

PLANE GEOMETRY

Tuesday, January 28, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

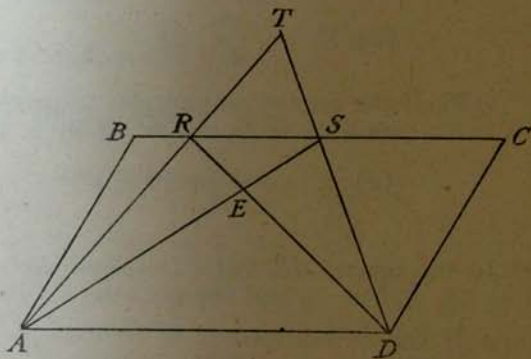
26 Prove that the diameter perpendicular to a chord of a circle bisects the chord. [10]

27 AB is the base of isosceles triangle ABC , D is any point on AC , and DB is drawn. Prove that DB is greater than DA . [10]

28 $ABCD$ is a parallelogram. Lines AT and DT intersect BC at points R and S respectively. Lines AS and DR intersect at E .

a Prove that triangle ARD is equal in area to triangle ASD . [6]

b There are other triangles in this figure which are equal in area. Name one such pair and prove them equal. [4]



29 Prove that if from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between the secant and its external segment. [10]

Part III

Answer one question from part III.

30 A ladder leaning against a wall is said to be in its safest position when the angle formed by the ladder and the level ground is approximately 75° . A ladder 32 feet long is in this position. Find, correct to the nearest foot,

a The distance from the top of the ladder to the ground [6]

b The distance from the foot of the ladder to the wall [4]

31 The side of an equilateral triangle is 6 inches. A line is drawn parallel to one side of the triangle and forming a trapezoid one of whose nonparallel sides is 4 inches.

- a Find the altitude of the trapezoid. [4]
 b Find, correct to the nearest square inch, the area of the trapezoid. [6]

Part IV

Answer one question from part IV.

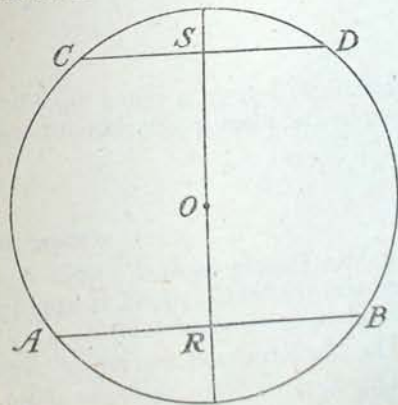
32 Each of the five parts of this question is a statement that may be correctly completed by one or more of the given choices. Write the numbers (1) to (5) on your answer paper and after each indicate the correct answer or answers to the corresponding question by writing one or more of the letters *a, b, c, d*. [10]

[In each of the five parts of the question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

- (1) In any regular polygon (a) the exterior angles are equal (b) a circle may be inscribed (c) the diagonals are equal (d) the area is equal to the product of its perimeter and its apothem
- (2) In any rhombus (a) the diagonals are equal (b) the diagonals are perpendicular to each other (c) the diagonals bisect the angles through which they pass (d) the area is equal to one half the product of one-side and the altitude upon that side
- (3) If the area of a square is represented by K , a diagonal by d and a side by s , then (a) $K = d^2$ (b) $d = s\sqrt{2}$ (c) $s = \sqrt{K}$ (d) $K = 4s$
- (4) If the area of an equilateral triangle is represented by K , a side by s and its altitude by h , then (a) $h = s \sin 60^\circ$ (b) $K = \frac{s^2 \sin 60^\circ}{2}$ (c) $s = \frac{K}{2h}$
 (d) $K = \frac{s^2}{4} \sqrt{3}$
- (5) If the area of an isosceles trapezoid is represented by K , its diagonals by d and d' , its altitude by h and a leg by s , then (a) $d = d'$ (b) $h = s \sin 45^\circ$
 (c) $K = \frac{h}{2} (d + d')$ (d) $K = \frac{s}{2} (d + d')$

33 In the figure, AB and CD are parallel chords on opposite sides of the center of circle O . $CD = 12$, $AB = 16$ and the distance RS between the chords is 14.

- a If OR is represented by x , express OS in terms of x . [1]
 b What is the length of RB ? of SD ? [1]
 c In terms of x , write two expressions for the square of the radius of the circle. [3]
 [Suggestion: Draw OB and OD .]
 d Using the results obtained in answer to c, find the length of OR . [3]
 e Find the area of the circle.
 [Answer may be left in terms of π .] [2]



Fill in the following lines:

Name of school..... Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 If the line segment joining the mid-points of two sides of a triangle is 6 inches long, find in inches the length of the third side of the triangle. 1.....
- 2 A line parallel to the base AB of triangle ABC intersects AC at D and BC at E . If $CD = 6$, $AD = 4$ and $BC = 15$, find EB . 2.....
- 3 Two tangents, PA and PB , are drawn from point P to a circle. The major arc intercepted by the tangents is three times the minor arc. Find the number of degrees in angle APB . 3.....
- 4 Inscribed angle ABC of circle O is 75° . Radii AO and CO are drawn. Find the number of degrees in angle AOC . 4.....
- 5 One acute angle of a right triangle is 60° and the hypotenuse is 10 inches. Find, in inches, the length of the shorter leg. 5.....
- 6 Two parallel lines are cut by a transversal. The two interior angles on the same side of the transversal are represented by x and $5x$. Find the number of degrees in the smaller angle. 6.....
- 7 In quadrilateral $ABCD$, $AB = DC$ and $AD = BC$. Diagonal BD is drawn and angle $ADB = 72^\circ$. Find the number of degrees in angle DBC . 7.....
- 8 Find the sum of the interior angles of a polygon of nine sides. 8.....
- 9 The bases of two parallelograms are 8 and 10 respectively and their areas are equal. The altitude of the first parallelogram is 5. Find the altitude of the second parallelogram. 9.....
- 10 Find in square feet the area of a rhombus whose diagonals are 8 feet and 10 feet. 10.....
- 11 The diagonals of a rhombus are 16 and 30. Find a side of the rhombus. 11.....
- 12 The segments made by the altitude on the hypotenuse of right triangle ABC are 4 and 5. Find the shorter leg of triangle ABC . 12.....
- 13 John defined a parallelogram as a polygon whose opposite sides are parallel. Henry objected to this definition, saying that the definition should also include the fact that the opposite sides are equal. James objected to both definitions and said that neither was correct. Who was right? 13.....
- 14 Is proposition B a converse of proposition A ? [Answer yes or no.]
 A The diagonals of a rectangle are equal.
 B If a parallelogram is a rectangle, its diagonals are equal. 14.....
- Directions (questions 15-17) — Indicate whether each of the following statements is true or false by writing the word *true* or *false* on the line at the right.
- 15 The locus of points equidistant from two intersecting lines consists of two lines which bisect the angles formed by the given lines. 15.....
- 16 If the radius of a circle is 2 inches, the number of square inches in the area of the circle is the same as the number of inches in its circumference. 16.....
- 17 The areas of two similar polygons are to each other as any two corresponding sides. 17.....

Directions (questions 18-21) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement is true. Select the word that will correctly complete each statement and write that word on the line at the right.

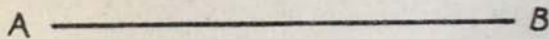
- 18 If AB is the longest side of triangle ABC , then angles A and B are . . . acute. 18.....
- 19 If in triangle ABC median AD is not perpendicular to BC , then AC is . . . equal to AB . 19.....
- 20 Two triangles are . . . congruent if they have equal bases and equal altitudes. 20.....
- 21 The point of intersection of the altitudes of a right triangle is . . . outside the triangle. 21.....

Directions (questions 22-23) — Indicate the correct answer to *each* question by writing on the line at the right the letter a , b , c or d .

- 22 If a point is equidistant from the vertices of a triangle, it must be the intersection of (a) the perpendicular bisectors of the sides (b) the angle bisectors (c) the medians (d) the altitudes 22.....
- 23 Given circle O and line l of unlimited length. If it can be proved that line l is not tangent to circle O and also that l is not a secant, it follows that l has no point in common with circle O .
The above statement is (a) true and is an illustration of direct reasoning (b) false and is an illustration of direct reasoning (c) true and is an illustration of indirect reasoning (d) false and is an illustration of indirect reasoning 23.....

Directions (questions 24-25) — Leave all construction lines on the paper.

24 Line segment AB is the hypotenuse of a right triangle. Determine by construction the length of the median on the hypotenuse.



25 Angle ABC is the vertex angle of an isosceles triangle. Find by construction one of the base angles of the triangle.

