

ELEVENTH YEAR MATHEMATICS

Tuesday, June 16, 1964 — 1:15 to 4:15 p.m., only

Name of pupil.....Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\frac{3}{4 + \sqrt{3}}$ as an equivalent fraction with a rational denominator. 1.....

2 Express the sum of $4 + \sqrt{-16}$ and $5 - \sqrt{-25}$ in terms of i . 2.....

3 If the number 0.00000058 is written in the form 5.8×10^n , what is the value of n ? 3.....

4 In $\triangle ABC$, $a = 18$, $b = 24$ and $\sin B = 0.8$. Find $\sin A$. 4.....

5 If θ is a positive acute angle, express $\sin \theta$ in terms of $\tan \theta$. 5.....

6 Express in simplest form $\frac{2 - \frac{2}{x}}{x - 1}$ 6.....

7 One root of the equation $2x^2 - 5x + k = 0$ is $\frac{3}{2}$. Find the value of k . 7.....

8 Find the positive root of the equation $x^2 - 5x = 24$. 8.....

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- 9 Find the value of $2(x^6 - 1) + x^{-4}$ when $x = 8$. 9.....
- 10 If x varies inversely as the square of y and if $x = 2$ when $y = 6$, find x when $y = 3$. 10.....
- 11 Write an equation of the axis of symmetry of the graph of the equation $y = 2x^2 - 3x + 7$. 11.....
- 12 Express $\cos 205^\circ$ as a function of a positive acute angle. 12.....
- 13 What is the positive value of $\cos (\arcsin \frac{1}{2})$? 13.....
- 14 The sum of an arithmetic progression is 240. The first term is 2 and the last term is 13. Find the number of terms. 14.....
- 15 If three positive geometric means are to be inserted between $\frac{1}{3}$ and 45, find the positive value of the common ratio of the progression. 15.....
- 16 In $\triangle ABC$, $a = 5$, $b = 6$ and $c = 7$. Find the value of $\cos A$. 16.....
- 17 Express $\frac{7\pi}{6}$ radians in degrees. 17.....
- 18 Find the logarithm of 663.2. 18.....
- 19 If $\log \cos \theta = 9.9514 - 10$, find θ to the nearest minute. 19.....
- 20 Write an equation of the line parallel to the line $3x + y = 8$ and passing through the point (0,4). 20.....
- 21 The graphs of $y = \sin x$ and $y = \cos x$ intersect in two points as x increases from 0° to 360° . Find in degrees the value of x greater than 180° and less than 270° for which $\sin x = \cos x$. 21.....
- 22 The area of $\triangle ABC = 15\sqrt{2}$. If $b = 5$ and $C = 45^\circ$, find a . 22.....
- 23 Combine into a single fraction: $\frac{x}{3} - \frac{2x-1}{6}$ 23.....

Directions (24–30): Write on the line at the right of each of the following the number preceding the expression that best completes the statement.

24 The expression $\frac{1}{2} \log x - \log y$ is equivalent to

(1) $\log \frac{x^2}{y}$

(3) $\log \frac{\sqrt{x}}{y}$

(2) $\log y\sqrt{x}$

(4) $\log (\sqrt{x} - y)$

24.....

25 The number of points common to the graphs of the equations

$x^2 + y^2 = 36$ and $y = x + 2$ is

(1) 1

(3) 3

(2) 2

(4) 4

25.....

26 In $\triangle ABC$, $\angle A = 38^\circ$, $a = 18$ and $c = 24$. Then $\angle C$

(1) must be acute

(3) may be either acute or obtuse

(2) must be obtuse

(4) may be a right angle

26.....

27 If the largest of four consecutive odd integers is represented by n , the smallest of these integers is represented by

(1) $n - 4$

(3) $n - 6$

(2) $n - 5$

(4) $n - 7$

27.....

28 A function having the period π is

(1) $y = 2 \sin x$

(3) $y = \sin \frac{1}{2}x$

(2) $y = \frac{1}{2} \sin x$

(4) $y = \sin 2x$

28.....

29 If A and B are positive acute angles and if $\sin A = \frac{3}{5}$ and $\sin B = \frac{4}{5}$, then $\sin(A + B)$ is equal to

(1) 1

(3) $\frac{7}{5}$

(2) 0

(4) $\frac{24}{25}$

29.....

30 The expression $(\sin x - \cos x)^2$ is equivalent to

(1) 1

(3) $-\cos 2x$

(2) $1 - \sin 2x$

(4) $1 - \cos 2x$

30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 31 Find to the *nearest tenth* the roots of the equation $5x^2 - 7x + 1 = 0$. [10]
- 32 a Starting with the formula for $\cos(x + y)$, derive a formula for $\cos 2x$ in terms of $\sin x$. [5]
 b Show that the following equality is an identity: [5]
- $$\frac{2 \cos x}{\sin 2x} = \frac{1}{\sin x \cos^2 x} - \frac{\tan^2 x}{\sin x}$$
- 33 Write the equation or equations that would be used to solve the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]
- a The tens digit of a two-digit number is 1 more than twice the units digit. The number obtained by reversing the digits is 27 less than the original number. Find the original number. [5]
- b A man drove his car at a uniform rate from his home toward Buffalo. The man's son left the same house 45 minutes later than his father and traveled along the same road at a rate 12 miles per hour faster than his father. The son overtook his father 130 miles from home. What was the rate at which the father drove? [5]
- 34 a Sketch the graph of $y = \cos 2x$ as x varies from 0 to 2π radians. [6]
 b On the set of axes used in answer to part a, draw the graph of $y = \frac{1}{2}$. [2]
 c From the graphs made in answer to parts a and b, determine the number of values of x from 0 to 2π radians which satisfy the equation $\cos 2x = \frac{1}{2}$. [2]
- 35 Using *logarithms*, compute to the *nearest tenth* the value of a if

$$a = 2 \sin 70^\circ \sqrt{\frac{29.8}{3.14}}. \quad [10]$$

- 36 Answer *either a or b* but *not both*:
- a Point B is 20 miles directly west of point A . The bearing of point C from A is $237^\circ 10'$ ($S 57^\circ 10' W$), and C is 65 miles from A . Find to the *nearest mile* the distance from B to C . [4, 6]
- OR
- b Two forces of 362 pounds and 529 pounds, respectively, act upon a body at an acute angle with each other. The angle between the resultant force and the 362-pound force is $35^\circ 40'$. Find to the *nearest ten minutes* the angle formed by the two given forces. [4, 6]
- *37 Using the formula $\tan \frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$, find C to the *nearest ten minutes* when the sides of a triangle are $a = 631$, $b = 457$ and $c = 318$. [3, 7]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit unless equations are written in 11 and 20. For questions 24-30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{12 - 3\sqrt{3}}{13}$

(2) $9 - i$

(3) -7

(4) 0.6

(5) $\frac{\tan \theta}{\sqrt{\tan^2 \theta + 1}}$ or $\frac{\tan \theta \sqrt{\tan^2 \theta + 1}}{\tan^2 \theta + 1}$

(6) $\frac{2}{x}$

(7) 3

(8) 8

(9) $\frac{1}{4}$

(10) 8

(11) $x = \frac{3}{4}$

(12) $-\cos 25^\circ$ or $-\sin 65^\circ$

(13) $\frac{\sqrt{3}}{2}$

(14) 32

(15) 3

(16) $\frac{5}{4}$

(17) 210

(18) 2.8216

(19) $26^\circ 37'$

(20) $3x + y = 4$

(21) 225

(22) 12

(23) $\frac{1}{4}$

(24) 3

(25) 2

(26) 3

(27) 3

(28) 4

(29) 1

(30) 2

[OVER]

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Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 0.2 and 1.2 [10]

(33) a u = units digit; t = tens digit

$$t = 2u + 1$$

$$10u + t = 10t + u - 27 \quad [5]$$

b x = father's rate

$$\frac{130}{x} = \frac{130}{x + 12} + \frac{3}{4} \quad [5]$$

OR

r = father's rate

t = father's time (in hrs.)

$$rt = 130$$

$$(r + 12)(t - \frac{3}{4}) = 130 \quad [5]$$

(34) c 4 [2]

(35) 5.8 [10]

(36) a Analysis [4]

49 [6]

b Analysis [4]

$59^\circ 10'$ [6]

(37) Analysis [3]

$28^\circ 40'$ [7]