

The University of the State of New York

312TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, June 20, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find to the *nearest tenth* the roots of the equation $3x^2 + 2x - 6 = 0$. [10]

27 Solve the following system of equations and check *one* set of answers: [8, 2]

$$\begin{aligned}2x + y &= 1 \\ x^2 - xy &= 10\end{aligned}$$

28 a Draw the graph of $y = x^2 - 4x - 3$ from $x = -1$ to $x = 5$ inclusive. [6]

b Write the equation of the axis of symmetry of the graph drawn in answer to a. [1]

c Find the minimum value of y . [1]

d From the graph, estimate to the *nearest tenth* the roots of the equation $x^2 - 4x - 3 = 0$. [2]

29 The radius R of a circle circumscribed about a regular decagon whose area is A is given by the formula $R = \sqrt{\frac{A}{5 \sin 36^\circ}}$. Using logarithms, find R to the *nearest hundredth of an inch* when $A = 71.70$ square inches. [10]

*30 Solve the following system of equations for x , y and z : [10]

$$\begin{aligned}2x - y + 2z &= 7 \\ 4x + 2y + 3z &= 3 \\ 2x - 3y - z &= 2\end{aligned}$$

*31 Solve for x : $6x^3 + 5x^2 - 17x - 6 = 0$ [10]

* This question is based upon one of the optional topics in the syllabus.

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Part III

Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [Solution of the equations is not required.]

a How many pounds of water must be evaporated from 100 pounds of a 3% solution of salt in order to make the result a 5% solution of salt? [5]

b Paul can do a certain job in half the time that Jim requires to do it. Jim worked alone for one hour and stopped; then Paul completed the job in ten hours. How many hours would it take each working alone to do the whole job? [5]

33 At his usual rate, a man can travel 60 miles by automobile in a certain time. If he were to increase his usual rate by 10 miles per hour, then he could travel the 60 miles in $\frac{1}{2}$ hour less time. Find his usual rate. [6, 4]

Part IV

Answer one question from part IV.

34 If the blank in each of the following statements is filled by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Write the numbers (1) to (5) on your answer paper and opposite *each* write the word that will correctly complete the corresponding statement. [In each case *a*, *b* and *c* are real, unequal numbers.]

(1) When drawn on the same axes, the graphs of the equations $y = ax$ and $y = bx + c$ will ... intersect. [2]

(2) If the graph of the equation $ax^2 + by^2 = c$ is an ellipse, then *a*, *b* and *c* will ... have the same sign. [2]

(3) If the graph of the equation $y = ax^2 + bx + c$ has a minimum point, then *a* will ... be negative. [2]

(4) If the graph of the equation $y = ax^2 + bx + c$ intersects the *x*-axis in 2 distinct points, then the discriminant of the equation $ax^2 + bx + c = 0$ is ... positive. [2]

(5) If the common solutions of the equations $x^2 + y^2 = a^2$ and $x^2 - y^2 = b^2$ are imaginary, then *b* will ... be greater than *a*. [2]

35 *a* Given $\log 2 = x$ and $\log 3 = y$

(1) Express $\log 12$ in terms of *x* and *y* [2]

(2) Express $\log \frac{\sqrt{2}}{\sqrt[3]{3}}$ in terms of *x* and *y* [2]

b Given $\log_{10} m = x$ and $\log_{10} n = y$

(1) Express *m* and *n* each as a power of 10 [2]

(2) Express $\log_{10} mn$ in terms of *x* and *y* [2]

c If $\log_{10} a + \log_{10} b = 2$, find the value of *ab*. [2]

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

- 1 Find the factors of $2x^2 - 5x - 12$. 1.....

- 2 Write the expression $2\sqrt{-25} - 3\sqrt{-1}$ as a single term. 2.....

- 3 Express $\frac{1}{4 + \sqrt{3}}$ as an equivalent fraction with a rational denominator. 3.....

- 4 Solve the equation $2\sqrt{x + 2} = 3$. 4.....

- 5 Write an equation of the straight line whose slope is 2 and whose y-intercept is -3 . 5.....

- 6 Write an equation expressing the relationship between x and y shown by the following table:

x	1	2	4	7
y	-1	1	5	11

 6.....

- 7 Solve for a the equation $r = \frac{a}{1-a}$ 7.....

- 8 To buy an article costing c dollars, a man makes a down payment of d dollars. If he is to pay the balance in n equal installments, express one installment in terms of c , d and n . 8.....

- 9 If s varies directly as t , and $s = 3$ when $t = 5$, find s when $t = 8$. 9.....

- 10 Find the 25th term of the progression $-8, -5, -2, \dots$ 10.....

- 11 Find *two* numbers that when inserted between 2 and 128 form with those numbers a geometric progression of four terms. 11.....

- 12 Find the sum of the infinite progression $2, -1, \frac{1}{2}, \dots$ 12.....

- 13 Find the value of $(27)^{-\frac{1}{3}} + (27)^0$ 13.....

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- 14 Write the *third* term in the expansion of $(b - c)^6$ 14.....
- 15 Simplify the complex fraction $\frac{1 - \frac{1}{m}}{\frac{1}{m}}$ 15.....
- 16 Find $\log 26.16$ 16
- 17 Find the number whose logarithm is $9.8030 - 10$ 17.....
- 18 At a point 150 feet from the foot of a building, the angle of elevation of the top of the building is 20° . Find the height of the building to the *nearest foot*. 18
- 19 Find the sum of the roots of the equation $2x^2 - 5x + 6 = 0$. 19.....
- 20 Find the product of the roots of the equation $px^2 + qx = 0$. 20
- 21 What is the name of the graph of the equation $x^2 - \frac{y^2}{4} = 9$? 21.....

Directions (questions 22-25) — Indicate the correct answer to *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

- 22 If 93,000,000 is expressed in the form 9.3×10^n , then *n* equals (a)6
 (b)7 (c)8 22.....
- 23 If $y = 2x - x^2$ and *x* varies from $x = -3$ to $x = 1$, then *y* (a)increases
 (b)decreases (c)decreases and then increases 23.....
- 24 The expression $(x^2)^4$ is equal to (a) x^8 (b) x^6 (c) x^{16} 24.....
- 25 The expression $\frac{x-1}{1-x^2}$ is equal to (a) $\frac{x+1}{1+x^2}$ (b) $\frac{1}{x+1}$
 (c) $-\frac{1}{x+1}$ 25.....