The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, August 16, 2012 — 8:30 to 11:30 a.m., only

Student Name: *Mr, Sibol*

School Name: *JMAP*

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.
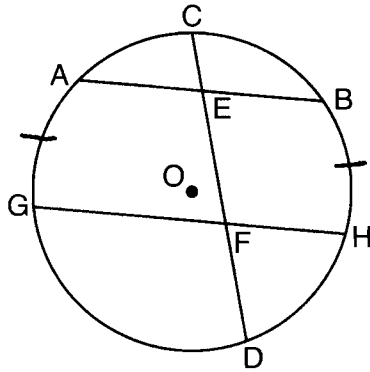
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers on your separate answer sheet. [56]

Use this space for computations.

1 In the diagram below of circle O, chord $\overline{AB}$ is parallel to chord $\overline{GH}$. Chord $\overline{CD}$ intersects $\overline{AB}$ at E and $\overline{GH}$ at F.

Which statement must always be true?

(1) $\overset{\frown}{AC} \cong \overset{\frown}{CB}$

(2) $\overset{\frown}{DH} \cong \overset{\frown}{BH}$

(3) $\overset{\frown}{AB} \cong \overset{\frown}{GH}$

(4) $\overset{\frown}{AG} \cong \overset{\frown}{BH}$

(handwritten: D'(5,0), A'(2,0), B'(0,3), C'(3,3))

2 The vertices of parallelogram $ABCD$ are $A(2,0)$, $B(0,-3)$, $C(3,-3)$, and $D(5,0)$. If $ABCD$ is reflected over the x-axis, how many vertices remain invariant?

(1) 1

(2) 2

(3) 3

(4) 0

3 Point M is the midpoint of $\overline{AB}$. If the coordinates of A are $(-3,6)$ and the coordinates of M are $(-5,2)$, what are the coordinates of B?

(1) $(1,2)$

(2) $(7,10)$

(3) $(-4,4)$

(4) $(-7,-2)$

(handwritten:
$\dfrac{-3+x}{2} = -5$ $\dfrac{6+y}{2} = 2$

$-3+x = -10$ $6+y = 4$

$x = 7$ $y = -2$ *)*

4 When a dilation is performed on a hexagon, which property of the hexagon will *not* be preserved in its image?

(1) parallelism   (3) length of sides

(2) orientation   (4) measure of angles

5 As shown in the diagram below of $\triangle ABC$, a compass is used to find points D and E, equidistant from point A. Next, the compass is used to find point F, equidistant from points D and E. Finally, a straight-edge is used to draw $\overrightarrow{AF}$. Then, point G, the intersection of $\overrightarrow{AF}$ and side $\overline{BC}$ of $\triangle ABC$, is labeled.
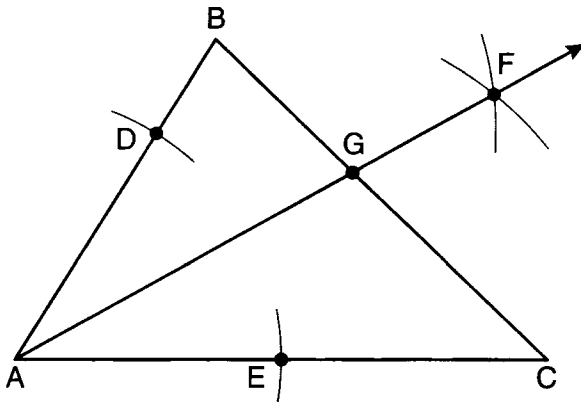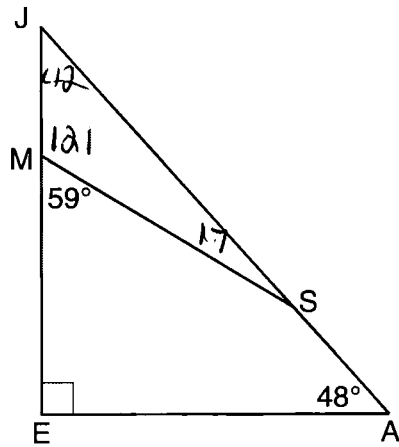

Which statement must be true?

(1) $\overrightarrow{AF}$ bisects side $\overline{BC}$ (3) $\overrightarrow{AF} \perp \overline{BC}$

(2) $\overrightarrow{AF}$ bisects $\angle BAC$ (4) $\triangle ABG \sim \triangle ACG$
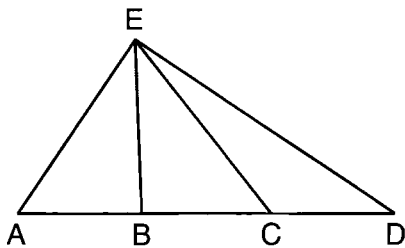
6 In the diagram of $\triangle JEA$ below, $m\angle JEA = 90$ and $m\angle EAJ = 48$. Line segment MS connects points M and S on the triangle, such that $m\angle EMS = 59$.

What is $m\angle JSM$?

(1) 163

(2) 121

(3) 42

(4) 17

7 In $\triangle AED$ with $\overline{ABCD}$ shown in the diagram below, $\overline{EB}$ and $\overline{EC}$ are drawn.
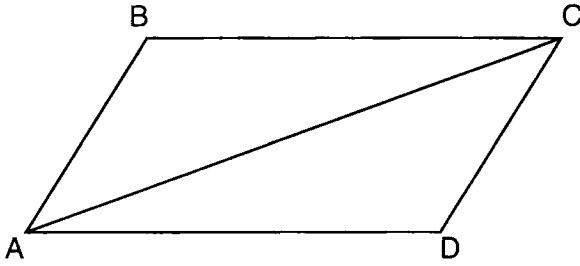

If $\overline{AB} \cong \overline{CD}$, which statement could always be proven?

(1) $\overline{AC} \cong \overline{DB}$

(2) $\overline{AE} \cong \overline{ED}$

(3) $\overline{AB} \cong \overline{BC}$

(4) $\overline{EC} \cong \overline{EA}$

8 Given that *ABCD* is a parallelogram, a student wrote the proof below to show that a pair of its opposite angles are congruent.

| Statement	Reason
1. ABCD is a parallelogram.	1. Given
2. $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{DC}$	2. Opposite sides of a parallelogram are congruent.
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Postulate of Congruency
4. $\triangle ABC \cong \triangle CDA$	4. Side-Side-Side
5. $\angle B \cong \angle D$	5. _CP CTC_

What is the reason justifying that $\angle B \cong \angle D$?

(1) Opposite angles in a quadrilateral are congruent.
(2) Parallel lines have congruent corresponding angles.
(3) Corresponding parts of congruent triangles are congruent.
(4) Alternate interior angles in congruent triangles are congruent.

9 The equation of a circle with its center at $(-3,5)$ and a radius of 4 is

(1) $(x + 3)^2 + (y - 5)^2 = 4$
(2) $(x - 3)^2 + (y + 5)^2 = 4$
(3) $(x + 3)^2 + (y - 5)^2 = 16$
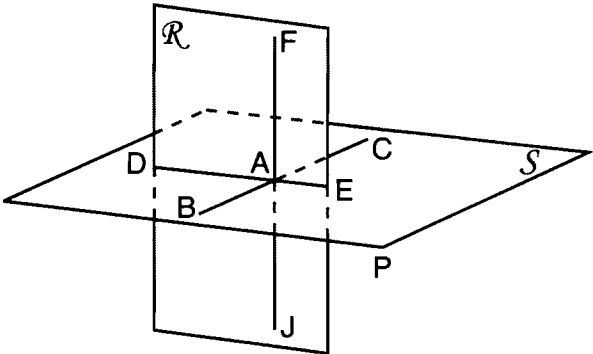(4) $(x - 3)^2 + (y + 5)^2 = 16$

10 In the diagram below of $\triangle DAE$ and $\triangle BCE$, $\overline{AB}$ and $\overline{CD}$ intersect at E, such that $\overline{AE} \cong \overline{CE}$ and $\angle BCE \cong \angle DAE$.
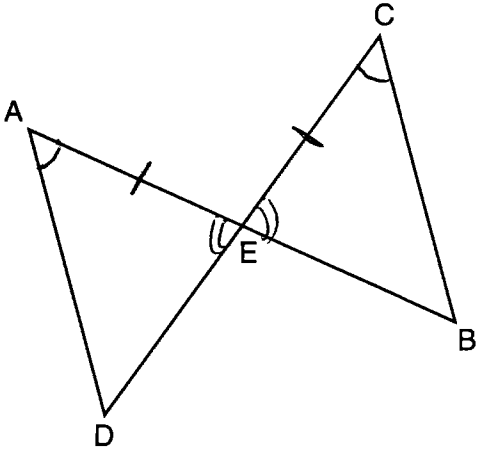
Triangle DAE can be proved congruent to triangle BCE by

(1) ASA

(3) SSS

(2) SAS

(4) HL

11 As shown in the diagram below, $\overline{FJ}$ is contained in plane $\mathcal{R}$, $\overline{BC}$ and $\overline{DE}$ are contained in plane S, and $\overline{FJ}$, $\overline{BC}$, and $\overline{DE}$ intersect at A.
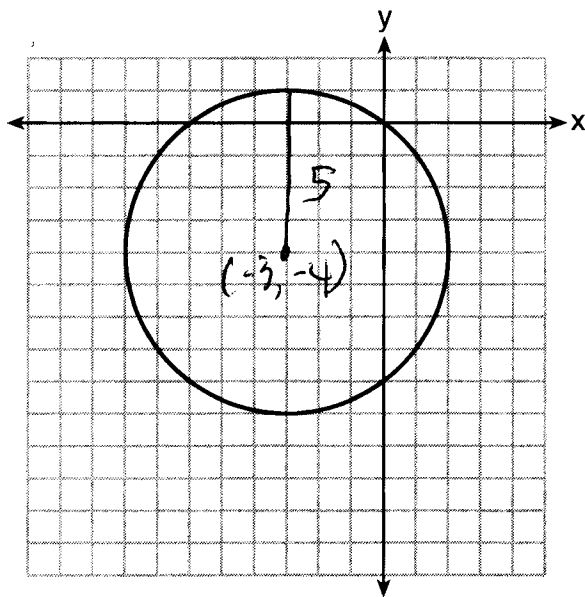

Which fact is *not* sufficient to show that planes $\mathcal{R}$ and S are perpendicular?

(1) $\overline{FA} \perp \overline{DE}$

(3) $\overline{BC} \perp \overline{FJ}$

(2) $\overline{AD} \perp \overline{AF}$
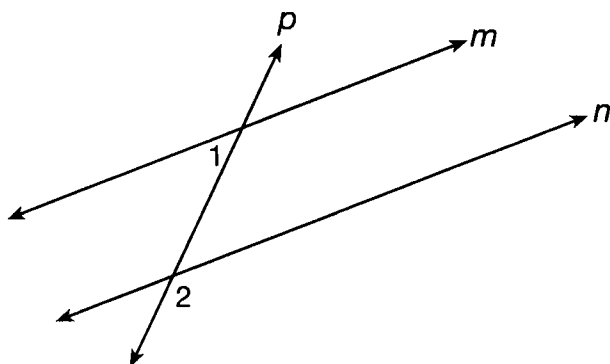
(4) $\overline{DE} \perp \overline{BC}$

12 What is an equation of the circle shown in the graph below?

(1) $(x - 3)^2 + (y - 4)^2 = 25$

(2) $(x + 3)^2 + (y + 4)^2 = 25$

(3) $(x - 3)^2 + (y - 4)^2 = 10$

(4) $(x + 3)^2 + (y + 4)^2 = 10$

13 As shown in the diagram below, lines m and n are cut by transversal p.

$4x + 14 + 8x + 10 = 180$

$12x + 24 = 180$

$\dfrac{12x}{12} = \dfrac{156}{12}$

$x = 13$

If $m\angle 1 = 4x + 14$ and $m\angle 2 = 8x + 10$, lines m and n are parallel when x equals

(1) 1   (3) 13

(2) 6   (4) 17

14 The angle formed by the radius of a circle and a tangent to that circle has a measure of

(1) 45°

(2) 90°

(3) 135°

(4) 180°

$r = 3$

15 A sphere is inscribed inside a cube with edges of 6 cm. In cubic centimeters, what is the volume of the sphere, in terms of π?

(1) 12π

(2) 36π

(3) 48π

(4) 288π

$V = \frac{4}{3} \pi (3)^3$

$V = 36\pi$

16 Scalene triangle ABC is similar to triangle DEF. Which statement is *false*?

(1) $AB : BC = DE : EF$

(2) $AC : DF = BC : EF$

(3) $\angle ACB \cong \angle DFE$

(4) $\angle ABC \cong \angle EDF$

$\angle ABC \cong \angle DEF$
$\angle BAC \cong \angle EDF$

$M = \frac{3}{2}$

17 Which equation represents a line that is parallel to the line whose equation is $y = \frac{3}{2}x - 3$ and passes through the point (1,2)?

(1) $y = \frac{3}{2}x + \frac{1}{2}$

(2) $y = \frac{2}{3}x + \frac{4}{3}$

(3) $y = \frac{3}{2}x - 2$

(4) $y = -\frac{2}{3}x + \frac{8}{3}$

$y = mx + b$

$2 = \left(\frac{3}{2}\right)(1) + b$

$\frac{1}{2} = b$

18 Lines a and b intersect at point P. Line c passes through P and is perpendicular to the plane containing lines a and b. Which statement must be true?

(1) Lines a, b, and c are coplanar.

(2) Line a is perpendicular to line b.

(3) Line c is perpendicular to both line a and line b.

(4) Line c is perpendicular to line a or line b, but not both.

19 As shown in the diagram of $\triangle ACD$ below, B is a point on $\overline{AC}$ and $\overline{DB}$ is drawn.
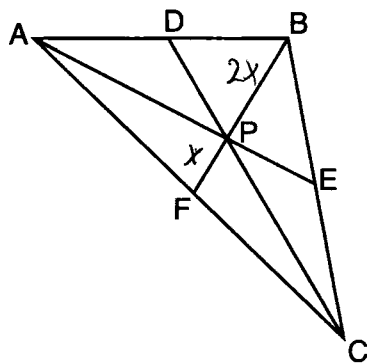
D
72° 18°
66° 42° 24°
A   B   C
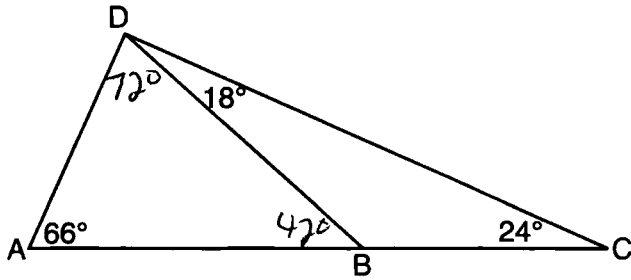
If m$\angle A$ = 66, m$\angle CDB$ = 18, and m$\angle C$ = 24, what is the longest side of $\triangle ABD$?

(1) $\overline{AB}$ (3) $\overline{AD}$

(2) $\overline{DC}$ (4) $\overline{BD}$

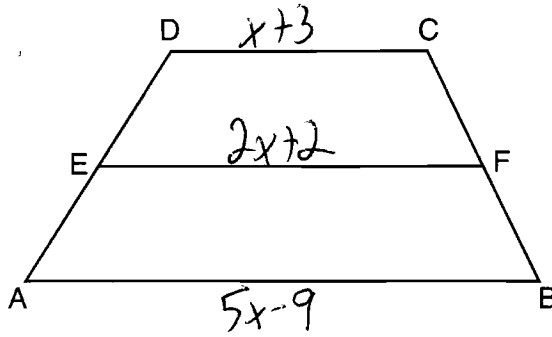**20** In $\triangle ABC$ shown below, P is the centroid and $BF = 18$.

A   D   B
2x
P
x   E
F
C

$2x + x = 18$

$3y = 18$

$x = 6$

What is the length of $\overline{BP}$?

(1) 6 (3) 3

(2) 9 (4) 12

21 In the diagram below, $\overline{EF}$ is the median of trapezoid $ABCD$.

D $x+3$ C

E $2x+2$ F

A $5x-9$ B

If $AB = 5x - 9$, $DC = x + 3$, and $EF = 2x + 2$, what is the value of x?

(1) 5

(2) 2

(3) 7

(4) 8

Handwritten computations:

$$\frac{x+3+5x-9}{2} = 2x+2$$

$$6x-6 = 4x+4$$

$$2x = 10$$

$$x = 5$$

22 In the diagram below of $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $m\angle A = 3x$, and $m\angle B = x + 20$.

A
$(3x)°$

$(x + 20)°$ $x+20$

B   C

What is the value of x?

(1) 10

(2) 28

(3) 32

(4) 40

Handwritten computations:
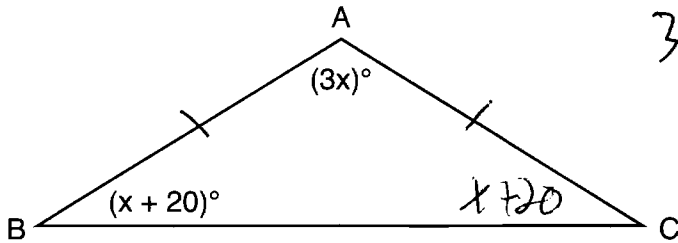
$$3x + x+20 + x+20 = 180$$

$$5x = 140$$

$$x = 28$$

23 For which polygon does the sum of the measures of the interior angles equal the sum of the measures of the exterior angles?

(1) hexagon

(2) pentagon

(3) quadrilateral

(4) triangle

Handwritten computations:

$$180(n-2) = n\left(180 - \frac{(n-2)180}{n}\right)$$

$$180n - 360 = 180n - 180n + 360$$

$$180n = 720$$

$$\frac{180n}{180} = \frac{720}{180}$$

$$n = 4$$

24 For a triangle, which two points of concurrence could be located outside the triangle?

(1) incenter and centroid

(2) centroid and orthocenter

(3) incenter and circumcenter

(4) circumcenter and orthocenter

25 The slope of line ℓ is $-\frac{1}{3}$. What is an equation of a line that is perpendicular to line ℓ?

(1) $y + 2 = \frac{1}{3}x$ $M = \frac{1}{3}$ (3) $9x - 3y = 27$ $M = 3$

(2) $-2x + 6 = 6y$ (4) $3x + y = 0$

$M = -\frac{1}{3}$ $M = -3$ $M_{\perp} = 3$

(handwritten notes: $M = 3$, $M = -\frac{1}{3}$)

26 Which type of triangle can be drawn using the points $(-2,3)$, $(-2,-7)$, and $(4,-5)$?

(1) scalene   (3) equilateral

(2) isosceles   (4) no triangle can be drawn
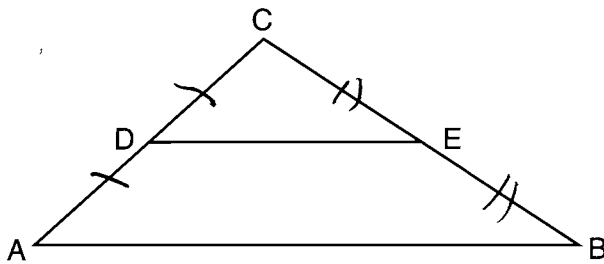
$\sqrt{(-2--2)^2 + (3--7)^2} = \sqrt{(-2-4)^2 + (3--5)^2}$

$10 = \sqrt{36 + 64}$

$10 = \sqrt{100}$

27 In the diagram below, $\overline{DE}$ joins the midpoints of two sides of $\triangle ABC$.

Which statement is *not* true?

(1) $CE = \frac{1}{2}CB$

(2) $DE = \frac{1}{2}AB$

(3) area of $\triangle CDE = \frac{1}{2}$ area of $\triangle CAB$

(4) perimeter of $\triangle CDE = \frac{1}{2}$ perimeter of $\triangle CAB$

$y = \frac{1}{2}x + 1 \quad M = \frac{1}{2} \quad M_\perp = -2$

28 Which equation represents the line that is perpendicular to $2y = x + 2$ and passes through the point $(4,3)$?

$\frac{2y}{2} = \frac{x}{2} + \frac{1\cdot2}{2}$

(1) $y = \frac{1}{2}x - 5$

(2) $y = \frac{1}{2}x + 1$

(3) $y = -2x + 11$

(4) $y = -2x - 5$

$y = mx + b$

$3 = (-2)(4) + b$

$11 = b$

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 Write the negation of the statement "2 is a prime number," and determine the truth value of the negation.

2 is not a prime number

false

30 The coordinates of the vertices of △ABC are A(1,2), B(−4,3), and C(−3,−5). State the coordinates of △A'B'C', the image of △ABC after a rotation of 90° about the origin. [The use of the set of axes below is optional.]

$A'(-2,1)$ $B'(-3,-4)$, $C'(5,-3)$
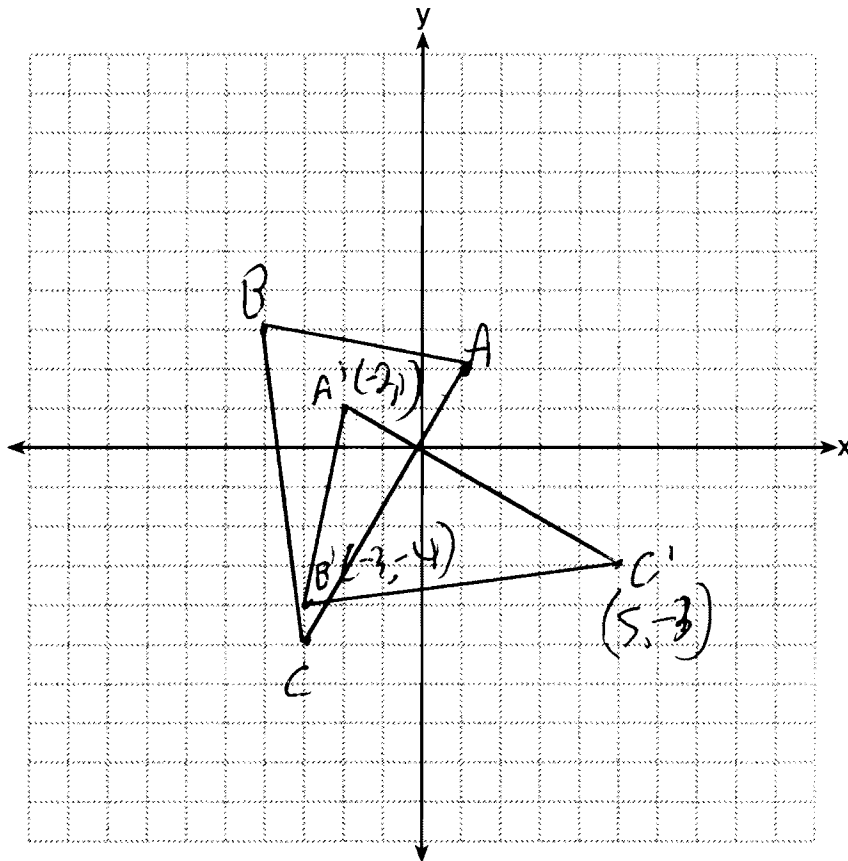
31 A cylinder has a height of 7 cm and a base with a diameter of 10 cm. Determine the volume, in cubic centimeters, of the cylinder in terms of π.

$r = 5$

$$V = \pi \cdot 5^2 \cdot 7$$

$$= 175\pi$$

32 The coordinates of the endpoints of $\overline{FG}$ are $(-4,3)$ and $(2,5)$. Find the length of $\overline{FG}$ in simplest radical form.
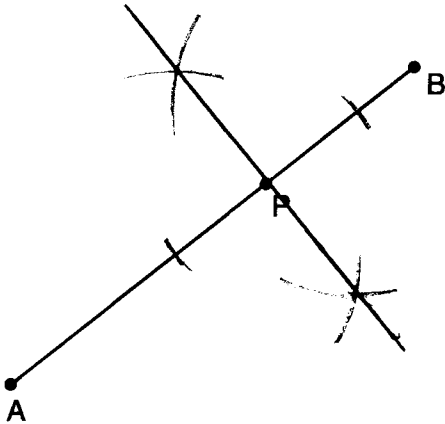
$$\sqrt{(-4-2)^2 + (3-5)^2}$$

$$\sqrt{36 + 4}$$

$$\sqrt{40}$$

$$\sqrt{4}\sqrt{10}$$
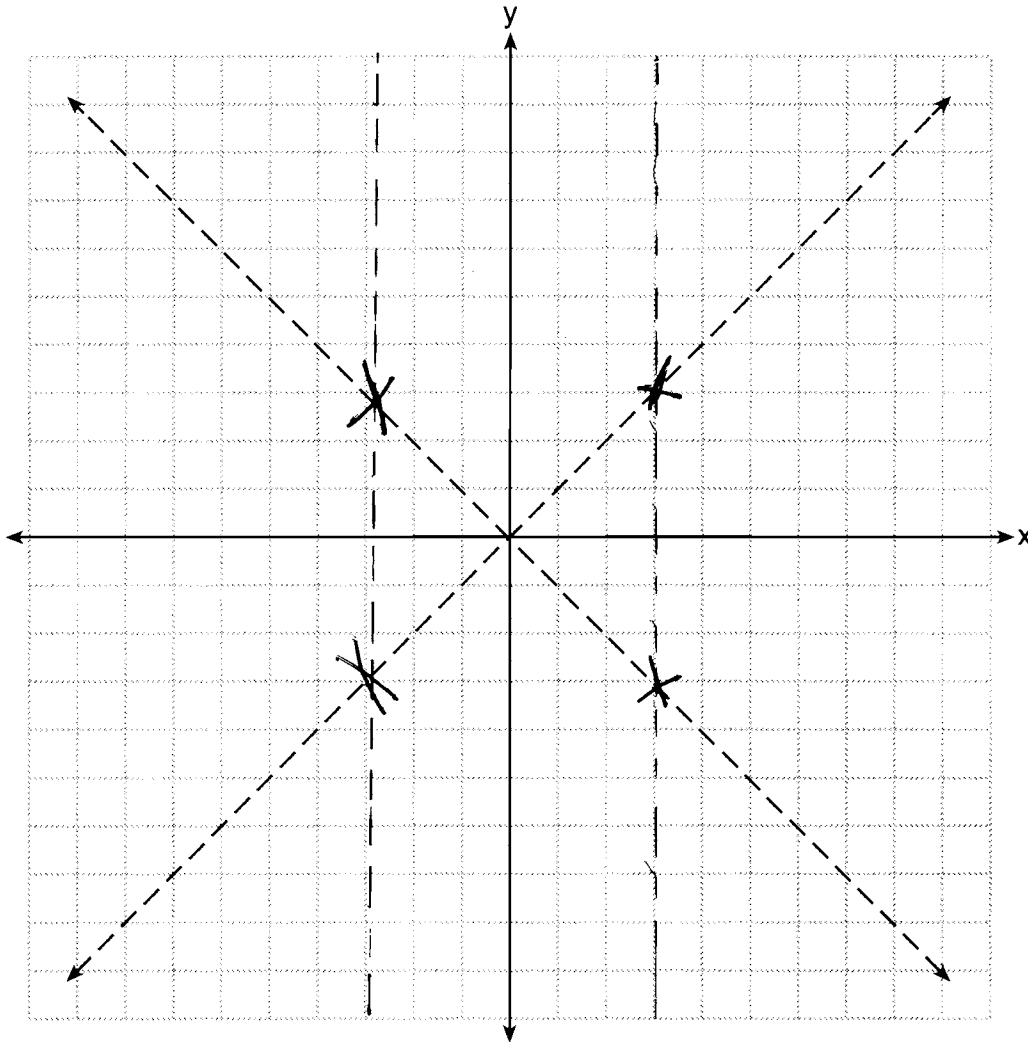
$$2\sqrt{10}$$

33 Using a compass and straightedge, construct a line perpendicular to $\overline{AB}$ through point P. [Leave all construction marks.]

34 The graph below shows the locus of points equidistant from the x-axis and y-axis. On the same set of axes, graph the locus of points 3 units from the line $x = 0$. Label with an **X** *all* points that satisfy both conditions.

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 As shown in the diagram below, the diagonals of parallelogram $QRST$ intersect at E. If $QE = x^2 + 6x$, $SE = x + 14$, and $TE = 6x - 1$, determine TE algebraically.
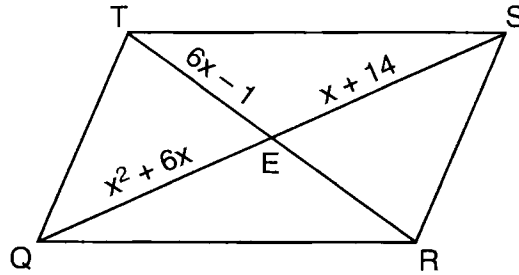
$$x^2 + 6x = x + 14$$

$$x^2 + 5x - 14 = 0$$

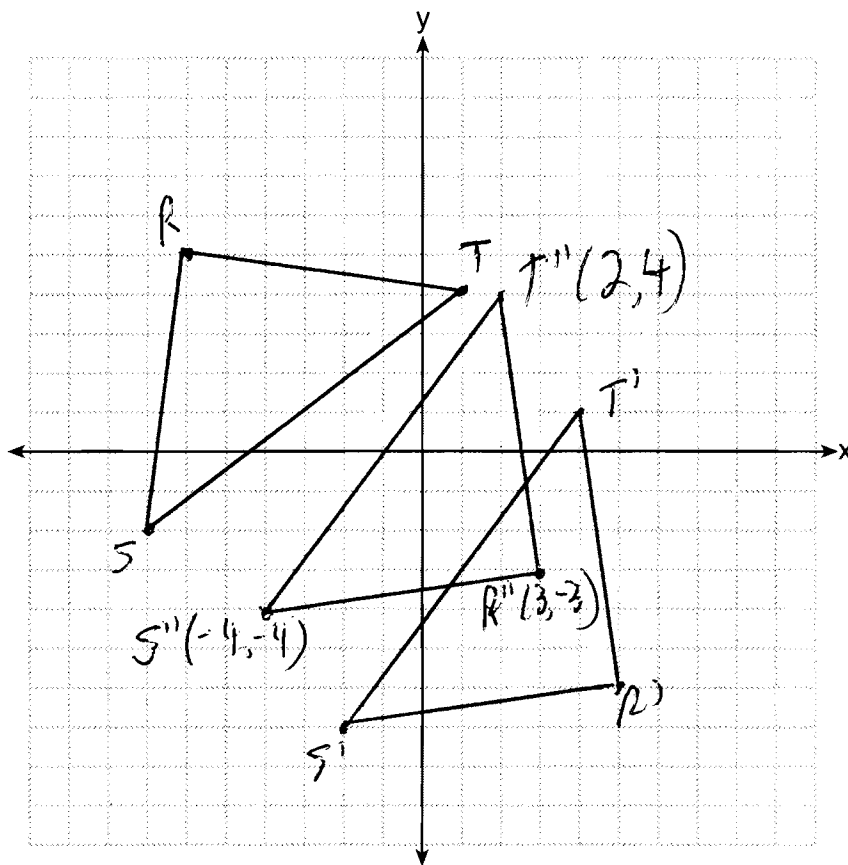$$(x + 7)(x - 2) = 0$$

$$x = -7 \qquad x = 2$$

$$6(2) - 1 = 11$$

36 The vertices of $\triangle RST$ are $R(-6,5)$, $S(-7,-2)$, and $T(1,4)$.

The image of $\triangle RST$ after the composition $T_{-2,3} \circ r_{y=x}$ is $\triangle R''S''T''$.

State the coordinates of $\triangle R''S''T''$.

[The use of the set of axes below is optional.]

$R'(5,-6)$, $S'(-2,-7)$, $T'(4,1)$
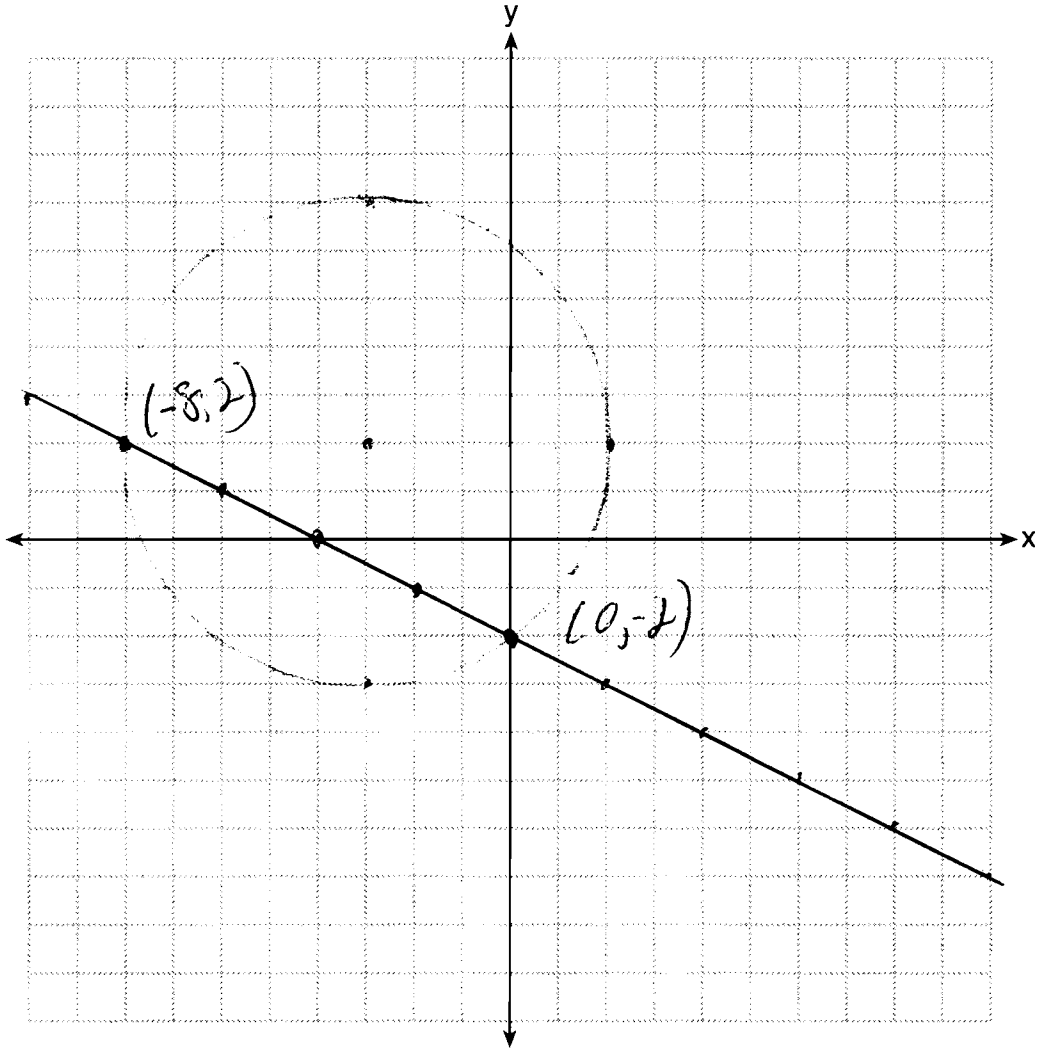
$R''(3,-3)$, $S''(-4,-4)$, $T''(2,4)$

37 On the set of axes below, solve the following system of equations graphically and state the coordinates of *all* points in the solution.

$$(x + 3)^2 + (y - 2)^2 = 25$$

$$2y + 4 = -x$$

$$2y = -x - 4$$

$$y = -\frac{1}{2}x - 2$$

(-8, 2)

(0, -2)

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

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38 Chords $\overline{AB}$ and $\overline{CD}$ intersect at E in circle O, as shown in the diagram below. Secant $\overline{FDA}$ and tangent $\overline{FB}$ are drawn to circle O from external point F and chord $\overline{AC}$ is drawn. The $m\widehat{DA} = 56$, $m\widehat{DB} = 112$, and the ratio of $m\widehat{AC} : m\widehat{CB} = 3:1$.

$\frac{}{4}$

56^0 A

114

D

O

E

112^0

C

F B 48^0
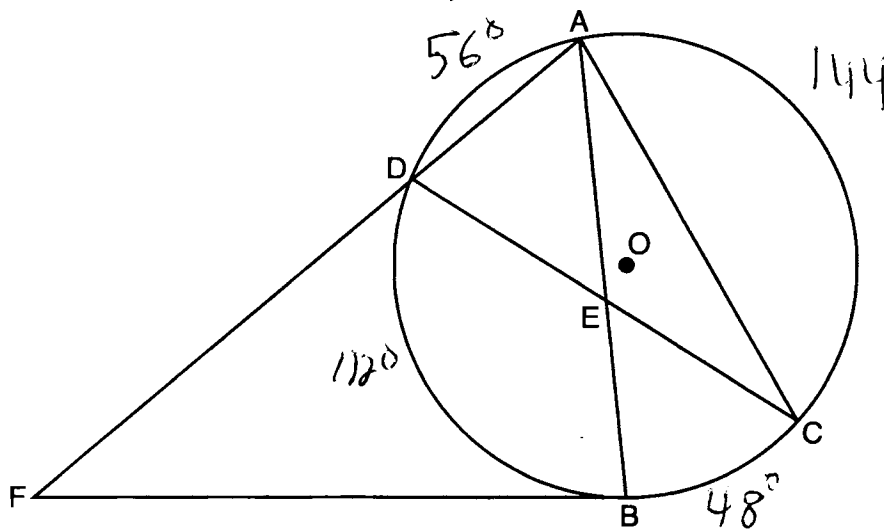
Determine m∠CEB.

$\frac{}{4} \times 192 = 48$

$\frac{56+48}{2} = \frac{104}{2} = 52$

Determine m∠F.

$\frac{192 - 112}{2} = \frac{80}{2} = 40$

Determine m∠DAC.

$\frac{112 + 48}{2} = \frac{160}{2} = 80$