

The University of the State of New York

289TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Thursday, August 19, 1943—8.30 to 11.30 a. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1943 or number and length in minutes of lessons taken in the summer of 1943 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1943 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

26 Prove that the diameter perpendicular to a chord of a circle bisects the chord and the arcs determined by the chord. [10]

27 In quadrilateral  $ABCD$ ,  $E$  is the mid-point of  $AB$  and  $F$  is the mid-point of  $DC$ . Diagonal  $AC$  and the line segment  $EF$  bisect each other.

a Prove that  $AB$  is equal to  $DC$ . [6]

b Prove that  $AB$  is parallel to  $DC$ . [2]

c Prove that  $ABCD$  is a parallelogram. [2]

28 Prove that the area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

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### Part III

Answer two questions from part III.

29 A railroad embankment is 30 feet wide at the top, 54 feet wide at the bottom and  $13\frac{1}{2}$  feet high. If a vertical cross section through the embankment is a trapezoid, find its area in square yards. [10]

30 In an isosceles triangle  $ABC$ , the vertex angle  $C = 120^\circ$  and  $AC = 8$ .

a Find the altitude on  $AB$ . [4]

b Find  $AB$  correct to the nearest integer. [4]

c Find the area of triangle  $ABC$ . [2]

31  $PC$  is a tangent to a circle whose center is  $O$  and  $PAB$  is a secant passing through the center of the circle. Arc  $BC$  equals  $126^\circ$  and  $PO$  equals 18. Find:

a The number of degrees in angle  $CPB$  [4]

b  $OC$ , the radius of the circle, correct to the nearest tenth [Use numerical trigonometry.] [6]

### Part IV

Answer one question from part IV.

32  $AB$ , a diameter of a circle whose center is  $O$ , is extended through  $B$  to point  $C$  so that  $BC$  is equal to the radius of the circle. From  $C$  a line is drawn tangent to the circle at  $E$ .  $AE$  and  $OE$  are drawn. A line perpendicular to  $AB$  at  $B$  intersects  $EC$  at  $F$  and  $AE$  extended at  $D$ .

a Prove that angle  $COE = 60^\circ$  and angle  $OCE = 30^\circ$ . [4]

b Prove that angle  $ADB = 60^\circ$ . [3]

c Prove that triangle  $EDF$  is equilateral. [3]

33 a An inscribed triangle intercepts arcs represented by  $2x - 24^\circ$ ,  $2x + 12^\circ$  and  $3x + 36^\circ$ . Prove that the triangle is a right triangle. [5]

b Two tangents to a circle meet in a point 16 inches from the center of the circle, whose radius is 12 inches. How many inches from the center is the chord joining the two points of contact of the tangents? [5]

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Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 The radius of a circle is 14. Find the area of the circle. [Answer may be left in terms of  $\pi$ .] 1.....
- 2 The perimeter of an equilateral triangle is 12. Find the area of the triangle. [Answer may be left in radical form.] 2.....
- 3 Two angles are complementary and one angle is five times the other. Find the smaller angle. 3.....
- 4 An exterior angle of a regular polygon equals  $40^\circ$ . Find the number of sides of the polygon. 4.....
- 5 From point  $P$  outside circle  $O$ , tangent  $PD$  and secant  $PCA$  are drawn. If tangent  $PD = 8$  and  $PC = 4$ , find secant  $PCA$ . 5.....
- 6 In triangle  $ABC$ , angle  $A = 65^\circ$  and angle  $B = 45^\circ$ . Which is the longest side? 6.....
- 7 In triangle  $ABC$ , angle  $C = 90^\circ$ ,  $BC = 8$  and  $AC = 15$ . Find angle  $A$  correct to the nearest degree. 7.....
- 8 In triangle  $ABC$ , a line parallel to  $AB$  intersects  $AC$  at  $D$  and  $BC$  at  $E$ . If  $CD = 4$ ,  $DA = 2$  and  $BE = 3$ , find  $CE$ . 8.....
- 9 Find the number of miles in the length of the equator if the radius of the earth is taken as 4000 miles: [Use  $\pi = 3.14$ ] 9.....
- 10 The areas of two similar triangles are in the ratio 1:25. Find the ratio of any two corresponding sides of these triangles. 10.....
- 11 Two chords intersect within a circle and form an angle of  $60^\circ$ . If one of the intercepted arcs is  $80^\circ$ , how many degrees are there in the other intercepted arc? 11.....
- 12  $CD$  is the altitude upon the hypotenuse  $AB$  of right triangle  $ABC$ . If  $CD = 6$  and  $AD = 4$ , find  $BD$ . 12.....
- 13 The sides of a triangle are 6, 9 and 12. Find the perimeter of a similar triangle whose shortest side is 4. 13.....

Directions (questions 14–19) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement is true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

- 14 Quadrilateral  $ABCD$  is circumscribed about a circle whose center is  $O$ . The bisector of angle  $A$  ... passes through  $O$ . 14.....
- 15 If two polygons have corresponding sides proportional they are ... similar. 15.....
- 16 The center of the circle circumscribed about a triangle is ... outside the triangle. 16.....
- 17 If all the sides of a triangle are unequal, the median to any side is ... equal to the altitude on that side. 17.....
- 18 Two triangles are inscribed in equal circles and have their angles respectively equal. The triangles are ... congruent. 18.....
- 19 A quadrilateral whose angles are equal and whose adjacent sides are unequal is ... a rectangle. 19.....

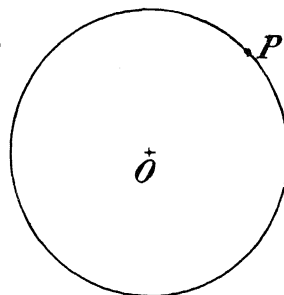
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Directions (questions 20–23) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

- 20 In circle  $O$ , if chord  $AB$  is one half chord  $DE$ , then  $AB$  is nearer the center of the circle. 20.....
- 21 The apothem of a square equals one half the side of the square. 21.....
- 22 In unequal circles equal central angles intercept arcs of the same number of degrees. 22.....
- 23 The locus of points within the square  $ABCD$  and equidistant from the sides  $AB$  and  $BC$  is the diagonal  $BD$ . 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

- 24 Construct a tangent to circle  $O$  at point  $P$ .



- 25 Construct a rhombus, using  $s$  as one side and angle  $A$  as one of the angles.

