The University of the State of New York

274TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Friday, January 20, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

See instructions for groups II, III and IV on page 1.

Group II

Answer two questions from this group.

26 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [10]

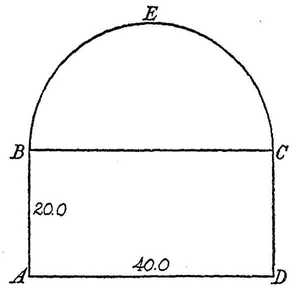
27 Prove that two right triangles are congruent if the hypotenuse and a leg of one are equal respectively to the hypotenuse and a leg of the other. [10]

28 Triangle ABC is inscribed in a circle. The bisector of angle C intersects side AB at D and arc AB at E. Prove: $AC \times BC = CD \times CE$ [10]

Group III

Answer two questions from this group.

29 The accompanying figure $ABECD$ represents the cross section of an underground tunnel. $ABCD$ is a rectangle 40.0 feet by 20.0 feet, surmounted by the semicircle BEC. Find, correct to the *nearest square foot,* the area of the cross section. (Use $\pi = 3.14$) [10]



30 The area of an equilateral triangle is equal to that of a trapezoid whose bases are 4 and 14 and whose altitude is $4\sqrt{3}$. Find a side of the triangle. [10]

31 A side of a regular pentagon is 8. Find its area correct to the *nearest integer.* [10]

Group IV

Answer one question from this group.

32 Consider *each* of the following statements and tell whether it is *always* true, *sometimes* true or *never* true. *Give reasons for your answers.*

a The area of a rectangle is equal to one half the product of its diagonals. [1, $1\frac{1}{2}$]

b If two triangles have a side and any two angles of one equal to the corresponding parts of the other, the triangles are congruent. [1, $1\frac{1}{2}$]

c If the radius of a circle is increased by x, then the circumference of the circle is increased by $2\pi x$. [1, $1\frac{1}{2}$]

d If the legs of a right triangle are represented by a and b and the hypotenuse by c, then $c^2 = (a + b)^2$ [1, $1\frac{1}{2}$]

33 A square whose side is s and a rectangle whose base is $s + a$ have equal perimeters.

a Express the altitude of the rectangle in terms of s and a. [4]

b Express the area of the rectangle in terms of s and a. [1]

c Indicate whether the following statement is true or false: Of all rectangles which have equal perimeters, that which is equilateral has the greatest area. Give reasons for your answer. [2, 3]

Plane Geometry

Fill in the following lines:

Name of school...Name of pupil...

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–17) — Indicate the correct answer to each of the following questions by writing on the dotted line at the right the letter *a*, *b* or *c*.

1 The diagonals of a rectangle are always (*a*) equal to each other, (*b*) perpendicular to each other or (*c*) bisectors of the angles through which they pass. 1...............

2 If two adjacent angles have their exterior sides in the same straight line, they are always (*a*) equal, (*b*) complementary or (*c*) supplementary. 2...............

3 The sum of the exterior angles of a polygon of n sides is (*a*) n straight angles, (*b*) 2 straight angles or (*c*) $(n - 2)$ straight angles. 3...............

4 If two parallel lines are cut by a transversal, the corresponding angles are always (*a*) supplementary, (*b*) equal or (*c*) acute. 4...............

5 In triangle ABC, AD is an altitude and AM is a median. If AB and AC are unequal, (*a*) $AM = AD$, (*b*) $AM > AD$ or (*c*) $AM < AD$ 5...............

6 A median of a triangle divides it into two triangles which are always (*a*) congruent, (*b*) similar or (*c*) equal in area. 6...............

7 The area of a rhombus is equal to (*a*) one half the sum of its diagonals, (*b*) one half the product of its diagonals or (*c*) the product of its diagonals. 7...............

8 If the corresponding sides of two similar triangles are in the ratio 1 : 2, the areas of the two triangles are in the ratio (*a*) 1 : 4, (*b*) 1 : 2 or (*c*) $1:\sqrt{2}$ 8...............

9 If the segments of one of two chords, not diameters, intersecting within a circle, are r and s and the segments of the other chord are v and w, then (*a*) $r \times s = v \times w$, (*b*) $r + s = v + w$ or (*c*) $\frac{r}{s} = \frac{v}{w}$ 9...............

10 If from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between (*a*) the whole secant and its internal segment, (*b*) the whole secant and its external segment or (*c*) the external and the internal segments of the secant. 10...............

11 If in the right triangle ABC, AB is the hypotenuse and CD is the altitude upon the hypotenuse, then (*a*) $(CD)^2 = AD \times DB$, (*b*) $(CD)^2 = AB \times AD$ or (*c*) $(CD)^2 = AC \times CB$ 11...............

12 Similar polygons are defined as polygons which have (*a*) their corresponding angles equal, (*b*) their corresponding sides proportional or (*c*) their corresponding angles equal and their corresponding sides proportional. 12...............

13 If in triangle ABC, BD is the altitude upon AC, then BD equals (*a*) $AB \times \sin A$, (*b*) $AB \times \tan A$ or (*c*) $AB \times \cos A$. 13...............

14 The center of the circle circumscribed about a triangle is always the intersection of (*a*) the bisectors of two angles, (*b*) two altitudes or (*c*) the perpendicular bisectors of two sides. 14...............

15 An inscribed angle of 80° intercepts an arc of (*a*) 80°, (*b*) 40° or (*c*) 160°. 15...............

16 The locus of the centers of all circles which pass through two given points is (*a*) a circle, (*b*) a straight line or (*c*) a point. 16...............

17 Converses of propositions are (*a*) always true, (*b*) sometimes true or (*c*) never true. 17...............

Directions (questions 18–22) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

18 The formula for the altitude h of an equilateral triangle in terms of its side s is $h =$ .... 18...............

19 The formula for the diagonal d of a square in terms of its side s is $d =$ .... 19...............

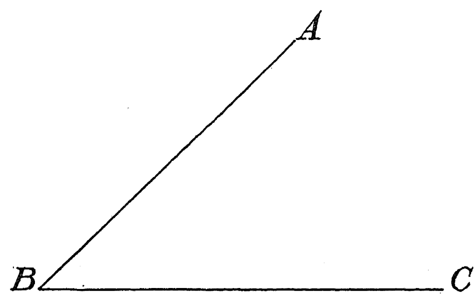
20 The formula for the circumference c of a circle in terms of its radius r is $c =$ .... 20...............

21 The hypotenuse of a right triangle is 17 and one leg is 15; the length of the other leg is .... 21...............

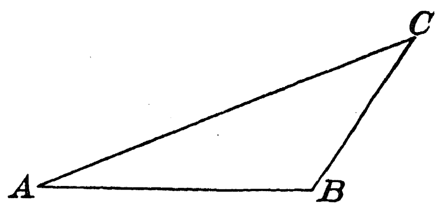
22 If the base of a triangle is 24, the length of the line segment joining the midpoints of the other two sides is .... 22...............

Directions (questions 23–25) — Leave all construction lines on your paper.

23 Find by construction the locus of points within angle ABC and equidistant from the sides of the angle.



24 Construct the altitude of triangle ABC upon side AB.



25 Construct the fourth proportional to the three line segments a, b and c.