The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, August 19, 1959 — 12 m. to 3 p.m., only

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express as a single term the sum of $5i$ and $-2\sqrt{-1}$. 1.....

2 Write an equation of the line whose y-intercept is 9 and whose slope is the same as that of the line whose equation is $y = 3x - 4$. 2.....

3 If y varies directly as x and if $y = \frac{1}{2}$ when $x = 6$, find the value of x when $y = 3$. 3.....

4 Simplify completely: $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{x} + \dfrac{1}{y}}$ 4.....

5 The first term of an arithmetic progression is 6 and the twenty-fifth term is 22. Find the common difference. 5.....

6 Find a geometric mean between 6 and 7. 6.....

7 Find the sum of the roots of the equation $9x^2 - 5x + 2 = 0$. 7.....

8 Given: $y = -x^2 + 10x - 16$. The maximum value of y occurs when $x = m$. Find the value of m. 8.....

9 Point P is the intersection of the graph of $4x^2 + 9y^2 = 36$ and the positive portion of the y-axis. Write the coordinates of P. 9.....

10 Solve the equation $\sqrt{4 \cos x + 7} = 3$ for the smallest positive value of x. 10.....

11 If A is a first quadrant angle, express $\cos A$ in terms of $\tan A$. 11.....

12 Express $\sin 280°$ as a trigonometric function of a positive acute angle. 12.....

13 Find n if $\log n = 0.4950$. 13.....

14 Find $\log \cos 21^\circ 14'$. 14.....

15 In triangle ABC, $a = 7$, $b = 6$ and $c = 2$. Find $\cos B$. 15.....

16 In triangle ABC, $a = 7$, $b = 6$, $\sin A = 0.7$. Find $\sin B$. 16.....

Directions (17–25): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

17 $\dfrac{1}{2-\sqrt{11}}$ is equivalent to (*a*) $\dfrac{2+\sqrt{11}}{9}$ (*b*) $\dfrac{2+\sqrt{11}}{7}$ (*c*) $-\dfrac{2+\sqrt{11}}{7}$ (*d*) $-\dfrac{2+\sqrt{11}}{9}$ 17.....

18 The value of $3x^0 + (3x)^{-\frac{1}{2}}$ when $x = 3$ is (*a*) $1\frac{1}{3}$ (*b*) $3\frac{1}{3}$ (*c*) 6 (*d*) 4 18.....

19 The roots of the equation $2x^2 - 8x + 7 = 0$ are (*a*) rational and unequal (*b*) irrational and equal (*c*) irrational and unequal (*d*) imaginary 19.....

20 If $z = \dfrac{x^3}{y^2}$ then $\log z$ is equal to (*a*) $\dfrac{3 \log x}{2 \log y}$ (*b*) $\dfrac{3}{2} \log \dfrac{x}{y}$ (*c*) $\dfrac{3}{2}(\log x - \log y)$ (*d*) $3 \log x - 2 \log y$ 20.....

21 In a circle whose radius is x inches, an arc of length y inches is intercepted by a central angle of (*a*) $\dfrac{x}{y}$ radians (*b*) $\dfrac{y}{x}$ radians (*c*) $\dfrac{x}{y}\pi$ radians (*d*) $\dfrac{y}{x}\pi$ radians 21.....

22 The period of the function $3 \sin 2x$ is (*a*) 120° (*b*) 180° (*c*) 3 (*d*) 360° 22.....

23 If $\sin x = \frac{3}{5}$ and $\cos x = \frac{4}{5}$, then $\sin^2 \frac{1}{2}x$ is equal to (*a*) $\frac{1}{10}$ (*b*) $\frac{1}{5}$ (*c*) $\frac{4}{5}$ (*d*) $\frac{9}{10}$ 23.....

24 The positive value of sin (arc tan 1) is (*a*) 0 (*b*) $\dfrac{\sqrt{2}}{2}$ (*c*) $\dfrac{\pi}{4}$ (*d*) $\dfrac{\pi}{2}$ 24.....

25 An example of a trigonometric identity is (*a*) $(\sec A)(\csc A) = 1$ (*b*) $\cos^2 A - \sin^2 A = 1$ (*c*) $\tan^2 A - \sec^2 A = 1$ (*d*) $\sec^2 A - \tan^2 A = 1$ 25.....

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 Solve the following set of equations, group your answers and check: [7, 1, 2]

$$x^2 + xy = -2x$$
$$x - y = 5$$

27 *a* *Sketch* the graph of $y = \sin 2x$ as x varies from 0 to 2π radians. [5]

b On the same set of axes used in *a*, *sketch* the graph of $y = -\frac{1}{2}$. [2]

c From the graphs made in answer to *a* and *b*, find the number of solutions of the equation $\sin 2x = -\frac{1}{2}$ when x is

(1) greater than 0 but less than $\frac{\pi}{2}$ [1]

(2) greater than $\frac{3\pi}{4}$ but less than $\frac{5\pi}{4}$ [1]

(3) greater than $\frac{3\pi}{2}$ but less than 2π [1]

28 Write the equation or equations that would be used to solve the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a How many ounces of water must be added to 24 ounces of a 10-percent solution of disinfectant to reduce it to an 8-percent solution? [5]

b A man buys a set of tools costing \$50. He makes a down payment of \$5 and arranges to pay the remainder of his bill by paying \$1 at the end of the first week, \$1.50 at the end of the second week and by increasing his payments by \$.50 each successive week. How many weeks will it take him to pay off the debt? [5]

29 *a* Starting with the formula for $\sin(x - y)$ and the formula for $\cos(x - y)$, *derive* the formula for $\tan(x - y)$ in terms of $\tan x$ and $\tan y$. [6]

b If $\sin x = \frac{3}{5}$, $\tan y = \frac{5}{12}$ and both x and y are in the first quadrant, use the formula derived in *a* to find the value of $\tan(x - y)$. [4]

30 *a* Solve the equation $3 \sin x - 2 \cos^2 x = 0$ for all positive values of x less than 180°. [6]

b Prove the following equation to be an identity: [4]

$$(\cot x)(\sec x) + \frac{2 \sin x}{\sin 2x} = \sec x + \csc x$$

Part III

Answer two questions from this part. Show all work.

31 The base edge of a regular pentagonal prism expressed in terms of its volume V and its altitude h is given by the formula $s = \sqrt{\frac{V \tan 36°}{1.25h}}$.

If $V = 775$ and $h = 11.0$, using logarithms find s to the *nearest tenth.* [10]

32 Point B is 37 miles directly north of point A. From B, point C bears N 75° 30′ E. From A, point C bears N 36° 40′ E. Find to the *nearest mile* the distance from A to C. [6, 4]

33 Two sides of a triangle are 21.0 inches and 32.0 inches in length, and the angle between these sides is 44°.

a Find the area of the triangle to the *nearest square inch.* [4]

b Find the length of the third side of the triangle to the *nearest inch.* [6]

*34 In triangle ABC, $AC = 23.7$, $BC = 36.3$, $C = 58°$. Using the law of tangents, find angle A to the *nearest degree.* [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III only.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

Wednesday, August 19, 1959 — 12 m. to 3 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–25, allow credit if the pupil has written the correct answer instead of the letters *a, b, c* or *d.*

(1) $3i$ *or* $3\sqrt{-1}$

(2) $y = 3x + 9$

(3) 36

(4) $y - x$

(5) $\frac{2}{3}$

(6) $\sqrt{42}$ *or* $-\sqrt{42}$

(7) $\frac{5}{9}$

(8) 5

(9) (0, 2)

(10) 60°

(11) $\frac{1}{\sqrt{\tan^2 A + 1}}$

(12) $-\sin 80°$ *or* $-\cos 10°$

(13) 3.126

(14) $9.9695 - 10$ *or* -0.0305

(15) $\frac{17}{28}$

(16) 0.6

(17) *c*

(18) *b*

(19) *c*

(20) *d*

(21) *b*

(22) *b*

(23) *a*

(24) *b*

(25) *d*

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(26) Solution [7]

x	0	$\frac{3}{2}$
y	-5	$-\frac{7}{2}$

[1]

Check [2]

(27) *c* (1) none [1]
(2) one [1]
(3) two [1]

(28) *a* x = number of ounces of water added
$.08(24 + x) = 2.4$ [5]
b n = number of weeks
$45 = \frac{n}{2}[2 + (n - 1).50]$ [5]
or
$4500 = \frac{n}{2}[200 + (n - 1)\ 50]$

(29) *b* $\frac{16}{63}$ [4]

(30) *a* 30° and 150° [6]

Part III

(31) 6.4 [10]

(32) Analysis [6]
57 miles [4]

(33) *a* 233 sq. in. [4]
b 22 in. [6]

(34) 82° [10]