The University of the State of New York

270TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, August 24, 1937 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in plane geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1937, (*d*) author of textbook used.

The minimum time requirement is five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1937 is required.

[1]

See instructions for groups II and III on page 1.
Group II
#### Answer three questions from this group.

21 Prove that if one angle of a triangle is greater than a second angle, the side opposite the first angle is greater than the side opposite the second angle. [10]

22 Prove that the area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

23 $ABCD$ is a parallelogram with side BC extended through C to any point E. AE is drawn, intersecting DC in F.

Prove $CF:FD = CE:BC$ [10]

24 In the figure at the right, the leg AB of right triangle ABC is the diameter of circle O. At D, the point where the circle cuts the hypotenuse, a tangent FE is drawn, meeting the leg AC at E. AD is drawn. Then angle EDC is equal to angle C.
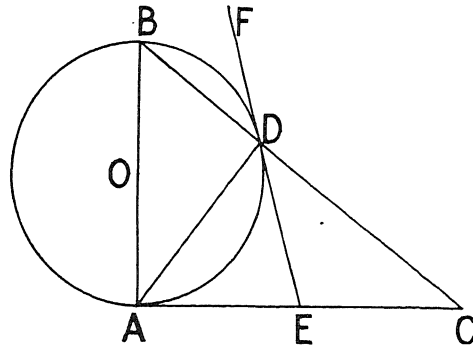
Below is a possible proof for this exercise. Give a reason for *each* of the following statements:

1	$\angle BDF$ is measured by $\frac{1}{2}$ arc BD	[1]
2	$\angle BAD$ is measured by $\frac{1}{2}$ arc BD	[1]
3	$\angle BDF = \angle BAD$	[1]
4	$\angle BDF = \angle EDC$	[1]
5	$\angle BAD = \angle EDC$	[1]
6	$\angle BDA$ is a right angle	[1]
7	$\angle BAC$ is a right angle	[1]
8	$\angle BAD = \angle C$	[2]
9	$\angle EDC = \angle C$	[1]

25 In triangle ABC, the medians BE and CD intersect in O. Prove that triangles BDO and CEO are equal in area. [10] [Suggestion: Draw DE.]

Group III
Answer two questions from this group.

26 P is a point outside a circle whose center is O and the line OP is 29 inches long. From P a tangent is drawn, touching the circle at B and forming an angle of 43° with OP.
  a Find the radius of the circle correct to the *nearest inch*. [5]
  b Find the length of the tangent. [5]

27 In a circle whose center in O and whose radius is 6 inches, a chord AB is drawn. The radii OA and OB form an angle of 60° with each other.
  a Find the area of triangle AOB. [5]
  b Find, correct to the *nearest tenth of a square inch,* the area of the minor segment cut off by the chord AB. [5]
    [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

28 $ABCD$ is a trapezoid with bases AB and DC. Diagonals AC and BD intersect in O. AB equals 20, DC equals 4 and the area of the trapezoid is 72.
  a Find the altitude of the trapezoid. [4]
  b Find the length of the perpendicular drawn from O to AB. [6]

[2]

Fill in the following lines:

Name of school..Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–8) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true. No work need be shown.

1 If the altitude of an equilateral triangle is $9\sqrt{3}$, the length of one side is .... *Ans*..............

2 Point P is 6 inches from the center of a circle whose radius is 10 inches. The numerical value of the product of the segments of any chord drawn through P is.... *Ans*..............

3 The altitude on the hypotenuse of a right triangle is 4. The segments of the hypotenuse made by this altitude are 2 and .... *Ans*..............

4 Two parallel lines are cut by a transversal. If one of the two interior angles on the same side of the transversal is three times the other, the number of degrees in the smaller angle is .... *Ans*..............

5 Corresponding bases of two similar triangles are 2 and 3. If the area of the first triangle is 12, the area of the second is .... *Ans*..............

6 Angle A of parallelogram $ABCD$ is 45° and the altitude on the base AB is represented by h; then the side AD expressed in terms of h is .... *Ans*..............

7 The radius of a certain circle is 12. A central angle of 90° intercepts an arc whose length in terms of π is .... *Ans*..............

8 The area of a rhombus whose diagonals are 6 and 12 is .... *Ans*..............

Directions (questions 9–13) — Indicate the correct answer to each of the following questions by writing on the dotted line at the right the letter (a), (b) or (c):

9 The locus of points at a given distance from a given straight line is (a) a circle, (b) a parallel line or (c) two parallel lines. *Ans*..............

10 The medians of a triangle meet in a point which is (a) $\frac{1}{3}$, (b) $\frac{2}{3}$ or (c) $\frac{1}{2}$ the distance from each vertex to the mid-point of the opposite side. *Ans*..............

11 In proving that the opposite sides of a parallelogram are equal, a diagonal is drawn. The triangles thus formed can be proved congruent by (a) SAS, (b) SSS or (c) ASA. *Ans*..............

12 If the altitude of a triangle is doubled and the base remains constant, the area of the triangle is (a) doubled, (b) halved or (c) multiplied by 4. *Ans*..............

13 A central angle and an inscribed angle intercept the same arc. The central angle is (a) equal to, (b) less than or (c) greater than the inscribed angle. *Ans*..............

[3] [OVER]

Directions (questions 14–17) — Indicate whether each of the following statements is *always true, sometimes true* or *never true* by writing the word *always, sometimes* or *never* on the dotted line at the right.

14 A circle can be circumscribed about any regular polygon.   *Ans*............

15 Two of the exterior angles of a right triangle may be acute angles.   *Ans*............

16 The sum of two angles of a triangle is greater than the third angle.   *Ans*............

17 A line perpendicular to a chord bisects the chord and its arc.   *Ans*............

Directions (questions 18–20) — Leave all construction lines on the paper.

18 Construct the locus of the centers of all circles lying above line *m* and tangent to *m* at point *P*.

$$\text{———————————}\underset{m}{}\underset{P}{}\text{———————}$$

19 Construct the mean proportional between the given line segments *a* and *b*.

$$\text{————}\underset{a}{}\;|\;\underset{b}{}\text{————}$$
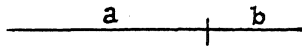
20 On line *DE* corresponding to side *AB*, construct a triangle similar to triangle *ABC*.

[4]