

The University of the State of New York

298TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Wednesday, August 21, 1946 — 8.30 to 11.30 a. m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) names of schools where you have studied, (b) number of weeks and recitations a week in plane geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1946 or number and length in minutes of lessons taken in the summer of 1946 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1946 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Prove that an angle formed by a secant and a tangent intersecting outside a circle is measured by one half the difference of the intercepted arcs. [10]

27 In isosceles triangle  $ABC$ , the equal sides  $CA$  and  $CB$  are produced through  $A$  and  $B$  to points  $D$  and  $E$  respectively, so that  $AD = BE$ .  $AE$  and  $BD$  are drawn. Prove that triangles  $ABD$  and  $ABE$  are congruent. [10]

28 In triangle  $ABC$ ,  $CD$  and  $AE$  are medians intersecting at  $O$ .  $DE$  is drawn.

a Why is  $DE$  parallel to  $AC$ ? [3]

b Prove triangles  $ACO$  and  $DEO$  similar. [5]

c Write a proportion involving the sides of these triangles. [2]

29 Prove that the area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

PLANE GEOMETRY

Part III

Answer one question from part III.

30 A trapezoid is inscribed in a circle whose radius is 13 inches. The bases are 12 inches and 5 inches from the center of the circle and on opposite sides of the center.

- a Find the bases of the trapezoid. [6]
- b Find the area of the trapezoid. [4]

31 Quadrilateral  $ABCD$  is inscribed in circle  $O$  and arcs  $AB$ ,  $BC$ ,  $CD$  and  $DA$  are in the ratio 3:4:5:6 respectively.

- a Find the number of degrees in *each* arc. [3]
- b Find the number of degrees in angle  $BAD$ . [1]
- c If side  $AB$  of the quadrilateral equals 10, find the area of the circle. [Suggestion: Draw  $BD$ . Use  $\pi = 3.14$ ] [6]

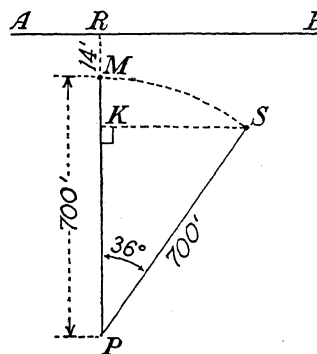
Part IV

Answer one question from part IV.

32 Each of the following statements may be correctly completed by *one or more* of the given choices. Write the numbers (1)–(5) on your answer paper and after *each* indicate the correct answer to the corresponding question by writing one or more of the letters  $a$ ,  $b$ ,  $c$ ,  $d$ . [10] [In each of the five parts of this question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

- (1) Only one right triangle can be constructed if we are given (a) the two legs, (b) the hypotenuse, (c) a leg and an acute angle, (d) the hypotenuse and the median to the hypotenuse.
- (2) Two parallelograms are congruent if they have equal, respectively, (a) two sides and the included angle, (b) two angles and the included side, (c) the four sides, (d) the four angles
- (3) Two isosceles triangles are similar if (a) an angle of one equals the corresponding angle of the other, (b) the ratio of the base to a leg in one is equal to the ratio of the base to a leg in the other, (c) they are inscribed in the same circle, (d) they are formed in two similar rhombuses, one in each rhombus, when two corresponding diagonals are drawn
- (4) At least one point can always be found that is equidistant from two given parallel lines and also (a) equidistant from two given points on one of the lines, (b) equidistant from any two given points, (c) equidistant from two given intersecting lines, (d) at a given distance from a given point
- (5) A polygon is a regular polygon if (a) it is inscribed in a circle and is equiangular, (b) it is circumscribed about a circle and is equiangular, (c) it is inscribed in a circle and is equilateral, (d) it is circumscribed about a circle and is equilateral

33 The figure at the right represents a mine  $M$  attached to a fixed anchor  $P$  by means of a 700-ft cable  $MP$ . When the cable is vertical, the mine is 14 feet below the surface of the water  $AB$ . How far below the surface is it when the current has swung it to the position  $S$ ? [10]



## PLANE GEOMETRY

Fill in the following lines:

Name of pupil.....Name of school.....

### Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.**

- |  |         |
|--|---------|
| 1 In triangle $MNP$ , angle $M = 59^\circ$ and angle $P = 60^\circ$ . Which is the longest side of the triangle?                           | 1.....  |
| 2 Find the diameter of a circle whose circumference is $8\pi$ .  | 2.....  |
| 3 Find the side of an equilateral triangle whose area is $\sqrt{3}$ .  | 3.....  |
| 4 Find the area of a rhombus whose side is 5 and one of whose diagonals is 8.  | 4.....  |
| 5 In triangle $ABC$ , angle $A = 30^\circ$ , angle $B = 60^\circ$ and $AC = 3\sqrt{3}$ . Find $BC$ .                                       | 5.....  |
| 6 Angle $C$ in triangle $ABC$ is a right angle and $CD$ is the altitude upon $AB$ . If $AD = 12$ and $DB = 4$ , find $BC$ .                | 6.....  |
| 7 How many sides has a polygon if the sum of its interior angles equals the sum of its exterior angles?                                    | 7.....  |
| 8 What is the locus of points outside circle $O$ and at a distance $d$ from it?  | 8.....  |
| 9 From an external point $P$ , tangent $PD$ and secant $PCA$ are drawn to a circle. If $PD = 6$ and $PC = 4$ , find $PCA$ .                | 9.....  |
| 10 Chords $AB$ and $CD$ intersect within a circle at point $E$ . If $CE = 3$ , $ED = 8$ and $AE = 6$ , find $EB$ .                         | 10..... |
| 11 Find the side of a square inscribed in a circle whose diameter is 6. [Answer may be left in radical form.]                              | 11..... |
| 12 In triangle $ABC$ the medians $AD$ , $BE$ and $CF$ intersect in point $P$ . If $AP = 8$ , find median $AD$ .                            | 12..... |
| 13 Two similar triangles have areas of 16 and 36. A side of the smaller triangle is 8. Find the corresponding side of the larger triangle. | 13..... |
| 14 A sector of a circle whose angle is $120^\circ$ has an area of $12\pi$ . Find the radius of the circle.                                 | 14..... |
| 15 The legs of a right triangle are 3 and 4. Find the smallest angle of this triangle, correct to the <i>nearest degree</i> .              | 15..... |

Directions (questions 16–18) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

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|---|---------|
| 16 If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are ... equal. | 16..... |
| 17 A circle can ... be circumscribed about any regular polygon.   | 17..... |
| 18 If the dimensions of a rectangle are doubled, the area is ... doubled.   | 18..... |

PLANE GEOMETRY

Directions (questions 19–23) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

19 Which of the following can *not* be constructed with a ruler and compasses?  
 (a) an angle of  $5^\circ$ , (b) an angle of  $15^\circ$ , (c) an angle of  $75^\circ$  19.....

20 The most direct method of proving the proposition: "All points on the perpendicular bisector of a line segment are equidistant from the end points of the segment," uses (a) *sss*, (b) *sas*, (c) *asa* 20.....

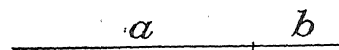
21 If the diagonals of a quadrilateral bisect each other, the quadrilateral is always  
 (a) a rhombus, (b) a rectangle, (c) a parallelogram 21.....

22 The mid-point of the hypotenuse of a right triangle is the point of intersection of (a) the three medians, (b) the bisectors of the three angles, (c) the perpendicular bisectors of the three sides 22.....

23 If a vehicle is an automobile, it has four wheels. John's vehicle has four wheels. Hence his vehicle is an automobile. This reasoning (a) is unsound and is an example of indirect reasoning, (b) is sound and is an example of reasoning from a converse, (c) is unsound and is an example of reasoning from a converse 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct the mean proportional between line segments *a* and *b*.



25 Find by construction the center of the circle that can be inscribed in triangle *ABC*.

