INTERMEDIATE ALGEBRA

Wednesday, June 21, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 *a* Solve the following set of equations and group your answers:
$$x^2 + 5y^2 = 25$$
$$x^2 + y^2 = 9 \qquad [5, 2]$$
   b What is the name of the graph of $x^2 + 5y^2 = 25$?   [1]
   c Name and describe the graph of $x^2 + y^2 = 9$ [2]

27 The velocity u of a bullet in flight is given by the formula $u = k\,d\,\sqrt{\dfrac{g}{R}}$. Using logarithms, find the value of u if $k = 834$, $d = 19.7$, $g = 980$ and $R = 265$.   [10]

28 Find, correct to the *nearest tenth*, the roots of the equation $3x^2 + 5x - 1 = 0$ [10]

29 *a* Draw the graph of the equation $y = -x^2 + 4x - 2$ from $x = 0$ to $x = 4$.   [5]
   b Write the coordinates of the maximum point of the graph made in answer to *a*.   [1]
   c On the set of axes used in answering *a*, draw the graph of the equation $y = 2$.   [1]
   d Explain how the graphs made in answer to *a* and *c* show that, for any value of *y* less than 2, the roots of the equation in *a* are real.   [2]
   e What is the smallest integral value of *y* for which the roots of the equation in *a* are imaginary?   [1]

*30 Find the roots of the equation $x^3 - 4x^2 + x + 6 = 0$ [10]

* This question is based on one of the optional topics in the syllabus.

[1]

[OVER]

Part III

Answer one question from part III.

31 Write the equations that would be used in solving *each* of the following problems. In each case state what the unknown letter or letters represent. [Solution of the equations is not required.]

  a The length of a rectangle is three times its width. If the width is increased 2 inches and the length is increased 4 inches, its area is doubled. Find the width of the original rectangle in inches.   [3]

  b An alloy of copper and tin is 20% copper. How many pounds of copper must be added to 20 pounds of the alloy in order that the resulting alloy may be 50% copper?   [3]

  c The units digit of a two-digit number is 1 more than twice the tens digit. If the digits are reversed and the resulting number is added to the original number, the sum is 110. Find the number.   [4]

32 A train traveling at 8 miles per hour less than its usual rate arrived at its destination 5 hours late. The destination was 800 miles from the starting point. What was the usual rate of the train?   [7, 3]

Part IV

Answer one question from part IV.

33 Given the formula $A = P(1 + r)^n$

  a Express $\log (1 + r)$ in terms of n and the logarithms of A and P.   [3]

  b If $A = 1466$, $P = 900$ and $n = 10$, find the value of $1 + r$.   [5]

  c At what rate must $900 be invested for 10 years to amount to $1466, if interest is compounded annually?   [2]

34 *a* Prove that the difference between the squares of any two consecutive integers is always an odd integer.   [4]

  b Given the equation $x^2 + 3x + k = 0$

  (1) Write the discriminant of the equation.   [2]

  (2) For what value of k are the roots equal?   [2]

  c Prove that $a^m \times a^n = a^{m+n}$ for the case in which m and n are positive integers.   [2]

[2]

Fill in the following lines:

Name of school.................................. Name of pupil..............................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Factor: $3x^2 - 5x - 12$

2 Reduce to lowest terms: $\dfrac{2x - 1}{2x^2 - x}$

3 Using the formula $C = \frac{5}{9}(F - 32°)$, find the Centigrade reading (C) when the Fahrenheit reading (F) is 68°.

4 Solve for I: $E = IR + Ir$

5 If $3x$ represents the length of the side of a square, what represents the area of the square?

6 If $\log x^2 = 2.3108$, find $\log x$.

7 Find the logarithm of 0.8168

8 Find, correct to the *nearest tenth*, the number whose logarithm is 2.8340

9 In triangle ABC, angle $C = 90°$, angle $A = 37°$ and $AB = 30$. Find the length of AC, correct to the *nearest integer*.

10 Express in terms of i the sum of $3\sqrt{-4}$ and $2\sqrt{-9}$

11 Insert *two* geometric means between 3 and 192.

12 Which term of the series 3, 7, 11, ... is 47?

13 What must be the value of c in the equation $ax^2 + bx + c = 0$, if one of the roots is 0?

14 If the roots of the equation $ax^2 + bx + c = 0$ are equal, what is the value of $b^2 - 4ac$?

15 Solve for x: $\dfrac{2x + 6}{\sqrt{x}} = 3\sqrt{x}$

16 If the parcel post rate is a cents for the first pound and b cents for each additional pound, express in cents the cost (c) of sending a package of six pounds.

17 In the following table, the ratio $h : d$ is constant. Find the missing numbers.

h	1	2	5	?
d	40	80	?	360

18 Write the first *two* terms of the expansion $(x + y^2)^4$

19 If a line makes an angle of 45° with the positive direction of the x axis, what is the slope of the line?

20 Express $\dfrac{\sqrt{2}}{\sqrt{3}}$ as an equivalent fraction with a rational denominator.

21 If $3^x = \frac{1}{3}$, find the value of x.

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[3]

22 If the graph of the equation $y = ax^2 + bx + c$ intersects the x axis, the roots of the equation $ax^2 + bx + c = 0$ must be (a) rational (b) real (c) equal (d) imaginary

22......

23 The expression $3^{-1} + 2^0$ is equal to (a) -3 (b) $2\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $1\frac{1}{3}$ 23......

24 Which of the following statements is true? (a) $x^2 \cdot x^3 = x^6$ (b) $(-y)^2 = -y^2$ (c) $x^2 - y^2 = (x - y)^2$ (d) $\cdot(-x)^2 = -x^2$

24......

25 If $y = x^2$ and if x increases from -5 to $+5$, then y will (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase

25......