

The University of the State of New York

311TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, January 24, 1951 — 9.15 a. m. to 12.15 p. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find, to the *nearest tenth*, the roots of the equation  $3x^2 + 2x - 4 = 0$ . [10]

27 Solve the following system of equations, group your answers and check *both* sets:

$$\begin{aligned}x^2 - xy - 4y &= 4 \\ x - 2y &= 1\end{aligned}\quad [7, 1, 2]$$

28 The radius  $R$  of the base of a right circular cone in terms of its volume  $V$  and angle  $A$  between an element and the base of the cone is given by the formula  $R = \sqrt[3]{\frac{3V}{\pi \tan A}}$ . Using logarithms find  $R$  to the *nearest hundredth* if  $V = 59.7$  and  $A = 37^\circ$ . [Use  $\pi = 3.14$ ] [10]

29 a Draw the graph of  $y = x^2 - 2x - 4$  from  $x = -2$  to  $x = +4$  inclusive. [6]

b From the graph made in answer to a.

(1) estimate to the *nearest tenth* the roots of the equation  $x^2 - 2x - 4 = 0$  [2]

(2) find the value of  $k$  for which the roots of the equation  $x^2 - 2x - 4 = k$  are equal [2]

\*30 Solve the following system of equations:

$$\begin{aligned}x - 4z &= 5 \\ 2x - y + 4z &= -3 \\ 6x - y + 2z &= 10\end{aligned}\quad [10]$$

\*31 Solve the equation  $x^3 - 7x^2 + 4x + 12 = 0$  [10]

\*This question is based upon one of the optional topics in the syllabus.

[1]

[OVER]

INTERMEDIATE ALGEBRA

Part III

Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [Solution of the equations is not required.]

- a* The perimeter of a right triangle is 60 inches. If the hypotenuse is 25 inches, find the length of each leg of the triangle. [5]
- b* The sum of the digits of a two-digit number is 12. If 8 is added to one third of the number, the sum is equal to the number obtained by interchanging the digits. Find the original number. [5]

33 A fruit dealer bought some boxes of berries for a total of \$6.00. Later 4 boxes of the berries spoiled and he sold all of the remaining boxes at an advance of 10 cents on the cost of each. If he made a total profit of \$1.00, how many boxes of berries did he buy? [6, 4]

Part IV

Answer one question from part IV.

34 If the blank in each of the following statements is filled by one of the words, *always*, *sometimes* or *never*, the resulting statement will be true. Write the numbers (1) to (5) on your answer paper and opposite *each* write the word that will correctly complete the corresponding statement. [Consider only the cases where  $a$ ,  $b$ ,  $c$  and  $d$  are real numbers other than 0.]

- (1) The circle  $x^2 + y^2 = a^2$  is ... tangent to the line  $y = a$ . [2]
- (2) If the roots of the equation  $ax^2 + bx + c = 0$  are real, the graph of  $y = ax^2 + bx + c$  ... cuts the  $x$  axis in two points. [2]
- (3) The equations  $ax + by = c$  and  $ax + by = d$  ... have a common solution if  $c$  is not equal to  $d$ . [2]
- (4) The graphs of  $x = y$  and  $xy = a$  ... intersect. [2]
- (5) The point of intersection of the graphs of  $ax + by = c$  and  $ax - by = c$  ... lies on the  $x$  axis. [2]

35 In the following questions,  $a$ ,  $d$  and  $r$  are positive numbers.

*a* Express  $\log ar^2$  in terms of  $\log a$  and  $\log r$ . [2]

*b* Is the following statement true or false?

$$\log(a + d) = \log a + \log d \quad [2]$$

*c* The first term of a progression is  $a$ .

- (1) If the progression is arithmetic and the common difference is  $d$ , write the first three terms. [1]
- (2) If the progression is geometric and the common ratio is  $r$ , write the first three terms. [1]
- (3) The logarithms of the terms written in answer to (1) always form (a) an arithmetic progression, (b) a geometric progression, (c) neither an arithmetic nor a geometric progression. Which is correct (a), (b) or (c)? [2]
- (4) If  $r$  does not equal 1, the logarithms of the terms given in answer to (2) always form (a) an arithmetic progression, (b) a geometric progression, (c) neither an arithmetic nor a geometric progression. Which is correct (a), (b) or (c)? [2]

INTERMEDIATE ALGEBRA

Fill in the following lines:

Name of pupil.....Name of school .....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

- 1 Express in terms of  $i$  the sum of  $\sqrt{-9}$  and  $\sqrt{-25}$ . 1.....
- 2 Factor  $y^{2n} - 16$ . 2.....
- 3 Solve for  $x$  the equation  $3 + 2\sqrt{x} = 9$ . 3.....
- 4 Find the positive root of the equation  $2y^2 - 3y - 5 = 0$ . 4.....
- 5 Write the fraction  $\frac{1}{3 - \sqrt{6}}$  with rational denominator. 5.....
- 6 Find the product of the roots of the equation  $x^2 - 3x + 5 = 0$ . 6.....
- 7 Find the value of  $3x^0 + x^{\frac{1}{2}}$  when  $x = 16$ . 7.....
- 8 If  $y = x - \frac{1}{x}$  and  $x$  is positive, does  $y$  increase or does it decrease as  $x$  increases? [Answer *increase* or *decrease*.] 8.....
- 9 Express  $\frac{3}{a - 5} + \frac{2}{5 - a}$  as a single fraction in lowest terms. 9.....
- 10 Solve the following formula for  $f$ :  $T = mg - mf$ . 10.....
- 11 Find the logarithm of 0.3716 11.....
- 12 Find the number whose logarithm is 2.9059 12.....
- 13 Find to the *nearest degree* the angle of elevation of the sun when a vertical pole 20 feet long casts a shadow 15 feet long. 13.....
- 14 Find the sum of the first 20 terms of an arithmetic progression in which the first term is 12 and the 20th term is 36. 14.....
- 15 Find two numbers that, when inserted between 5 and 135, form with those numbers a geometric progression. 15.....
- 16 Find the sum of the infinite progression 9, 6, 4, .... 16.....
- 17 Write an equation which expresses the relationship between  $x$  and  $y$  in the following table:  

$x$	-2	0	4	10
$y$	-1	3	11	23

17.....
- 18 Find the  $y$ -intercept of the straight line  $3y = 2x + 12$ . 18.....
- 19 A salesman's basic weekly salary is \$85. In addition, if his weekly sales exceed \$500, he is given a commission of 3% on the excess. Express his total earnings for a week in which his sales amounted to  $V$  dollars where  $V$  is more than \$500. 19.....
- 20 If  $x$  varies inversely as  $y$  and  $x = 24$  when  $y = 6$ , find  $x$  when  $y = 18$ . 20.....
- 21 Write the first two terms in the expansion of  $(x - 2y)^6$  21.....

INTERMEDIATE ALGEBRA

22 Simplify the complex fraction  $\frac{\frac{1}{x}}{\frac{1}{x} - x}$  22.....

Directions (questions 23–25) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

23 The discriminant of a quadratic equation is 1. The roots of the equation are  
 (a)irrational and unequal (b)rational and unequal (c)rational and equal 23.....

24 The graph of  $y = x^2 + 4$  is (a)a parabola (b)an hyperbola (c)a straight line 24.....

25 The number .000016 is equal to (a) $1.6 \times 10^{-4}$  (b) $1.6 \times 10^{-5}$   
 (c) $1.6 \times 10^{-6}$  25.....