

PLANE GEOMETRY

Thursday, January 20, 1916—1.15 to 4.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including the eleventh.

Assign 16 credits to the eleventh question and 12 credits to each of the others.

1 Prove that the locus of points equidistant from two given points is the perpendicular bisector of the line joining them.

2 Prove that if in the same circle, or in equal circles, two chords are unequal, the shorter is at the greater distance from the center.

3 Prove that the product of the segments of a chord that passes through a fixed point within a circle is the same for all directions of the chord.

4 Prove that the area of a regular polygon is equal to half the product of its perimeter and its apothem.

5 a What is the sum of the interior angles of a polygon of 16 sides?

b The vertical angle of an isosceles triangle is $15^{\circ} 24'$; how large is each base angle?

c One angle of a parallelogram is 35° ; what is the value of each of the other angles?

d The smaller arc intercepted by two secants which meet at an angle of 44° is 71° ; how many degrees are in the greater intercepted arc?

6 Prove that if a trapezoid is isosceles the lower base angles are equal.

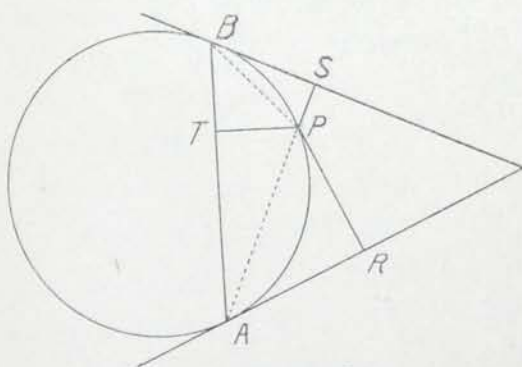
7 For each of the following theorems (no proof required) draw the figure and state the hypothesis and conclusion in terms of letters on the figure:

a The perpendicular from the intersection of the medians of a triangle to any straight line in the plane of the triangle, not intersecting it, is equal to one third the sum of the perpendiculars from the vertices of the triangle to the same line.

- b* The perpendiculars drawn from the vertices of a triangle to the opposite sides are the bisectors of the angles of the triangle formed by joining the feet of the perpendiculars.
- 8 The sides of a triangle are 16, 24, 32.
a Find the length of the altitude to the side 24.
b Find the length of the bisector of the largest angle, or, find the segments into which the bisector of this angle divides the opposite side.
- 9 Prove that if from any point on a circle a chord and a tangent are drawn, the perpendiculars let fall upon them from the mid-point of the intercepted arc are equal.
- 10 Given a side and the two diagonals of a parallelogram; construct the parallelogram. [To receive credit construction lines must be shown.]
- 11 Answer *each* of the eight questions pertaining to the following proof:

THEOREM

The rectangle of the perpendiculars drawn from any point of a circle to two tangents to the circle is equal to the square on the perpendicular from the given point to the chord of contact.



- 1 Why is $\angle PBT = \angle PAR$?
- 2 Why is $\angle ARP = \angle PTB$?
- 3 Why is $\triangle ARP$ similar to $\triangle PTB$?
- 4 Why does $\frac{PR}{PT} = \frac{PA}{PB}$?
- 5 Likewise two other triangles in the figure are similar; name these triangles.
- 6 $\frac{PT}{PS} =$ what ratio?
- 7 Why does $\frac{PR}{PT} = \frac{PT}{PS}$?
- 8 Why does $\overline{PT}^2 = PR \times PS$?