The University of the State of New York

241st High School Examination

PLANE GEOMETRY

Wednesday, January 18, 1928 — 9.15 a. m. to 12.15 p. m., only

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Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry.

The minimum time requirement is five recitations a week for a school year.

Name the author of the textbook you have used in plane geometry.

Answer eight questions, including not more than two from each of groups I and II, and at least two from each of groups III and IV.

Group I

Do not answer more than two questions from this group.

1 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [12½]

2 Prove that the area of a triangle is equal to one half the product of its base and its altitude. [12½]

3 *a* If the opposite sides of a quadrilateral are equal and a diagonal is drawn, why are the triangles thus formed congruent? [2½]

b What construction line is drawn in proving that the sum of the angles of a triangle is a straight angle? [2½]

c State the theorem about the angles of a triangle used in proving that if one side of a triangle is greater than a second side, the angle opposite the first side is greater than the angle opposite the second side. [2½]

d State the theorem about similar triangles used in proving that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [2½]

e State the theorem about areas used in proving that the area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. [2½]

Group II

Do not answer more than two questions from this group.

Problems in this group should be constructed accurately with ruler and compasses. Leave all construction lines on the paper.

4 Given a line AB and a point P not in AB

a Through P construct a line parallel to AB. [7]

b From P drop a line perpendicular to AB. [5½]

5 *a* Given two unequal line segments; construct a rectangle having these line segments for its dimensions. [5½]

b Construct a square equal in area to the rectangle obtained in answer to *a*. [7]

6 *a* Given a trapezoid; locate a point that shall be equidistant from the lower base and the two nonparallel sides. [10]

b When will this point lie in the upper base of the trapezoid? [2½]

Group III

Answer at least two questions from this group.

7 The bisector of angle A meets side DC of parallelogram $ABCD$ in point F and the bisector of angle C meets side AB in E; prove that AF equals CE. [12½]

8 Point P outside a circle is joined to the center O. Line PO cuts the circle in point A. If PB is any other line from P to the circle, prove that PB is greater than PA. [12½]

9 AB and AC are secants to a circle from the common point A, the points B and C being on the circle. If the secants cut the circle in D and E respectively, prove that $AB : AC = AE : AD$. [12½]

10 Prove that the area of an inscribed equilateral triangle is one half the area of a regular hexagon inscribed in the same circle. [12½]

Group IV

Answer at least two questions from this group.

Irrational results may be left in the form of π and radicals unless otherwise stated.

11 The vertex angle of an isosceles triangle is 30° and the equal sides are each 8 feet long.

a Find the length of the altitude on one of the equal sides. [3½]

[OVER]

b Find the lengths of the segments of the side made by this altitude. [4½]

c Find the area of the given triangle. [4½]

12 The area of a circle is 81 π square inches; find the length of an arc of 40°. [12½]

13 If a, b and c are the sides of a triangle, of which c is the longest, and A, B and C respectively the opposite angles, then

(1) when $c^2 = a^2 + b^2$, angle C = a right angle.
(2) when $c^2 < a^2 + b^2$, angle $C <$ a right angle.
(3) when $c^2 > a^2 + b^2$, angle $C >$ a right angle.

Using the above facts, classify *each* of the following triangles as acute, right or obtuse:

a A triangle whose sides are 24, 7, 25. [4]

b A triangle whose sides are 18, 8, 16. [4½]

c A triangle whose sides are 12, 9, 8. [4]

14 *A* and *B* are two trees 200 feet apart on the same side of a straight path. *A* is directly on the path while *B* is 120 feet from the path. How many feet from *A* should a point be taken on the path so that the point will be the same distance from *A* and *B*? [12½]