ELEVENTH YEAR MATHEMATICS

Monday, June 17, 1963 — 1:15 to 4:15 p.m., only

Name of pupil...Name of school...

Name of teacher...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\dfrac{3}{4-\sqrt{3}}$ as an equivalent fraction with a rational denominator.

1...................

2 Express cos $(-250°)$ as a function of a positive acute angle.

2...................

3 Find the product of the roots of the equation $3x^2 - 13x + 12 = 0$.

3...................

4 In a circle of radius 3, a central angle of $\frac{2}{3}$ radian is drawn. Find the length of the arc intercepted by that central angle.

4...................

5 Express as an equivalent fraction in *simplest form*:

$$\dfrac{\dfrac{1}{a} - b}{\dfrac{1}{b} - a}$$

5...................

6 What is the smallest integral value of k for which the roots of $3x^2 - 4x + k = 0$ are imaginary?

6...................

7 If, in $\triangle ABC$, $a = 5$, $b = 7$ and $c = 8$, find cos B.
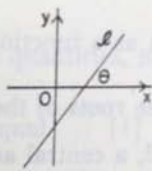
7...................

8 If A is a positive acute angle and $\sin A = \dfrac{5}{13}$, find $\sin 2A$.

8...................

9 Write an equation of the line which passes through $(0,5)$ and which has the same slope as the line $3x - y = 1$.

9...................

10 Write an equation of the axis of symmetry of the graph of $y = -x^2 + 6x + 1$.

10...................

11 If θ is a positive acute angle and $\sin \theta = \dfrac{4}{5}$, find $\sin \dfrac{\theta}{2}$.

11...................

[1]

12 Express cos $(270° - x)$ in terms of sin x.

12.................

13 Solve the following system of equations for s:

$$x - y = 4$$
$$s - x = 1$$
$$s + y = 3$$

13.................

14 Find the value of $2x° - (2x)^{-\frac{1}{2}}$, if $x = 2$.

14.................

15 If log $N = 1.1025$, find N to the *nearest hundredth*.

15.................

16 Find the value of log sin $76° 12'$.

16.................

17 Find in degrees the value of the positive acute angle θ which satisfies the equation $2 \cos^2 \theta - 1 = 0$.

17.................

18 If a and b are the first two terms, respectively, of an arithmetic progression, write the third term of the progression in terms of a and b.

18.................

19 Find to the *nearest degree* the inclination θ of the line l whose equation is $y = \dfrac{3}{2} x - 4$.

19.................

20 Evaluate cos [arc sin (-1)].

20.................

21 A new element has a half-life of 20 minutes; that is, if x ounces of the element exist at a given time, $\dfrac{x}{2}$ ounces exist 20 minutes later. The other half disintegrates into another element. If 12 ounces of this element existed, how many ounces would be left after eight 20-minute intervals?

21.................

Directions (22–30): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 The equation $\sqrt{1 - 2x} = 1 + x$ has
(1) 0 and -4 as its only roots
(2) 0 as its only root
(3) -4 as its only root
(4) no roots

22......

23 A value of x for which tan $(x + 20°)$ is undefined is
(1) $-20°$
(2) $70°$
(3) $160°$
(4) $340°$

23......

[2]

24 If an angle increases from $180°$ to $360°$, the cosine of that angle

    (1) increases throughout the interval   (3) increases, then decreases

    (2) decreases throughout the interval   (4) decreases, then increases

24......

25 Which equation has an ellipse as its graph?

    (1) $2x^2 = 8 - 3y$ (3) $2x^2 = 8 - 3y^2$

    (2) $2x^2 = 8 + 3y^2$ (4) $2x = 8 - 3y$

25......

26 If a and b are positive numbers and if $\log_a b = 2$, then

    (1) $a = b^2$ (2) $a = 2^b$ (3) $b = a^2$ (4) $b = 2^a$

26......

27 The expression $2 - \log a$ is equivalent to

    (1) $\log \dfrac{100}{a}$ (3) $\log \dfrac{2}{a}$

    (2) $\dfrac{2}{\log a}$ (4) $\sqrt{\log a}$

27......

28 If x varies inversely with y, then when x is

    (1) increased by 2, y is decreased by 2

    (2) increased by 2, y is increased by 2

    (3) multiplied by 2, y is multiplied by 2

    (4) multiplied by 2, y is divided by 2

28......

29 In $\triangle ABC$, $\angle A = 30°$, $a = 6$ and $b = 10$. Then angle C

    (1) must be acute   (3) may be either acute or obtuse

    (2) must be obtuse   (4) may be a right angle

29......

30 Which computation uses the associative postulate for multiplication?

    (1) $5(6) = 6(5) = 30$

    (2) $4(10 + 2) = 40 + 8 = 48$

    (3) $7(4) + 7(6) = 7(10) = 70$

    (4) $\frac{1}{2}(6 \cdot 7) = 3(7) = 21$

30......

[3]

Part II

31 a Solve for all values of θ between $0°$ and $360°$, inclusive, $(0° \leqq \theta \leqq 360°)$: [6]

$$\sin 2\theta - \cos \theta = 0$$

b Show that the following equality is an identity: [4]

$$2 - \tan^2 A = 1 + \frac{\cos 2A}{\cos^2 A}$$

32 Find to the *nearest tenth* the roots of $2x^2 + 3x - 3 = 0$. [10]

33 a If the numerator n and the denominator d of a common fraction are each increased by the same amount x, the resulting fraction is equal to the reciprocal of the original fraction. Express x in terms of n and d. [7]

b Using the result obtained in part a, find the numerical value of x if the fraction is $\dfrac{14}{19}$. [3]

34 a On the same set of axes, *sketch* the graphs of $y = 2 \cos \frac{1}{2}x$ and $y = \sin 2x$ as x varies from 0 to 2π radians. [Label *each* curve with its equation.] [4, 4]

b What is the amplitude of the curve $y = 2 \cos \frac{1}{2}x$? [1]

c What is the period of the curve $y = \sin 2x$? [1]

35 Write the equation or equations that would be used to solve *each* of the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a A solution of iodine and alcohol which is $3\frac{1}{2}\%$ iodine must be so diluted as to reduce it to a solution which is 2% iodine. If the solution contains 10 ounces, how many ounces of pure alcohol must be added? [5]

b The total time available to do a certain job is 3 hours. After one machine had been working on this job for 1 hour, another machine which would have taken half as many hours to do the job alone was brought in. It was put to work with the first machine, and the job was finished exactly on time. How many hours would it have taken the first machine to do the job alone? [5]

36 Answer *either a or b* but *not* both: [10]

a Two forces of 40 pounds and 30 pounds, respectively, act on a body at the same point so that their resultant is a force of 38 pounds. Find to the *nearest degree* the angle between the two original forces.

OR

b From a ship, the angle of elevation of point A at the top of a cliff is $22°$. After the ship has sailed 2,000 feet directly toward the foot of the cliff, the angle of elevation of A is $48°$. Find to the *nearest ten feet* the height of the cliff.

*37 In $\triangle ABC$, $b = 215$, $c = 382$ and angle $A = 43° 20'$. Using the law of tangents, find angle B to the *nearest ten minutes*. [10]

* This question is based on an optional topic in the syllabus.

[4]

FOR TEACHERS ONLY

11

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\dfrac{3(4 + \sqrt{3})}{13}$

(2) $-\sin 20°$ *or* $-\cos 70°$

(3) 4

(4) 2

(5) $\dfrac{b}{a}$

(6) 2

(7) $\frac{1}{2}$

(8) $\dfrac{120}{169}$

(9) $3x - y = -5$

(10) $x = 3$

(11) $\dfrac{\sqrt{5}}{5}$

(12) $-\sin x$

(13) 4

(14) $\dfrac{3}{2}$

(15) 12.66

(16) 9.3776—10

(17) 45

(18) $2b - a$

(19) 56

(20) 0

(21) $\dfrac{3}{64}$

(22) 2

(23) 2

(24) 1

(25) 3

(26) 3

(27) 1

(28) 4

(29) 3

(30) 4

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 30°, 90°, 150°, 270°   [6]

(32) 0.7 and —2.2   [10]

(33) *a* —*n* — *d*   [7]
     b —33   [3]

(34) *b* 2   [1]
     c π or 180°   [1]

(35) *a* Let *x* = ounces of pure alcohol to be added.
     $.035(10) = .02(10 + x)$ [5]
     b Let *x* = number of hours it would have taken the first machine to do the job alone.

$$\frac{1}{x} + \frac{2}{x} + \frac{4}{x} = 1 \quad [5]$$

(36) *a* 116   [10]
     b 1,270   [10]

(37) 33° 10′   [10]