ELEVENTH YEAR MATHEMATICS

Monday, June 15, 1959—1:15 to 4:15 p.m., only

Part 1

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Express as a single term the sum of $\sqrt{-9}$ and $4i$.

1..............

2. Express $\dfrac{5}{\sqrt{3}-1}$ as an equivalent fraction with a rational denominator.

2..............

3. Solve the equation $4\sin^2 x - 1 = 0$ for a value of x which lies in the third quadrant.

3..............

4. If x is a positive acute angle and $\tan x = k$, express $\cos x$ in terms of k.

4..............

5. Find to *four* decimal places the logarithm of 39.56.

5..............

6. Find to the *nearest minute* the positive acute angle whose cotangent is 1.1758.

6..............

7. One root of the equation $x^2 - 12x + k = 0$ is 4. Find the value of k.

7..............

8. Write an equation of the straight line whose slope is -2 and which passes through the point $(3, 0)$.

8..............

9. Find the value of $(2a)^0 + a^{-\frac{1}{2}}$ when $a = 9$.

9..............

10. Express in radian measure the principal value of arc $\sin \dfrac{\sqrt{2}}{2}$.

10..............

11. If $\cos x = 2/3$, find the value of $\cos 2x$.

11..............

12. Find the sum of the infinite geometric progression 1, $-1/3$, $1/9$, ...

12..............

13. The first term of an arithmetic progression is a, the last term is l, the common difference is d and the number of terms is n. Express d in terms of a, l and n.

13..............

14. In triangle ABC, $a = 8$, $b = 6$, $A = 30°$. Find $\sin B$.

14..............

15. The sides of a triangle are 4, 5, 6. Find the cosine of the largest angle of the triangle.

15..............

16. If $\tan x = 2a$ and $\tan y = a$, express $\tan (x - y)$ in terms of a.

16..............

Directions (17-19): Indicate whether *each* of the following is (a) true for all values of x, (b) true for only certain values of x or (c) not true for any value of x, by writing a, b or c on the line at the right.

17. $2 \sin^2 \frac{1}{2}x = 1 - \cos x.$ 　　　　　　　17...............

18. $\sqrt{x} + 3 = 1.$ 　　　　　　　18...............

19. $\sin (90° + x) = \cos x.$ 　　　　　　　19...............

Directions (20-25): Indicate the correct completion for *each* of the following by writing the letter a, b, c or d on the line at the right.

20. The complex fraction $\dfrac{1 - \dfrac{x}{2}}{\dfrac{x}{2} - 1}$ when simplified becomes (a) 1

(b) —1　(c) $x - 2$　(d) $2 - x$ 　　　　　　　20........

21. The expression $(\cos x - \sin x)^2$ is equivalent to (a) 1 (b) $\cos 2x$ (c) $\sin 2x$　(d) $1 - \sin 2x$ 　　　　　　　21........

22. If a quadratic equation with integral coefficients has irrational roots, the discriminant of the equation may be (a) 1　(b) 2　(c) —2 (d) 0 　　　　　　　22........

23. T varies directly as the square of L. If L is multiplied by 2, T is (a) multiplied by 2　(b) multiplied by 4　(c) divided by 2　(d) divided by 4 　　　　　　　23........

24. In triangle ABC, $a = 6$, $b = 10$, $C = 120°$. The area of triangle ABC is　(a) $30\sqrt{3}$　(b) 30　(c) $15\sqrt{3}$　(d) 15 　　　　　24........

25. If log tan $x = m$, log cot x is equivalent to　(a) $\dfrac{1}{m}$　(b) —m

(c) $1 - m$　(d) —$\dfrac{1}{m}$ 　　　　　　　25........

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26. a. Solve the equation tan x + cot $x = 3$ for tan x to the *nearest tenth*. [9]
　　b. How many values of x are there between 0° and 360° that satisfy the equation tan x + cot $x = 3$? [1]

27. a. Draw the graph of the equation $y = x^2 + 2x - 5$ from $x = -4$ to $x = 2$, inclusive. [6]
　　b. From the graph drawn in answer to part a, estimate to *tenths* the roots of the equation $x^2 + 2x - 5 = 0$. [2]
　　c. Find the minimum value of k for which the roots of the equation $x^2 + 2x - 5 = k$ are real. [2]

28. *a. Sketch* the graph of $y = 2 \cos x$ for values of x from 0 to 2π radians. [4]

   b. On the same set of axes used in *a, sketch* the graph of $y = \sin 2x$ for values of x from 0 to 2π radians. [4]

   c. What is the amplitude of the function $2 \cos x$? [1]

   d. What is the period of the function $\sin 2x$? [1]

29. The sum of the digits of a two-digit number is 15, and the product of the digits is 56. If the tens digit is larger than the units digit, what is the number? [Only an algebraic solution will be accepted.] [3, 7]

30. Solve *each* of the following equations for the real value of x:

   a. $3^x = \dfrac{1}{27}$ [2]

   b. $\log_x 8 = 3$ [2]

   c. $\log_8 x = -2/3$ [3]

   d. $x^{3/2} = 125$ [3]

Part III

Answer two questions from this part. Show all work.

31. A pendulum formula is given by the equation $t = 6.28 \sqrt{\dfrac{L}{32.2}}$. Using logarithms, find t to the *nearest hundredth* if $L = 20.9$. [10]

32. *a.* Starting with the formula for $\sin (x + y)$, *derive* a formula for $\sin (x - y)$. [4]

   b. If $\sec \theta = x$ and $\csc \theta = y$, show that $x^2 + y^2 = x^2 y^2$. [6]

33. A ship sails from point A in a direction N 58° 30′ E for a distance of 28 miles to C. It then changes its course to N 82° 40′ E for a distance of 36 miles to B. Find to the *nearest mile* the distance from A to B. [5, 5]

34. In triangle ABC, $\angle A = 52°\ 10′$, $a = 54$ feet, $b = 63$ feet and $\angle B$ is obtuse. Find C to the *nearest degree*. [10]