

August 17, 1983

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the answer sheet.

- Express $\frac{3}{4 + \sqrt{2}}$ as an equivalent fraction with a rational denominator. 1 _____
- Solve for x : $2^{2x-1} = 4$ 2 _____
- The graph of the equation $2x + 3y = 17$ passes through the point $(1, k)$. What is the numerical value of k ? 3 _____
- Solve the inequality $\frac{3x-1}{4} > \frac{x+3}{2}$ for x . 4 _____
- In triangle ABC , $a = 24$, $b = 20$, and $\sin A = 0.24$. Find $\sin B$. 5 _____
- Express the sum of $2\sqrt{-9}$ and $3\sqrt{-16}$ as a monomial in terms of i . 6 _____
- If 25 is divided by 7 times a certain number, the quotient is 3 and the remainder is 4. What is the number? 7 _____
- Express in *lowest terms* the product of $\left(\frac{x^2-4}{x}\right)\left(\frac{x}{x^2+x-2}\right)$. 8 _____
- What is the numerical value of $\cos 25^\circ 45'$ correct to four decimal places? 9 _____
- How many distinct triangles can be constructed if $m\angle A = 30$, side $a = 3$, and side $b = 6$? 10 _____
- What is the total number of points in which the graph of $y = 4 - 4x + x^2$ intersects the x -axis? 11 _____
- If the number 0.0000058 is expressed in the form 5.8×10^n , what is the value of n ? 12 _____
- What is the amplitude of the function $y = \frac{1}{2} \sin 2\theta$? 13 _____
- What is the slope of a line which is perpendicular to the line whose equation is $y = 2x - 1$? 14 _____

Directions (15-30): Write in the space provided on the answer sheet the numeral preceding the expression that best completes each statement or answers each question.

15. What restriction is placed on the domain of $\{(x,y) | y = \frac{4x}{x-5}\}$?
- (1) $x \neq 0$ (2) $x > 0$ (3) $x \neq 5$ (4) $x \neq -5$ 15_____
16. The numerical value of $\sin 45^\circ + \sin (-45^\circ)$ is
- (1) 1 (2) 0 (3) $\sqrt{2}$ (4) $\frac{\sqrt{2}}{2}$ 16_____
17. The roots of the equation $x^2 - x - 1 = 0$ are
- (1) $\frac{1 \pm \sqrt{5}}{2}$ (2) $\frac{-1 \pm \sqrt{5}}{2}$
- (3) $\frac{1 \pm \sqrt{3}}{2}$ (4) $\frac{-1 \pm \sqrt{3}}{2}$ 17_____
18. As x varies from 0 to 2π radians, the graphs of the functions $y = \sin x$ and $y = \cos x$ intersect in
- (1) 1 point (2) 2 points
(3) 3 points (4) 4 points 18_____
19. The graph of the curve $4x^2 - 2y^2 = 12$ is
- (1) an ellipse
(2) a circle (3) a hyperbola (4) a parabola 19_____
20. The value of $\cos (\text{Arc sin } \frac{1}{2})$ is
- (1) 1 (2) $\frac{1}{2}$
(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{2}}{2}$ 20_____
21. If A is an acute angle and $\cos A = k$, then $\sin A$ equals
- (1) $\sqrt{k^2 - 1}$ (2) $\sqrt{1 - k^2}$ (3) $k^2 - 1$ (4) $1 - k^2$ 21_____
22. The equation $\sqrt{x+6} + x = 6$ has
- (1) neither 3 nor 10
as roots (2) both 3 and 10 as roots (3) 3 as its only root
(4) 10 as its only root 22_____
23. In circle O the length of a radius is 3, and the length of chord AB is 3. The measure of $\angle AOB$ in radians is
- (1) 1 (2) 2
(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$ 23_____

24. If $\tan \theta < 0$ and $\cos \theta > 0$, the terminal side of angle θ lies in Quadrant (1) I (2) II (3) III (4) IV 24_____

25. If V varies inversely as P , then when P is doubled, V is (1) multiplied by 2 (2) divided by 2 (3) multiplied by 4 (4) divided by 4 25_____

26. The expression $\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ is equivalent to
 (1) $\frac{1}{\sin x + \cos x}$ (2) $\frac{\cos 2x}{\sin x + \cos x}$
 (3) $\cos^2 x + \sin^2 x$ (4) $\frac{1}{\sin x \cos x}$ 26_____

27. Which expression has the smallest numerical value?
 (1) $\sin 30^\circ$ (2) $\csc 30^\circ$ (3) $\tan 45^\circ$ (4) $\cos 90^\circ$ 27_____

28. In triangle ABC , $a = 5$, $b = 6$, and $c = 7$. The value of $\cos B$ is (1) $3/35$ (2) $19/35$ (3) $1/5$ (4) 9 28_____

29. Which statement is true?
 (1) If $|a| > |b|$, then $a > b$.
 (2) $|-6| + |-6| = |-12|$
 (3) $|-6| - |-6| = -12$
 (4) If $|a| = 6$, then a must equal 6. 29_____

30. If $\log x = a$, $\log y = b$, and $\log z = c$, then $\log \frac{x^2y}{\sqrt{z}}$ is equivalent to (1) $2a + b + \frac{1}{2}c$ (2) $2ab - \frac{1}{2}c$
 (3) $a^2 + b - \frac{1}{2}c$ (4) $2a + b - \frac{1}{2}c$ 30_____

Part II

Answer four questions from this part.
 Show all work unless otherwise directed.

31. Find, to the nearest degree, all values of θ , in the interval $0^\circ < \theta < 360^\circ$, that satisfy the equation $4 \cos 2\theta - 3 \sin \theta + 1 = 0$. [10]

32. a On the same set of axes, sketch and label the graphs of $y = \sin \frac{1}{2}x$ and $y = \cos 2x$ as x varies from 0 to 2π radians. [8]

b From the graphs made in part a, determine the value of x between 0 and 2π radians for which $\sin \frac{1}{2}x + \cos 2x = 2$. [2]

33. a Using logarithms, find to the nearest tenth, the value of N if $N = 17\sqrt{\tan 31^\circ}$. [6]

b If $\log 3 = a$ and $\log 5 = b$, express $\log 45$ in terms of a and b . [2]

c Find x if $\log_8 x = -\frac{2}{3}$. [2]

34. *a* Starting with the formula for $\tan(x + y)$ derive the formula for $\tan(x - y)$. [3]

b If $\tan x = 2$ and $\tan y = \frac{1}{2}$, find $\tan(x - y)$. [2]

c Show that the following equality is an identity:

$$\frac{\cot x \sec^2 x}{1 + \cot^2 x} = \tan x \quad [5]$$

35. The distance from point *A* to point *B* is 24 meters and the distance from point *A* to point *C* is 30 meters. If the measure of angle *BAC* is 115° , find to the nearest meter the distance from *B* to *C*. [10]

36. Solve the following system of equations and check your answers.

$$\begin{aligned} y^2 + 5x^2 &= 6 \\ y - 2x &= 3 \end{aligned} \quad [8, 2]$$

*37. *a* Using vectors, represent on the complex coordinate axes the complex numbers *p*, *q*, and *r*, where $p = 4 + 6i$, $q = 5 - 2i$, and $r = -3 + 9i$. [3]

b Find graphically the vector sum $p + q$. [3]

c Find graphically the vector difference $r - p$. [4]

* This question is based on an optional topic in the syllabus.