The University of the State of New York

327TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Friday, June 15, 1956—1:15 to 4:15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 *a* Solve the equation $2\cos^2 x - 7\cos x + 4 = 0$ for $\cos x$, correct to the *nearest hundredth.* [7]

b Find to the *nearest degree* the smallest positive value of x that satisfies the given equation. [3]

27 Solve the following system of equations and check: [8, 2]

$$x^2 - xy + y^2 = 63$$
$$y - x = 3$$

28 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a A chemist has an 18% solution and a 45% solution of a disinfectant. How many ounces of *each* should be used to make 12 ounces of a 36% solution? [5]

b When a certain two-digit number is divided by the sum of its digits, the quotient is 7. If the digits are reversed, the resulting number is 18 less than the original number. Find the original number. [5]

29 *a* Starting with a formula for $\cos 2x$, *derive* the formula for $\cos \frac{1}{2}A$ in terms of $\cos A$. [5]

b Prove the identity: $(1 + \sec x)(1 - \cos x) = \tan x \sin x$. [5]

30 *a* *Sketch* the graph of $y = \sin x$ as x varies from 0 to 2π radians. [4]

b On the same set of axes used in part *a*, *sketch* the graph of $y = \cos 2x$. [5]

c From the graphs, determine the number of values of x from 0 to 2π radians which satisfy the equation $\sin x = \cos 2x$. [1]

Part III

Answer two questions from this part. Show all work.

31 The area K of a regular octagon in terms of its side s is given by the formula $K = 2s^2 \tan 67° 30'$.

a Solve the formula for s. [3]

b Find by logarithms the value of s to the *nearest tenth* if $K = 880$. [7]

32 In triangle ABC, $b = 16$ feet, $c = 14$ feet and angle $A = 136° 28'$. Find a to the *nearest foot*. [10]

33 Airfield B is directly north of airfield A. A plane takes off from A at noon and travels at a speed of 150 miles per hour in a direction N 20° 10′ W. At the same time a second plane takes off from airfield B in a direction S 15° 40′ W. At 2 p.m. the plane from A is directly above the plane from B. Find to the *nearest ten miles* the speed of the second plane. [5, 5]

*34 In triangle ABC, $BC = 84$, $AC = 36$ and angle $C = 93°$. Using the law of tangents, find angle A to the *nearest minute*. [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III *only*.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Express $3 \sin^2 x - 3$ as the product of three factors. 1.....

2 Express $\frac{2}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 2.....

3 Express as a single term the sum of $3\sqrt{-4}$ and $3i$. 3.....

4 Solve for $\tan x$ the equation $2\sqrt{\tan x - 1} = 4$. 4.....

5 Find the value of $3x^0 + x^{-\frac{3}{2}}$ when $x = 4$. 5.....

6 Write the equation of the line whose y-intercept is -2 and which is parallel to the line through the points $(0, 0)$ and $(2, 5)$. 6.....

7 If x varies inversely as y and $x = 8$ when $y = 9$, find y when $x = 12$. 7.....

8 The sum of the roots of the equation $x^2 - kx + 7 = 0$ is 3. Find the value of k. 8.....

9 The first term of an arithmetic progression is 9 and the seventh term is 57. Find the common difference. 9.....

10 Express $\log \frac{a}{\sqrt{b}}$ in terms of $\log a$ and $\log b$. 10.....

11 Find $\log 312.3$ 11.....

12 Find to the *nearest minute* the positive acute angle A for which $\log \sin A = 9.9145 - 10$. 12.....

13 Find the value of $\cos \frac{2\pi}{3}$. 13.....

14 In a circle whose radius is 5, a central angle of 3 radians intercepts an arc AB. Find the length of arc AB. 14.....

[OVER]

15 In triangle ABC, $c = 7$, $b = 5$ and $a = 3$. Find the cosine of the smallest angle of the triangle. 15.....

16 In triangle ABC, $a = 18$, $b = 8$, and $\sin C = \frac{1}{3}$. Find the area of triangle ABC. 16.....

17 In triangle ABC, $\sin A = .2$, $a = 1.5$ and $b = 6$. Find $\sin B$. 17.....

18 If $\tan x = 1$ and $\tan y = \frac{1}{2}$, find $\tan (x + y)$. 18.....

19 If $\cos A = -\frac{4}{5}$ and $\tan A$ is positive, find $\sin A$. 19.....

Directions (20–25): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

20 In triangle ABC with angle C a right angle, (*a*) $\sin 2A = \sin 2B$ (*b*) $\sin 2A > \sin 2B$ (*c*) $\sin 2A < \sin 2B$ 20.....

21 The maximum value of $2 \cos \frac{1}{2}x$ is (*a*) 2 (*b*) 1 (*c*) $\frac{1}{2}$ 21.....

22 As angle A increases from 90° to 270°, the value of $\cos A$ (*a*) increases and then decreases (*b*) decreases and then increases (*c*) decreases 22.....

23 The coordinates of the minimum point of the graph of the equation $y = x^2 - 4x + 5$ are (*a*) (2, 1) (*b*) (−2, 17) (*c*) (4, 5) 23.....

24 The value of the discriminant of the equation $3x^2 - 5x - 4 = 0$ is (*a*) −23 (*b*) $\sqrt{73}$ (*c*) 73 24.....

25 The graph of the equation $y^2 = x^2 + 9$ is (*a*) a circle (*b*) an ellipse (*c*) a hyperbola 25.....

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING ELEVENTH YEAR MATHEMATICS

Friday, June 15, 1956 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 11 is not expressed to *four decimal places*. For questions 20–25, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $3(\sin x + 1)\ (\sin x - 1)$

(2) $\frac{2(3+\sqrt{2})}{7}$

(3) $9i$

(4) 5

(5) $3\frac{1}{8}$

(6) $2y = 5x - 4$

(7) 6

(8) 3

(9) 8

(10) $\log a - \frac{1}{2}\log b$

(11) 2.4946

(12) 55° 13′

(13) $-\frac{1}{2}$

(14) 15

(15) $\frac{13}{14}$

(16) 24

(17) .8

(18) 3

(19) $-\frac{1}{5}$

(20) *a*

(21) *a*

(22) *b*

(23) *a*

(24) *c*

(25) *c*