August 18, 1982

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the answer sheet.

1. From $7 + 6i$ subtract the sum of $5 + i$ and $1 + 2i$.   1_____

2. In which quadrant does the terminal side of an angle of 420° lie?   2_____

3. Find the number of degrees in $\dfrac{5\pi}{9}$ radians.   3_____

4. If the decimal 0.000000245 is written in the form 2.45×10^n, what is the value of n?   4_____

5. Factor: $3x^2 - x - 24$ 5_____

6. What is the sum of the roots of the equation $4y^2 - 3y = 1$?   6_____

7. Express tan $(-100°)$ as a function of a positive acute angle. 7_____

8. Find the numerical value of l in the formula $S = \dfrac{n}{2}(a + l)$ if $S = 50, n = 5$, and $a = 4$.   8_____

9. If x years ago, Jack was one-half as old as he is now, express Jack's current age in terms of x.   9_____

10. What is the positive value of tan $\left(\text{arc sin } \dfrac{\sqrt{3}}{2}\right)$?   10_____

11. If sin x cos $x = \frac{1}{2}$, what is the numerical value of sin $2x$? 11_____

12. Find the solution set of the equation $|x - 3| = 7$.   12_____

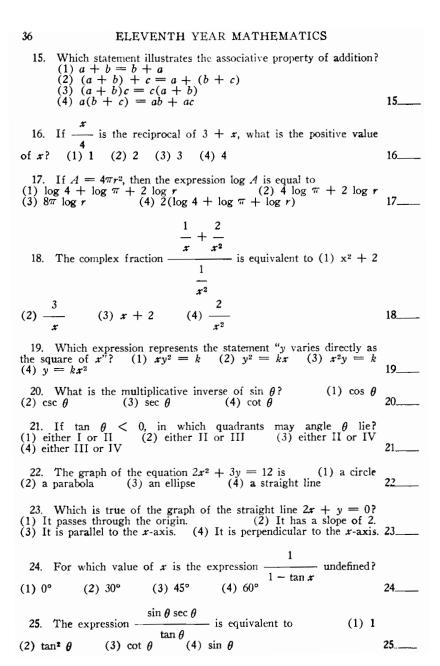13. If $n = \sqrt[4]{0.81}$, find the value of log n.   13_____

Directions (14-30): Write in the space provided on the answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

14. What is the solution of the following system of equations?
$$2x + y = 3$$
$$4x + 2y = 8$$

(1) (0,0)   (2) (1,1)   (3) (2,−1)   (4) ϕ 14_____

15. Which statement illustrates the associative property of addition?
(1) $a + b = b + a$
(2) $(a + b) + c = a + (b + c)$
(3) $(a + b)c = c(a + b)$
(4) $a(b + c) = ab + ac$ 15_____

16. If $\dfrac{x}{4}$ is the reciprocal of $3 + x$, what is the positive value
of x?   (1) 1   (2) 2   (3) 3   (4) 4   16_____

17. If $A = 4\pi r^2$, then the expression log A is equal to
(1) log 4 + log π + 2 log r (2) 4 log π + 2 log r
(3) 8π log r (4) 2(log 4 + log π + log r)   17_____

18. The complex fraction $\dfrac{\dfrac{1}{x} + \dfrac{2}{x^2}}{\dfrac{1}{x^2}}$ is equivalent to (1) $x^2 + 2$

(2) $\dfrac{3}{x}$ (3) $x + 2$ (4) $\dfrac{2}{x^2}$ 18_____

19. Which expression represents the statement "y varies directly as
the square of x"?   (1) $xy^2 = k$ (2) $y^2 = kx$ (3) $x^2y = k$
(4) $y = kx^2$ 19_____

20. What is the multiplicative inverse of sin θ?   (1) cos θ
(2) csc θ (3) sec θ (4) cot θ 20_____

21. If tan $\theta < 0$, in which quadrants may angle θ lie?
(1) either I or II   (2) either II or III   (3) either II or IV
(4) either III or IV   21_____

22. The graph of the equation $2x^2 + 3y = 12$ is   (1) a circle
(2) a parabola   (3) an ellipse   (4) a straight line   22_____

23. Which is true of the graph of the straight line $2x + y = 0$?
(1) It passes through the origin.   (2) It has a slope of 2.
(3) It is parallel to the x-axis.   (4) It is perpendicular to the x-axis. 23_____

24. For which value of x is the expression $\dfrac{1}{1 - \tan x}$ undefined?

(1) 0°   (2) 30°   (3) 45°   (4) 60°   24_____

25. The expression $\dfrac{\sin \theta \sec \theta}{\tan \theta}$ is equivalent to   (1) 1

(2) $\tan^2 \theta$ (3) cot θ (4) sin θ 25_____

26. In $\triangle ABC$, $a = 6$, $b = 4$, and $m \angle B = 30$. The numerical value of $\sin A$ is (1) $\dfrac{3\sqrt{3}}{4}$ (2) $\dfrac{3\sqrt{2}}{4}$ (3) $\dfrac{3}{4}$ (4) $-\dfrac{3}{4}$ 26____

27. The product of $(a^2b^{-3}x^2)$ and $(a^{-2}bx^{-1})$ is equivalent to
(1) ab^2x (2) $\dfrac{x}{b^2}$ (3) $\dfrac{ax}{b}$ (4) $\dfrac{b^3}{x^2}$ 27____

28. The cosecant of an angle may be equal to (1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) 0 28____

29. If $f(x) = \sin x$, what is the value of $f(\pi)$? (1) 1 (2) 2 (3) -1 (4) 0 29____

30. The graph of the equation $y = 2 \sin 2x$ has the same x-intercepts as the graph of the equation (1) $y = \sin 2x$ (2) $y = \cos 2x$ (3) $y = 2 \sin x$ (4) $y = 2 \cos x$ 30____

Part II

Answer four questions from this part.
Show all work unless otherwise directed.

31. *a* Solve the following equation for x to the *nearest tenth*:
$$3x^2 = 2x + 6 \quad [8]$$

   b If in the equation given in part *a*, x is replaced by $\sin \theta$, then the number of values of θ, $0 \le \theta \le 2\pi$, which satisfy the equation is
   (1) 1 (2) 2 (3) 0 (4) 4 [2]

32. *a* Sketch the graph of $y = 2 \sin x$ as x varies from $-\pi$ to π radians inclusive. [4]

   b On the same set of axes used in part *a*, sketch the graph of $y = \cos 2x$ as x varies from $-\pi$ to π radians inclusive. [4]

   c From the graphs made in answer to parts *a* and *b*, determine the number of values of x in the interval $-\pi \le x \le \pi$ which satisfy the equation $2 \sin x = \cos 2x$. [2]

33. *a* For all values of x for which the expression is defined, show that the following statement is an identity:
$$\frac{\sin x + \tan x}{1 + \cos x} = \tan x \quad [5]$$

   b Starting with a formula for $\cos (x + y)$, derive the formula for $\cos 2x$ in terms of $\sin x$ [5]

34. *a* Using logarithms, find the value of *n* to the *nearest tenth*:

$$n = \frac{427 \sin 72°}{36} \quad [5]$$

b If $\log 2 = p$ and $\log 3 = q$, express $\log 48$ in terms of p and q. [3]
c In which quadrant(s) does the graph of $y = \log_{10} x$ lie?   [2]

35. The members of a club were asked to contribute equally to provide a fund of $60. When two members failed to contribute, each of the others were asked to increase their contribution by $1.00 to obtain the needed $60. How many members are there in the club? [*Only an algebraic solution will be accepted.*]   [10]

36. Answer *either a* or *b* but *not* both.

   a Two forces act on an object. The first force has a magnitude of 20 pounds and makes an angle of 37 degrees with the resultant, whose magnitude is 30 pounds. Find, to the *nearest pound,* the magnitude of the second force.   [10]
                             OR

   b A scout and the base of a tower are on level ground. The scout finds that the angle of elevation to the top of the tower is 20°. After the scout walks 160 meters toward the base of the tower, he measures the angle of elevation to be 55°. What is the height of the tower to the *nearest meter?*   [10]

*37. Solve the following system of equations and check.

$$x + y + 4z = 5$$
$$3x - 2y - 2z = 2 \quad [8, 2]$$
$$2x - y + 6z = -1$$

* This question is based on an optional topic in the syllabus.