The University of the State of New York

309TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 21, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Solve the following system of equations, group your answers and check *one* set. [7, 2, 1]

$$x^2 + 4y^2 = 13$$
$$x - 2y = 5$$

27 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [Solution of the equations is not required.]

a A man invested $6000 in two enterprises. At the end of the first year he found that he had gained 6% on one of the sums invested and had lost 4% on the other. His net profit for the year was $160. How much did he invest at each rate? [5]

b Three numbers are in the ratio 1:2:5. If 3 is subtracted from the first number, the second number is left unchanged and 9 is added to the third, these three numbers taken in the same order then form a geometric progression. Find the numbers. [5]

28 Angle A of triangle ABC can be found by using the formula $\cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$ in which a, b and c are the sides of the triangle and s is one-half its perimeter. Using logarithms, find A to the *nearest degree* if $a = 26.6$, $b = 36.5$ and $c = 30.3$. [10]

29 *a* On the same set of axes draw the graphs of $y = \sin x$ and $y = \sin 2x$ as x varies from 0 to π radians at intervals of $\frac{\pi}{6}$. [3, 5]

b From the graphs made in answer to *a*, determine the values of x from 0 to π radians *inclusive* that satisfy the equation $\sin x = \sin 2x$. [2]

Part III

Answer two questions from part III.

30 Given the equation $\tan x - \cot x - 2 = 0$.

a Write this equation in terms of $\tan x$. [1]

b Find, to the *nearest thousandth,* the positive value of $\tan x$ which satisfies this equation. [8]

c From the result found in answer to *b,* find the acute angle x to the *nearest ten minutes.* [1]

31 *a* Starting with the formulas for $\sin (A + B)$ and $\cos (A + B)$, derive the formula for $\tan (A + B)$ in terms of $\tan A$ and $\tan B$. [4]

b Prove the following equality is an identity:

$$\frac{\cos x\ (1 - \cos 2x)}{\sin x} = \sin 2x \qquad [6]$$

32 Point B is 8 miles due east of point A. Point C is 3 miles from A and in the direction N 30° E from A.

a Find the distance from B to C. [4]

b Find, to the *nearest degree,* the direction of C from B. [6]

*33 In triangle ABC, $A = 55° \ 20'$, $b = 18.5$, and $c = 12.8$. Using the law of tangents, find B to the *nearest minute.* [10]

Part IV

Answer one question from part IV.

34 For each of the following statements, in which a, b and c are real numbers, indicate whether the information given is *too little, just enough* or *more than is necessary,* to justify the conclusion.

(1) If the graph of $y = mx + b$ is parallel to a line whose equation is given, then the value of m and the value of b are determined. [2]

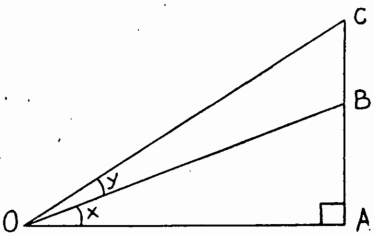
(2) If the center of a circle is at the origin and the circle passes through the point (a, b), then the radius of the circle is $\sqrt{a^2 + b^2}$. [2]

(3) If the graph of $ax^2 + by^2 = c$ is an ellipse, then a, b and c have the same sign. [2]

(4) If, in the equation $y = ax^2 + bx + c$, a and c are opposite in sign, then the graph of the equation intersects the x- axis. [2]

(5) If, in the equation $ax^2 - by^2 = c$, a, b and c are positive and a is not equal to b, then the graph of the equation is a hyperbola. [2]

35 In the accompanying figure AOC is a right triangle. Angles AOB and BOC are represented by x and y respectively.



a Show: $BC = \dfrac{OB \sin y}{\cos (x + y)}$ [4]

b Show: $AC = OB\left[\dfrac{\sin y}{\cos (x + y)} + \sin x\right]$ [4]

c Find AC if $x = 35° \ 10'$, $y = 24° \ 50'$ and $OB = 100$. [2]

* This question is based upon one of the optional topics in the syllabus.

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil...Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the number whose logarithm is 9.4356 — 10. 1................

2 Find the logarithm of tan 22° 18′. 2................

3 The hypotenuse of a right triangle is 12 and one of the acute angles is 28°. Find, to the *nearest tenth,* the longer leg of the triangle. 3................

4 Simplify the complex fraction $\dfrac{\dfrac{1}{\sin x}+\dfrac{1}{\cos x}}{\dfrac{1}{\sin x \cos x}}$ 4................

5 Write the fraction $\dfrac{1}{\sqrt{3}-1}$ with a rational denominator. 5................

6 Express in terms of i the sum of $\sqrt{-16}$ and $\sqrt{-9}$. 6................

7 Using the formula $A = P(1 + rt)$, find A when $P = 500$, $r = .03$ and $t = 15$. 7................

8 If r varies inversely as s and $r = 3$ when $s = 8$, find r when $s = 12$. 8................

9 Find the sum of all the integers from 1 to 100 inclusive. 9................

10 Solve the equation $\sqrt{\sin x + 3} = 2$ for the smallest positive value of x. 10................

11 Find the abscissa of the point in which the graphs of $y = 2x$ and $y = x + 4$, when drawn on the same set of axes, intersect. 11................

12 Write the equation of the straight line whose slope is 2 and which passes through the point (3, 5). 12................

13 Express tan 200° as a function of a positive angle less than 45°. 13................

14 Solve the equation $2 \cos^2 x + 3 \cos x - 2 = 0$ for the smallest positive value of x. 14................

15 Express $\sin A$ in terms of $\tan A$ where A is an angle in the first quadrant. 15................

16 If $\tan x = a$, express $\tan 2x$ in terms of a. 16................

17 If $\cos x = a$, express the positive value of $\cos \frac{1}{2}x$ in terms of a. 17................

Directions (questions 18–20) — Indicate whether each of the following statements is true or false by writing the word *true* or the word *false* on the line at the right. [In questions 18–19, x and y are real and not equal to zero.]

18 The expression $x^{-2} - y^0$ is equal to $\dfrac{1}{x^2} - 1$. 18................

19 The expression $\sqrt{x} + \sqrt{y}$ is equal to $\sqrt{x + y}$. 19................

20 An angle of an equilateral triangle is equal to one radian. 20................

Directions (questions 21–25) — Indicate the correct answer to *each* of the following by writing the letter *a, b* or *c* on the line at the right.

21 The roots of the equation $2x^2 - 8x + 3 = 0$ are (*a*) equal and rational (*b*) unequal and rational (*c*) unequal and irrational 21......

22 If $\log r + \log s = \log t$, then (*a*) $\log (r + s) = \log t$ (*b*) $r + s = t$ (*c*) $rs = t$ 22......

23 As A varies from 0 to π radians, the graphs of $y = \tan A$ and $y = 2$, when drawn on the same set of axes, (*a*) intersect in one point (*b*) intersect in two points (*c*) do not intersect 23......

24 The expression $\cos^4\theta - \sin^4\theta = \cos 2\theta$ is (*a*) true for all values of θ (*b*) true for only certain values of θ (*c*) not true for any value of θ 24......

25 The principal value of $\sin^{-1} (-\frac{1}{2})$ is (*a*) 30° (*b*) 210° (*c*) —30° 25......