

PLANE GEOMETRY

Wednesday, January 19, 1927 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including not more than three from group I and at least one from group II.

Group I

Do not answer more than three questions from this group.

1 Prove that if in the same circle or in equal circles two chords are equal, the arcs subtended by them are equal. [12½]

2 Prove that if in a right triangle the perpendicular is drawn from the vertex of the right angle to the hypotenuse,

a the two triangles thus formed are similar to the given triangle and to each other. [8½]

b the perpendicular is the mean proportional between the segments of the hypotenuse. [4]

3 Prove that the locus of points equidistant from the sides of an angle is the bisector of the angle. [6, 6½]

4 Prove that a circle may be circumscribed about any regular polygon. [12½]

Group II

Answer at least one question from this group.

Problems in this group should be constructed accurately with ruler and compass. Leave all construction lines on the paper.

5 Construct a parallelogram $ABCD$, given the base AB , the altitude h on AB , and angle A . [12½]

6 Find the locus of points between two parallel lines such that each point on the locus will be twice as far from one of these lines as from the other. [12½]

Group III

Irrational results may be left in the form of π and radicals unless otherwise stated.

7 A triangle that circumscribes a given circle has two of its angles 44° and 36° ; find the three angles of the inscribed triangle formed by joining the points of tangency of the sides of the given triangle. [12½]

8 ABC is a triangle in which side AB is greater than side AC . OB and OC bisect angles B and C respectively. Show that OB is greater than OC . [12½]

9 a Prove that if two sides of a triangle are equal, the medians to these two sides are equal. [9]

b State the converse of a. [Proof not required in b] [2½]

10 $ABCDEF$ is a regular hexagon inscribed in a circle whose radius is 8; ACE is the triangle formed by joining alternate vertices of the hexagon.

a Find the area of the hexagon $ABCDEF$. [4]

b Find the area of the triangle ACE . [8½]

11 A flywheel 30 inches in diameter is running at 35 revolutions per minute; find the speed in feet per minute of a point on the rim of the flywheel. [12½]

12 ABC is a triangle with base $AB = 10''$ and altitude $CD = 12''$; EF is a line $8''$ long parallel to AB and terminated by the other sides of the triangle.

a Find the area of triangle EFC . [6½]

b Find the distance of the line EF from base AB . [6]

13 The radii of two circles are 8 and 3 and the length of the common external tangent is 12; find (a) the distance between the centers of the two circles [6], (b) the distance from the center of the smaller circle to the point of intersection of the line through the centers and the external tangent extended to meet [6½].