

PLANE GEOMETRY

Tuesday, September 15, 1925 — 9.15 a. m. to 12.15 p. m., only

*Answer eight questions. Irrational results may be left in the form of  $\pi$  and radicals unless otherwise stated. Papers entitled to less than 75 credits will not be accepted.*

- 1 Prove that two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.
- 2 Prove that the tangents drawn to a circle from an external point are equal.
- 3 Prove that an equilateral polygon inscribed in a circle is a regular polygon.
- 4 Construct a circle touching a given line at a given point and passing through another given point not on the line.
- 5 Two tangents are drawn from point  $P$  to a circle whose center is  $O$ .  $PO$  produced meets the circle at  $M$  and the tangent at  $M$  meets the other two tangents extended at  $R$  and  $S$ . Prove  $PR = PS$ .
- 6 The base of a triangle is 15 and its altitude is 12; find the length of the side of a rhombus having the same area, if one of the diagonals of the rhombus is 10.
- 7 Construct an isosceles triangle, having given the altitude on the base and the vertex angle.
- 8 Prove that a chord is parallel to the tangent drawn through the mid-point of either arc of the chord.
- 9  $RS$ , the diameter of a circle, is extended to  $D$  and a perpendicular at  $D$  meets chord  $NS$  produced at  $M$ . Prove that  $RS:SM = NS:SD$ .
- 10 In a circle whose radius is 24 inches find the area of a segment whose arc is  $60^\circ$ .

Group III

*Irrational results may be left in the form of  $\pi$  and radicals unless otherwise stated.*

- 7 State and prove the converse of the following theorem: If the bisector of the exterior angle at  $C$  of triangle  $ABC$  is parallel to  $AB$ , then  $AC$  equals  $BC$ . [ $3, 9\frac{1}{2}$ ]
- 8 Two secants are drawn to a circle from a given external point; if the intercepted chords are equal, prove that the two secants are equal. [ $12\frac{1}{2}$ ]
- 9 On the longer diagonal  $AC$  of parallelogram  $ABCD$ , distance  $AP$  is taken equal to side  $AB$ ; show that  $BC$  is greater than  $PC$ . [ $12\frac{1}{2}$ ]
- 10 Two similar triangles have a combined area of 78 square inches. If two corresponding sides are 6 inches and 9 inches, find (a) the two areas, (b) the two altitudes corresponding to the given sides. [ $8, 4\frac{1}{2}$ ]
- 11 Two tangents to a circle from the same point include an angle of  $60^\circ$ . If the radius is 8 inches find (a) the length of one of the tangents, (b) the area bounded by the two radii to the points of contact and the minor arc determined by the tangents. [ $6, 6\frac{1}{2}$ ]  
Or  
In rectangle  $ABCD$  diagonal  $AC$  is drawn;  $BE$  is perpendicular to  $AC$  at  $E$ .  $BE$  is 12 and  $EC$  exceeds  $AE$  by 7. Find the area of the rectangle. [ $12\frac{1}{2}$ ]
- 12 A boat sails parallel to a straight coast at a distance of 5 miles from the shore and travels at the rate of 20 miles an hour. A coast defense gun situated on the shore has a range of 13 miles; for how long a time is the boat within the range of the gun? [ $12\frac{1}{2}$ ]