The University of the State of New York

11 REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, August 17, 1965 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now complete the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

Directions (1–30): For *each* statement or question, write on the separate answer sheet the *number* preceding the word or expression that, of those given, best completes the statement or answers the question.

1 The sum of $3i$ and $2\sqrt{-16}$ is

(1) $-5i$ (3) $9i$
(2) $7i$ (4) $11i$

2 To the *nearest hundredth*, what is the number whose logarithm is 1.5228?

(1) 3.33 (3) 1.83
(2) 33.33 (4) 18.27

3 The expression $\frac{1}{\sqrt{3}+1}$ when written with a rational denominator is equivalent to

(1) $\frac{\sqrt{3}+1}{4}$ (3) $\frac{\sqrt{3}+1}{2}$
(2) $\frac{\sqrt{3}-1}{4}$ (4) $\frac{\sqrt{3}-1}{2}$

4 There are 4 positive angles less than 360° which satisfy the equation $\sin^2 A = \frac{1}{4}$. Two of these angles are 30° and 330°. The other two angles are

(1) 60° and 300° (3) 120° and 240°
(2) 150° and 210° (4) 90° and 270°

5 The expression $x - x^3$ is equivalent to

(1) $x(x)(x-1)$
(2) $x(1+x)(1-x)$
(3) $x(1-x)(1-x)$
(4) $x(x-1)(x+1)$

6 The expression $\frac{\frac{s}{r}-\frac{r}{s}}{\frac{1}{r}+\frac{1}{s}}$ is equivalent to

(1) $s - r$ (3) rs
(2) $s + r$ (4) $\frac{s-r}{rs}$

7 If x varies inversely as y^2 and if $x = 2$ when $y = 6$, the value of x when $y = 12$ is

(1) 1 (3) $\frac{1}{2}$
(2) 2 (4) 8

8 The positive value of cos (arc tan $\frac{3}{4}$) is

(1) $\frac{3}{5}$ (3) $\frac{4}{3}$
(2) $\frac{5}{3}$ (4) $\frac{4}{5}$

9 If $\cos\theta = \frac{1}{8}$, the positive value of $\cos\frac{1}{2}\theta$ is

(1) $\frac{\sqrt{7}}{2}$ (3) $\frac{\sqrt{7}}{4}$
(2) $\frac{3}{2}$ (4) $\frac{3}{4}$

10 The eighth term of the geometric progression -8, -4, -2, ... is

(1) $-\frac{1}{8}$ (3) $-\frac{1}{32}$
(2) $-\frac{1}{16}$ (4) $-\frac{1}{64}$

11 If the reciprocal of $(x + 2)$ is equal to $\frac{2}{x}$, then x is equal to

(1) -2
(2) 2
(3) -4
(4) 4

12 What is an equation of the line which is parallel to the line $y = 4x - 1$ and which has the same y-intercept as the line $2x + 3y = 6$?

(1) $4x + y = 2$
(2) $4x - y = -2$
(3) $4x + y = 6$
(4) $4x - y = -6$

13 What is a positive value of A which satisfies both of the equations below?

$$\sin A + 2 \cos A = 1$$
$$\sin A + \cos A = 1$$

(1) 0°
(2) 45°
(3) 90°
(4) 180°

14 The value of log cos 61° 28′ is

(1) 9.9438 — 10
(2) 9.9628 — 10
(3) 9.6792 — 10
(4) 9.6805 — 10

15 The numerical value of sin (—210°) is

(1) $\frac{1}{2}$
(2) $-\frac{1}{2}$
(3) $\frac{\sqrt{3}}{2}$
(4) $-\frac{\sqrt{3}}{2}$

16 In triangle ABC, angle A is acute and $\sin A = \cos B$. The value of $\sin C$ is

(1) 1
(2) $\frac{1}{2}$
(3) $\frac{\sqrt{2}}{2}$
(4) $\frac{\sqrt{3}}{2}$

17 In the equation $x^2 - 12x + k = 0$, the sum of the roots exceeds the product of the roots by 2. The value of k is

(1) —10
(2) 10
(3) —14
(4) 14

18 In triangle ABC, side $a = 5$, side $b = 10$ and angle $A = 30°$. The numerical value of $\sin B$ is

(1) 1
(2) $\frac{1}{2}$
(3) $\frac{1}{4}$
(4) $\frac{1}{10}$

19 If $x = 8$, the real value of $x^{\frac{2}{3}} - x^{-1}$ is

(1) 12
(2) —4
(3) $\frac{31}{8}$
(4) $\frac{33}{8}$

20 What is the number of radians in a central angle which subtends an arc whose length is twice the radius of the circle?

(1) $\frac{1}{2}$
(2) 2
(3) $\frac{\pi}{2}$
(4) 2π

21 Three consecutive integers which satisfy the condition that the square of the second exceeds the product of the first and third by 1 are

(1) —3, —2, —1 only
(2) 1, 2, 3 only
(3) any three consecutive integers
(4) never possible

22 If $x - y = 2s$ and $y = k - s$, then $x + y$ equals

(1) $2k$
(2) $-2k$
(3) $2s + 2k$
(4) $s + k$

23 The graph of $y = \frac{1}{2} \sin 2x$ has the period

(1) $\frac{\pi}{2}$
(2) π
(3) $\frac{1}{2}$
(4) 2π

24 It is required that the roots of the equation $2x^2 + kx + 4 = 0$ be real numbers. A value of k which will satisfy this requirement is

(1) 0
(2) 6
(3) —4
(4) 4

25 Which is an equation of a circle?

(1) $2x^2 + y^2 = 7$ (3) $x^2 - y^2 = 10$

(2) $x = \frac{y}{8}$ (4) $5(x^2 + y^2) = 12$

26 If $\log 100\, ab^2$ is expressed as a function of $\log a$ and $\log b$, the result will be

(1) $2 \log a + 2 \log b$
(2) $100 \log a + 2 \log b$
(3) $2 + \log a + 2 \log b$
(4) $2 + 2 \log a + 2 \log b$

27 The truth of the statement $\sin A\ (\cos A + \sin B) = \sin A \cos A + \sin A \sin B$ is guaranteed for all values of A and B by the

(1) commutative law for addition
(2) commutative law for multiplication
(3) distributive law for multiplication over addition
(4) associative law for multiplication

28 Which statement is true for all real values of θ?

(1) $\cos^2 \theta - \sin^2 \theta = 1$
(2) $\cos \theta + \sin \theta = 1$
(3) $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
(4) $\cos 2\theta = 2 \cos \theta$

29 Two distinct triangles can be constructed if angle $A = 34°$, $b = 20$ and $a =$

(1) 10 (3) 20
(2) 15 (4) 25

30 In triangle ABC, if $a = 4$, $b = 5$ and $C = 120°$, then

(1) $c^2 > a^2 + b^2$ (3) $c^2 < 2ab$
(2) $c^2 > (a + b)^2$ (4) $c^2 < a^2 + b^2$

Answers to the following questions are to be written on paper furnished by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the roots of the equation $2x^2 - 7x = 3$. [8]

b If in part *a*, $x = \cos\theta$, determine the quadrant(s) in which angle θ lies. [2]

32 *a* On the same set of axes sketch the graphs of $y = 2\cos x$ and $y = \sin 2x$ as x varies from 0 to 2π radians. [Label each curve with its equation.] [4,4]

b For what value(s) of x greater than 0 and less than 2π radians does $2\cos x - \sin 2x = 0$? [2]

33 Given the formula $c^2 = \dfrac{4.18H}{rt}$. Using logarithms, find to the *nearest hundredth* the positive value of c if $r = 150$, $t = 45$ and $H = 892$. [10]

34 Write the equation or equations that would be used to solve *each* of the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a Three positive integers form an arithmetic progression whose common difference is 5. If the first integer is left unchanged, the second integer is increased by 7 and the third integer is tripled and increased by 6, the new numbers taken in the same order are the first three terms of a geometric progression. Find the three original numbers. [5]

b If the price of cookies were increased 10 cents per dozen, 5 dozen fewer could be bought for $6. Find the original price per dozen. [5]

35 *a* Starting with the formulas for $\sin(A + B)$ and $\cos(A + B)$, derive a formula for $\tan(A + B)$ in terms of $\tan A$ and $\tan B$. [5]

b Show that the following equality is an identity: [5]

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$$

36 Answer *either a* or *b* but *not* both:

a A man at point A observes a tower at point B in a direction 20° 20′ (N 20° 20′ E). After traveling 475 feet due north to point C, he observes the same tower in the direction 108° 20′ (S 71° 40′ E). Find to the *nearest foot* the distance from the tower to the first point of observation. [4, 6]

OR

b In triangle ABC, $a = 11$, $b = 17$ and $c = 12$. Find angle C to the *nearest degree*. [10]

*37 *a* Show without the use of trigonometric tables that

$$\frac{\sin 75° + \sin 15°}{\sin 75° - \sin 15°} = \sqrt{3}.$$ [7]

b In triangle ABC, $a = 2b$ and $\tan\frac{1}{2}(A + B) = 1$. Find the numerical value of $\tan\frac{1}{2}(A - B)$. [3]

*This question is based on optional topics in the syllabus.

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ELEVENTH YEAR MATHEMATICS

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ANSWER SHEET

Pupil.....Teacher.....

School.....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1....	11....	21....
2....	12....	22....
3....	13....	23....
4....	14....	24....
5....	15....	25....
6....	16....	26....
7....	17....	27....
8....	18....	28....
9....	19....	29....
10....	20....	30....

Total
Part I Score.....

Rater's
Initials.....

Your answers for Part II should be placed on paper supplied by the school.

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS

Tuesday, August 17, 1965 — 12:30 to 3:30 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit.

(1) 4
(2) 2
(3) 4
(4) 2
(5) 2
(6) 1
(7) 3
(8) 4
(9) 4
(10) 2
(11) 3
(12) 2
(13) 3
(14) 3
(15) 1
(16) 1
(17) 2
(18) 1
(19) 3
(20) 2
(21) 3
(22) 1
(23) 2
(24) 2
(25) 4
(26) 3
(27) 3
(28) 3
(29) 2
(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 3.9 and -0.4 [8]

b II and III [2]

(32) *b* $\frac{\pi}{2}, \frac{3\pi}{2}$ [2]

(33) 0.74 [10]

(34) *a* Let a = the first integer.

$$\frac{a+12}{a} = \frac{3a+36}{a+12}$$ [5]

b Let x = the original price per dozen in cents.

$$\frac{600}{x+10} + 5 = \frac{600}{x}$$

OR [5]

$$xy = 600$$
$$(x+10)(y-5) = 600$$

(36) *a* 451 [4, 6]

OR

b 45 [10]

*(37) *b* $\frac{1}{3}$ [3]