January 22, 1981

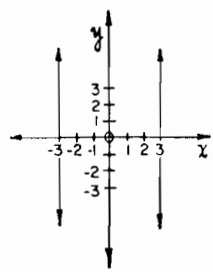
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

Directions (1-16): Write in the space provided the ***numeral*** preceding the word or expression that best completes *each* statement or answers *each* question.

1. The set of odd counting numbers is closed under which operation? (1) addition (2) subtraction (3) multiplication (4) division 1____

2. The accompanying graph is the graph of which equation?
(1) $|y| = 3$
(2) $|x| = 3$
(3) $|y| = x + 3$
(4) $|x| = y + 3$



2____

3. The expression $\dfrac{\tan x \cos x}{\csc x}$ is equivalent to (1) 1 (2) $\sin^2 x$ (3) $\csc^2 x$ (4) $\tan x$ 3____

4. Which equation has a circle as its graph? (1) $x^2 = 10 - y^2$ (2) $x^2 = 10 + y^2$ (3) $2x^2 = 10 - y^2$ (4) $2x^2 = 10 + y^2$ 4____

5. The expression $\dfrac{a^2+1}{a^2-1} - \dfrac{a}{a+1}$ is equivalent to (1) $\dfrac{-1}{a+1}$ (2) $\dfrac{1}{a+1}$ (3) $\dfrac{1}{a-1}$ (4) $\dfrac{1}{1-a}$ 5____

6. If $\sin B = \cos B$, what is the measure of angle B? (1) $\dfrac{\pi}{6}$ (2) $\dfrac{\pi}{2}$ (3) $\dfrac{\pi}{3}$ (4) $\dfrac{\pi}{4}$ 6____

7. The fraction $\dfrac{1 - \sqrt{3}}{6 - \sqrt{3}}$ is equivalent to (1) $\dfrac{3 - 5\sqrt{3}}{33}$ (2) $\dfrac{3 + 5\sqrt{3}}{-33}$ (3) $\dfrac{9 - 5\sqrt{3}}{33}$ (4) $\dfrac{9 + 5\sqrt{3}}{-33}$ 7____

8. The graphs of $y = x^2 - 4$ and $y = -4$ intersect at the point whose abscissa is (1) 1 (2) 2 (3) -2 (4) 0 8____

9. If $\sec \theta < 0$ and $\sin \theta < 0$, then angle θ lies in Quadrant (1) I (2) II (3) III (4) IV 9____

10. Which statement is true of the slope of the straight line that passes through the points (5, 2) and (−1, 2)? (1) It has no slope. (2) It has a slope of zero. (3) It has a slope of 3. (4) It has a slope of $\frac{1}{3}$. 10____

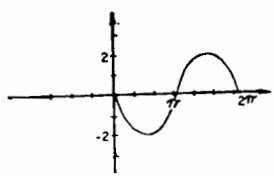
11. The product of $(6 - 5i)$ and $(3 + 2i)$, expressed in the form $a + bi$, is (1) $28 + 3i$ (2) $28 - 3i$ (3) $18 - 3i$ (4) $8 - 10i$ 11____

12. Which statement is an identity? (1) $\cos 2x = 1 - 2 \sin^2 x$ (2) $\sin 2x = 2 \sin^2 x$ (3) $\sin \frac{1}{2}x = \pm\sqrt{1 - \cos x}$ (4) $\cos \frac{1}{2}x = \pm\sqrt{1 + \cos x}$ 12____

13. What is the solution set of the equation $2\sqrt{x + 1} + x = 2$? (1) { } (2) {0} (3) {8} (4) {0, 8} 13____

14. Which relation is *not* a function? (1) $\{(x,y) \mid y = x\}$ (2) {(1,2), (2,3), (3,4)} (3) {(1,3), (2,5), (1,7)} (4) {(1,4), (2,4), (3,4)} 14____

15. The accompanying diagram is the graph of which equation? (1) $y = 2 \sin x$ (2) $y = -2 \sin x$ (3) $y = -\sin 2x$ (4) $y = \sin x - 2$



15____

16. Two distinct triangles can be constructed if $m\angle A = 40$, $b = 10$, and a is equal to (1) 16 (2) 10 (3) 8 (4) 4 16____

Directions (17-30): Write your answers in the spaces provided. Unless otherwise specified, answers may be left in terms of π or in radical form.

17. Find the positive acute angle θ if $2 \sin^2 \theta - \sin \theta = 0$. 17____

18. If x varies inversely as y and $x = 2$ when $y = 18$, find x when $y = 6$. 18____

19. Solve for x: $\frac{3}{x} + \frac{1}{2x} = \frac{7}{8}$ 19____

20. Given the equation $y = \frac{3}{2 + x}$. If $x = -\frac{1}{2}$, what is the numerical value of y? 20____

21. In triangle ABC, $a = 3$, $\sin A = 0.1$, and $\sin B = 0.2$. What is the length of side b? 21____

22. If $\log_a 100 = 2$, what is the positive value of x? 22____

23. If one of the roots of the equation $x^2 - x + q = 0$ is 3, what is the value of q? 23____

24. If $\sin x = \frac{3}{5}$ and x is an acute angle, what is the numerical value of $\sin 2x$? 24_____

25. If $\tan x = 1$ and $\tan y = 2$, find $\tan (x + y)$. 25_____

26. If $f(x) = x^{3/2} + x^0$, find $f(4)$. 26_____

27. Factor: $4 \tan^2 \theta - \tan \theta - 5$ 27_____

28. If $m\angle A = 30$ and $\cos A \cos B = \frac{\sqrt{3}}{4}$, what is the measure of acute angle B? 28_____

29. Express an angle of 315° in terms of π radians. 29_____

30. In isosceles $\triangle ABC$, vertex angle B measures 30° and each leg has length 10. Find the area of $\triangle ABC$. 30_____

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31. *a* Solve for all values of $\cot \theta$ to the *nearest hundredth*:
$\cot^2 \theta - 4 \cot \theta + 2 = 0$ [6]
b Using the answers in part *a*, find, to the *nearest degree*, all values of θ which satisfy $\cot^2 \theta - 4 \cot \theta + 2 = 0$, where $0° < \theta \leq 360°$. [4]

32. Solve the following system of equations algebraically.
$$x^2 - 2y^2 = 23$$
$$x - 2y = 7$$ [10]

33. *a* Sketch the graph of $y = \cos 2x$ as x varies from $-\pi$ to π radians. [4]
b On the same set of axes, sketch the graph of $y = 2 \sin x$ as x varies from $-\pi$ to π radians. [4]
c From the graphs made in answer to parts *a* and *b*, determine the number of values of x from $-\pi$ to π radians which satisfy the equation $\cos 2x = 2 \sin x$. [2]

34. Using logarithms, find the value of x to the *nearest whole number*:
$$x = \sqrt{\frac{(7.34)\ (56.2)^3}{\sin 5° 30'}}$$ [10]

35. Answer *either* *a* or *b* but *not* both.
a Two angles of a triangle are 25° and 58°. The longest side of the triangle is 39 meters. Find the length of the shortest side of the triangle to the *nearest meter*. [10]
OR
b Two forces act on an object. The first force has a magnitude of 80 pounds and makes an angle of 37° with the resultant. The magnitude of the resultant is 120 pounds. Find, to the *nearest pound*, the magnitude of the second force. [10]

36. Write an equation or system of equations that can be used to solve each of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a A two-digit number is 12 more than five times the sum of the digits. The tens digit is one more than the units digit. Find the number. [5]

b When flying in the same direction as a 30 kilometer-per-hour wind, a plane can travel 1,620 kilometers in the same time that it can travel 1,260 kilometers flying against a 30 kilometer-per-hour wind. Find the rate of the plane in calm air. [5]

*37. *a* On the complex coordinate axes, use vectors to represent the complex numbers r and s, such that $r = 4 - i$ and $s = 2 + 9i$. [4]

b Show graphically the vector sum $r + s$. [3]

c Find the magnitude (length) of the vector sum in part *b*. [3]

* This question is based on an optional topic in the syllabus.