 14................

15 Three numbers are inserted between 2 and 4 to form with these numbers an arithmetic progression. Find the common difference of this progression. 15................

16 Find two numbers that, when inserted between 6 and 162, form with these numbers a geometric progression. 16................

17 Find the area of triangle ABC, given $a = 12$, $b = 10$ and $C = 150°$. 17................

18 Express $\sin(-160°)$ as a function of a positive acute angle. 18................

19 If x is a first quadrant angle and $\cos x = \frac{1}{8}$, find $\cos \frac{1}{2} x$. 19................

20 Using the data $a = 6$, $b = 10$ and $A = 30°$, how many different triangles is it possible to construct? 20................

21 In triangle ABC, $\sin A = 0.5$, $\sin B = 0.3$ and $a = 10$. Find b. 21................

Directions (22–25) : Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b, c* or *d.*

22 The value of $\sqrt{x^2 - 9}$ is a real, irrational number when x is equal to
(*a*) 5 (*b*) 0 (*c*) —3 (*d*) 4 22................

23 The graph of the equation $y^2 = 6x$ is (*a*) a circle (*b*) an ellipse (*c*) a hyperbola (*d*) a parabola 23................

24 An angle of 1 radian is equal to an angle (*a*) between 50° and 55° (*b*) between 55° and 60° (*c*) equal to 60° (*d*) between 60° and 65° 24................

25 Given the fraction $\dfrac{4}{x - y}$. Which of the following substitutions leads to an operation that is *not* permitted by the laws of algebra?
(*a*) $x = 1, y = -1$ (*b*) $x = 2, y = 3$ (*c*) $x = -2, y = -3$
(*d*) $x = 1, y = 1$ 25................

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 *a* Find, to the *nearest tenth*, the values of $\tan x$ which satisfy the equation $2\tan^2 x - 6\tan x + 3 = 0$. [8]

b Using the larger answer found in part *a*, find to the *nearest degree* an acute angle which satisfies the equation given in *a*. [2]

27 Solve graphically the following set of equations: [Estimate the answers to *tenths.*] [4, 4, 2]

$$x^2 + y^2 = 16$$
$$y = x^2 + 2$$

28 Write the equations that would be used to solve the following problems. In each case state what the letter or letters represent. [*Solution of the equations is not required.*]

a A tailor paid $144 for material for some suits. The following season he sent the same amount of money for the same material but was informed that, due to an increase of $1 per yard, he would receive 2 yards less. Find the original price per yard of the material. [5]

b The perimeter of a rectangle is 102 inches. If a diagonal of the rectangle is 39 inches, find the length and the width of the rectangle. [5]

29 *a* Starting with the formula for $\tan(A + B)$, *derive* the formula for $\tan 2x$ in terms of $\tan x$. [4]

b Reduce $\dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$ to an equivalent single function of A. [6]

30 *a* Draw the graph of $y = 2\cos x$ for values of x from 0° to 180°. [4]

b On the same set of axes used in *a*, draw the graph of $y = \tan x$ for values of x from 0° to 180°. [4]

c From the graphs made in answer to *a* and *b*, estimate to within approximately 10° the values of x between 0° and 180° that satisfy the equation $\tan x = 2\cos x$. [2]

Part III

Answer two questions from this part. Show all work.

31 In triangle ABC, $A = 72° 17'$, $b = 243$ and $C = 49° 20'$. Find c to the *nearest integer.* [10]

32 At noon on a certain day, a ship sailed from port on a course N 80° E at a speed of 12 miles per hour. At 1 p.m. a second ship left the same port on a course S 62° E at a speed of 13.5 miles per hour. Find, to the *nearest mile,* the distance between the ships at 3 p.m. [5, 5]

33 The sides of a triangle are 18, 13 and 10.

a Find, to the *nearest degree,* the largest angle of the triangle. [6]

b Find, to the *nearest square unit,* the area of the triangle. [You may use the result found in answer to *a*.] [4]

*34 In a unit circle draw an angle θ which lies between 90° and 135°. In this figure, draw the line segments whose lengths represent $\sin\theta$, $\cos\theta$ and $\tan\theta$. Label these line segments to show which functions they represent. [3, 2, 2, 3]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III *only.*

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 12 is not expressed to the nearest minute. For questions 22–25, allow credit if the pupil has written the correct answer instead of the letters *a, b, c* or *d*.

(1) $\tan x(\tan x + 1)(\tan x - 1)$

(2) $\dfrac{6x - 1}{x(1 - x)}$

(3) 1

(4) 6

(5) $\dfrac{3(2 + \sqrt{2})}{2}$

(6) $-\frac{1}{3}$

(7) 2

(8) 76

(9) $\frac{2}{3}$

(10) $y = \frac{1}{2}x + 3$ *or* $2y - x = 6$

(11) $x = \frac{5}{6}$ *or* $x = \frac{1}{3}$

(12) 0.3957

(14) 2

(15) $\frac{1}{2}$

(16) 18, 54

(17) 30

(18) $-\sin 20°$ *or* $-\cos 70°$

(19) $\frac{3}{4}$

(20) two

(21) 6

(22) *d*

(23) *d*

(24) *b*

(25) *d*